11.5 Areas of Circles and Sectors

What *you should learn*

GOAL 1 Find the area of a circle and a sector of a circle.

GOAL 2 Use areas of circles and sectors to solve **real-life** problems, such as finding the area of a boomerang in **Example 6**.

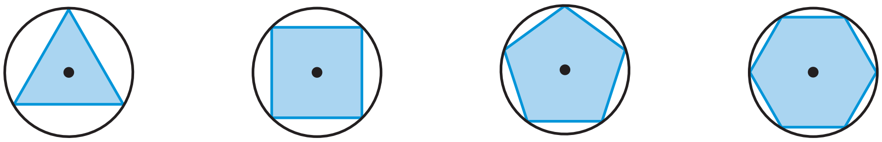
Why *you should learn it*

▼ To solve **real-life** problems, such as finding the area of portions of tree trunks that are used to build Viking ships in **Exs. 38 and 39**.

REAL LIFE

GOAL 1 AREAS OF CIRCLES AND SECTORS

The diagrams below show regular polygons inscribed in circles with radius r. Exercise 42 on page 697 demonstrates that as the number of sides increases, the area of the polygon approaches the value πr^2.

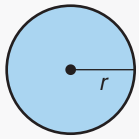


3-gon 4-gon 5-gon 6-gon

THEOREM

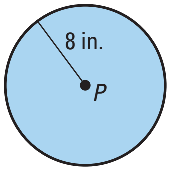
THEOREM 11.7 ***Area of a Circle***

The area of a circle is π times the square of the radius, or $A = \pi r^2$.

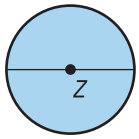


EXAMPLE 1 ***Using the Area of a Circle***

a. Find the area of $\odot P$.



b. Find the diameter of $\odot Z$.



Area of $\odot Z$ = 96 cm^2

SOLUTION

a. Use $r = 8$ in the area formula.

$$A = \pi r^2$$
$$= \pi \cdot 8^2$$
$$= 64\pi$$
$$\approx 201.06$$

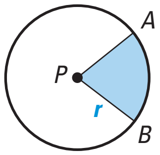
▶ So, the area is 64π, or about 201.06, square inches.

b. The diameter is twice the radius.

$$A = \pi r^2$$
$$96 = \pi r^2$$
$$\frac{96}{\pi} = r^2$$
$$30.56 \approx r^2$$
$$5.53 \approx r \qquad \text{Find the square roots.}$$

▶ The diameter of the circle is about 2(5.53), or about 11.06, centimeters.

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$. The following theorem gives a method for finding the area of a sector.



THEOREM

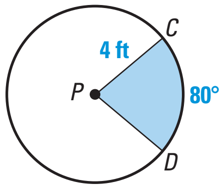
THEOREM 11.8 ***Area of a Sector***

The ratio of the area A of a sector of a circle to the area of the circle is equal to the ratio of the measure of the intercepted arc to 360°.

$$\frac{A}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or } A = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$

EXAMPLE 2 *Finding the Area of a Sector*

Find the area of the sector shown at the right.



SOLUTION

Sector CPD intercepts an arc whose measure is 80°. The radius is 4 feet.

$A = \frac{m\overset{\frown}{CD}}{360^\circ} \cdot \pi r^2$ — Write the formula for the area of a sector.

$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 4^2$ — Substitute known values.

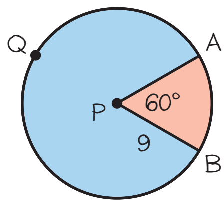
≈ 11.17 — Use a calculator.

▶ So, the area of the sector is about 11.17 square feet.

EXAMPLE 3 *Finding the Area of a Sector*

A and B are two points on a $\odot P$ with radius 9 inches and $m\angle APB = 60^\circ$. Find the areas of the sectors formed by $\angle APB$.

SOLUTION



Draw a diagram of $\odot P$ and $\angle APB$. Shade the sectors.

Label a point Q on the major arc.

Find the measures of the minor and major arcs.

Because $m\angle APB = 60^\circ$, $m\overset{\frown}{AB} = 60^\circ$ and $m\overset{\frown}{AQB} = 360^\circ - 60^\circ = 300^\circ$.

Use the formula for the area of a sector.

Area of small sector $= \frac{60^\circ}{360^\circ} \cdot \pi \cdot 9^2 = \frac{1}{6} \cdot \pi \cdot 81 \approx 42.41$ square inches

Area of larger sector $= \frac{300^\circ}{360^\circ} \cdot \pi \cdot 9^2 = \frac{5}{6} \cdot \pi \cdot 81 \approx 212.06$ square inches

STUDENT HELP

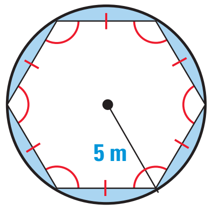
INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING AREAS OF CIRCLES AND REGIONS

You may need to divide a figure into different regions to find its area. The regions may be polygons, circles, or sectors. To find the area of the entire figure, add or subtract the areas of the separate regions as appropriate.

EXAMPLE 4 *Finding the Area of a Region*

Find the area of the shaded region shown at the right.



SOLUTION

The diagram shows a regular hexagon inscribed in a circle with radius 5 meters. The shaded region is the part of the circle that is outside of the hexagon.

$$\boxed{\text{Area of shaded region}} = \boxed{\text{Area of circle}} - \boxed{\text{Area of hexagon}}$$

$$= \pi r^2 - \frac{1}{2}aP$$

$$= \pi \cdot 5^2 - \frac{1}{2} \cdot \left(\frac{5}{2}\sqrt{3}\right) \cdot (6 \cdot 5)$$

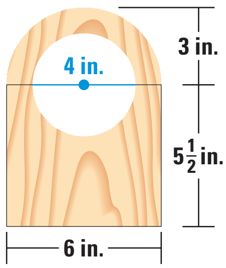
The apothem of a hexagon is $\frac{1}{2} \cdot$ side length $\cdot \sqrt{3}$.

$$= 25\pi - \frac{75}{2}\sqrt{3}$$

▶ So, the area of the shaded region is $25\pi - \frac{75}{2}\sqrt{3}$, or about 13.59 square meters.

EXAMPLE 5 *Finding the Area of a Region*

REAL LIFE **WOODWORKING** You are cutting the front face of a clock out of wood, as shown in the diagram. What is the area of the front of the case?



SOLUTION

The front of the case is formed by a rectangle and a sector, with a circle removed. Note that the intercepted arc of the sector is a semicircle.

$$\boxed{\text{Area}} = \boxed{\text{Area of rectangle}} + \boxed{\text{Area of sector}} - \boxed{\text{Area of circle}}$$

$$= 6 \cdot \frac{11}{2} + \frac{180^\circ}{360^\circ} \cdot \pi \cdot 3^2 - \pi \cdot \left(\frac{1}{2} \cdot 4\right)^2$$

$$= 33 + \frac{1}{2} \cdot \pi \cdot 9 - \pi \cdot (2)^2$$

$$= 33 + \frac{9}{2}\pi - 4\pi$$

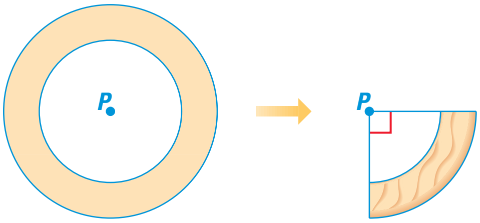
$$\approx 34.57$$

▶ The area of the front of the case is about 34.57 square inches.

FOCUS ON APPLICATIONS

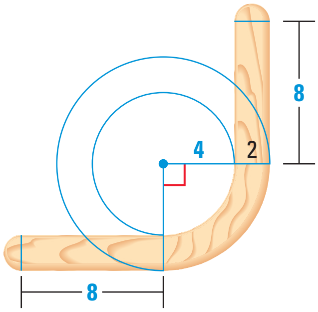
BOOMERANGS are slightly curved at the ends and travel in an arc when thrown. Small boomerangs used for sport make a full circle and return to the thrower.

Complicated shapes may involve a number of regions. In Example 6, the curved region is a portion of a ring whose edges are formed by concentric circles. Notice that the area of a portion of the ring is the difference of the areas of two sectors.



EXAMPLE 6 *Finding the Area of a Boomerang*

BOOMERANGS Find the area of the boomerang shown. The dimensions are given in inches. Give your answer in terms of π and to two decimal places.

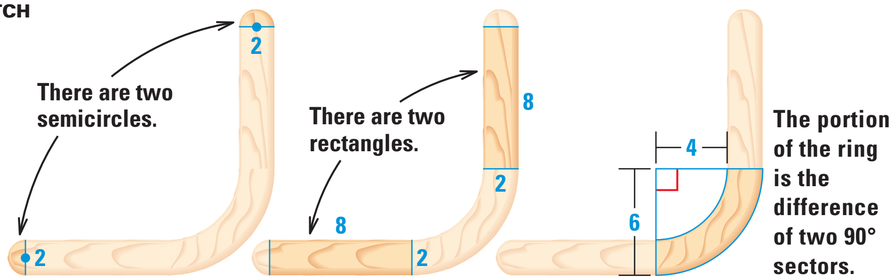


SOLUTION

Separate the boomerang into different regions. The regions are two semicircles (at the ends), two rectangles, and a portion of a ring. Find the area of each region and add these areas together.

PROBLEM SOLVING STRATEGY

DRAW AND LABEL A SKETCH Draw and label a sketch of each region in the boomerang.



VERBAL MODEL

$$\boxed{\text{Area of boomerang}} = 2 \cdot \boxed{\text{Area of semicircle}} + 2 \cdot \boxed{\text{Area of rectangle}} + \boxed{\text{Area of portion of ring}}$$

LABELS

Area of semicircle $= \frac{1}{2} \cdot \pi \cdot 1^2$ (square inches)

Area of rectangle $= 8 \cdot 2$ (square inches)

Area of portion of ring $= \frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2$ (square inches)

REASONING

$$\boxed{\text{Area of boomerang}} = 2\left(\frac{1}{2} \cdot \pi \cdot 1^2\right) + 2(8 \cdot 2) + \left(\frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2\right)$$

$$= 2\left(\frac{1}{2} \cdot \pi \cdot 1\right) + 2 \cdot 16 + \left(\frac{1}{4} \cdot \pi \cdot 36 - \frac{1}{4} \cdot \pi \cdot 16\right)$$

$$= \pi + 32 + (9\pi - 4\pi)$$

$$= 6\pi + 32$$

▶ So, the area of the boomerang is $(6\pi + 32)$, or about 50.85 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the boundaries of a *sector of a circle*.

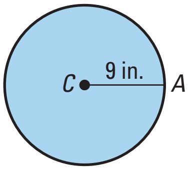
Concept Check ✓

2. In Example 5 on page 693, explain why the expression $\pi \cdot \left(\frac{1}{2} \cdot 4\right)^2$ represents the area of the circle cut from the wood.

Skill Check ✓

In Exercises 3–8, find the area of the shaded region.

3.



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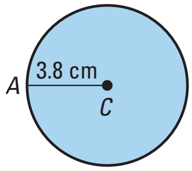

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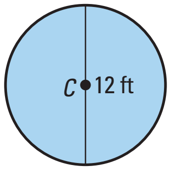

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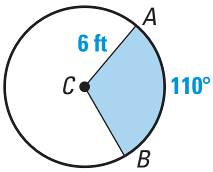

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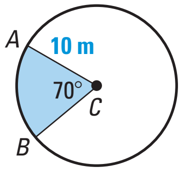

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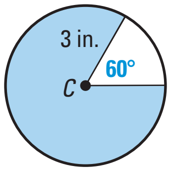
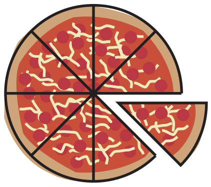


9. **PIECES OF PIZZA** Suppose the pizza shown is divided into 8 equal pieces. The diameter of the pizza is 16 inches. What is the area of one piece of pizza?



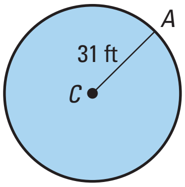
PRACTICE AND APPLICATIONS

STUDENT HELP

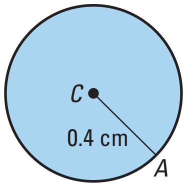
Extra Practice to help you master skills is on p. 824.

FINDING AREA **In Exercises 10–18, find the area of the shaded region.**

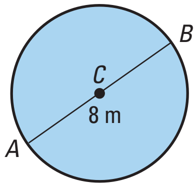
10.



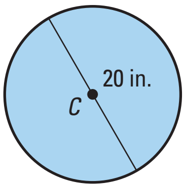
11.



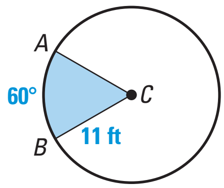
12.



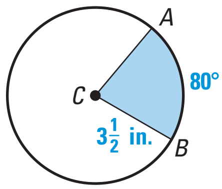
13.



14.



15.



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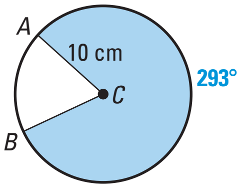
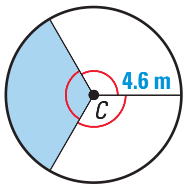
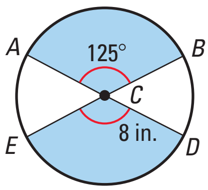


17.



18.



STUDENT HELP

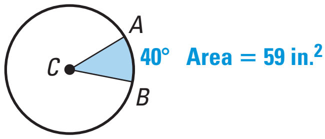
HOMEWORK HELP
Example 1: Exs. 10–13, 19, 20
Example 2: Exs. 14–18, 21, 22, 29
Example 3: Exs. 14–18, 21, 22, 29
Example 4: Exs. 23–28, 35–37
Example 5: Exs. 23–28, 35–37
Example 6: Exs. 38–40

19. **USING AREA** What is the area of a circle with diameter 20 feet?

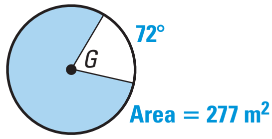
20. **USING AREA** What is the radius of a circle with area 50 square meters?

USING AREA **Find the indicated measure. The area given next to the diagram refers to the shaded region only.**

21. Find the radius of $\odot C$.

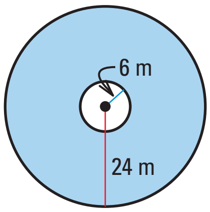


22. Find the diameter of $\odot G$.



FINDING AREA **Find the area of the shaded region.**

23.



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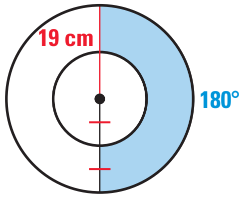

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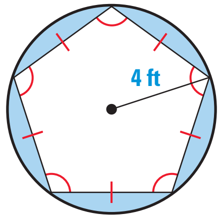

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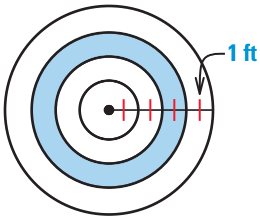
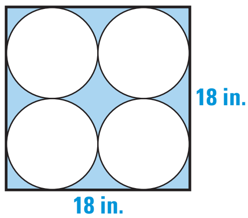
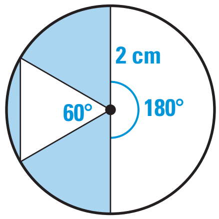


27.



28.



FINDING A PATTERN **In Exercises 29–32, consider an arc of a circle with a radius of 3 inches.**

29. Copy and complete the table. Round your answers to the nearest tenth.

Measure of arc, x	30°	60°	90°	120°	150°	180°
Area of corresponding sector, y	?	?	?	?	?	?

30. **USING ALGEBRA** Graph the data in the table.

31. **USING ALGEBRA** Is the relationship between x and y linear? Explain.

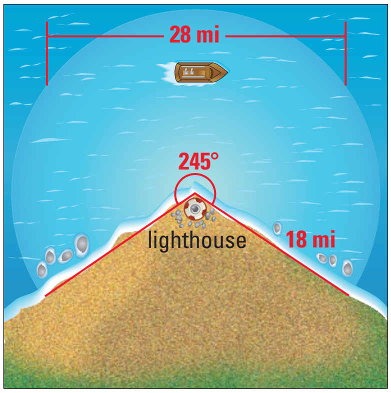
32. **LOGICAL REASONING** If Exercises 29–31 were repeated using a circle with a 5 inch radius, would the areas in the table change? Would your answer to Exercise 31 change? Explain your reasoning.

FOCUS ON APPLICATIONS

REAL LIFE

LIGHTHOUSES use special lenses that increase the intensity of the light projected. Some lenses are 8 feet high and 6 feet in diameter.

LIGHTHOUSES **The diagram shows a projected beam of light from a lighthouse.**

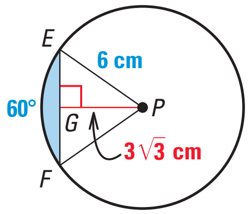


33. What is the area of water that can be covered by the light from the lighthouse?

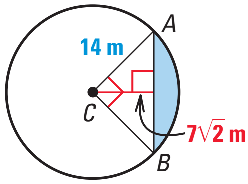
34. Suppose a boat traveling along a straight line is illuminated by the lighthouse for approximately 28 miles of its route. What is the closest distance between the lighthouse and the boat?

USING AREA **In Exercises 35–37, find the area of the shaded region in the circle formed by a chord and its intercepted arc. (*Hint:* Find the difference between the areas of a sector and a triangle.)**

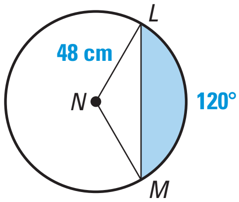
35.



36.



37.

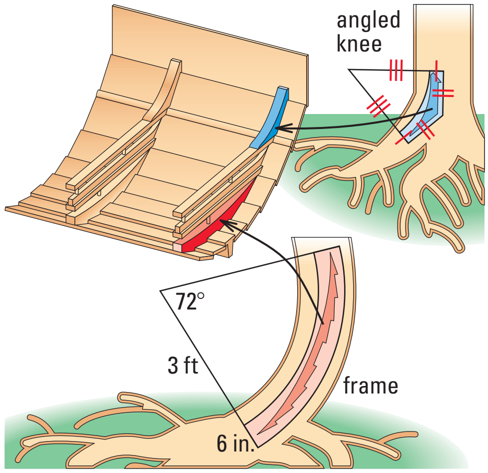


FOCUS ON APPLICATIONS

VIKING LONGSHIPS
The planks in the hull of a longship were cut in a radial pattern from a single green log, providing uniform resiliency and strength.

APPLICATION LINK
www.mcdougallittell.com

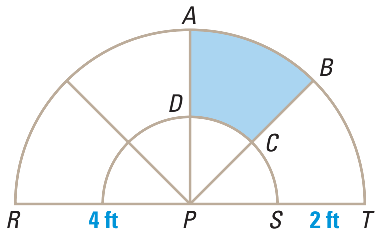
VIKING LONGSHIPS **Use the information below for Exercises 38 and 39.**
When Vikings constructed *longships*, they cut hull-hugging frames from curved trees. Straight trees provided angled knees, which were used to brace the frames.



38. Find the area of a cross-section of the frame piece shown in red.

39. *Writing* The angled knee piece shown in blue has a cross section whose shape results from subtracting a sector from a kite. What measurements would you need to know to find its area?

40. **WINDOW DESIGN** The window shown is in the shape of a semicircle with radius 4 feet. The distance from S to T is 2 feet, and the measure of $\overset{\frown}{AB}$ is 45°. Find the area of the glass in the region $ABCD$.

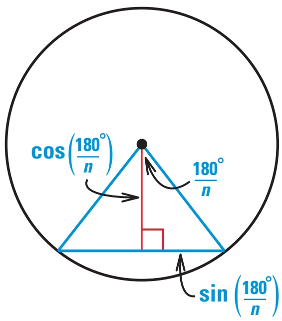


41. **LOGICAL REASONING** Suppose a circle has a radius of 4.5 inches. If you double the radius of the circle, does the area of the circle double as well? What happens to the circle's circumference? Explain.

42. **TECHNOLOGY** The area of a regular n-gon inscribed in a circle with radius 1 unit can be written as

$$A = \frac{1}{2}\left(\cos\left(\frac{180°}{n}\right)\right)\left(2n \cdot \sin\left(\frac{180°}{n}\right)\right).$$

Use a spreadsheet to make a table. The first column is for the number of sides n and the second column is for the area of the n-gon. Fill in your table up to a 16-gon. What do you notice as n gets larger and larger?

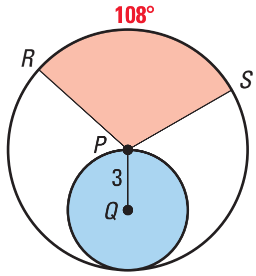


STUDENT HELP

Look Back
to Activity 11.4 on p. 690 for help with spreadsheets.

Test Preparation

⊙Q and ⊙P are tangent. Use the diagram for Exercises 43 and 44.



43. MULTIPLE CHOICE If ⊙Q is cut away, what is the remaining area of ⊙P?

Ⓐ 6π Ⓑ 9π Ⓒ 27π

Ⓓ 60π Ⓔ 180π

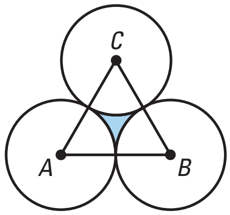
44. MULTIPLE CHOICE What is the area of the region shaded in red?

Ⓐ 0.3 Ⓑ 1.8π Ⓒ 6π

Ⓓ 10.8π Ⓔ 108π

★ Challenge

45. FINDING AREA Find the area between the three congruent tangent circles. The radius of each circle is 6 centimeters. (*Hint:* $\triangle ABC$ is equilateral.)



EXTRA CHALLENGE
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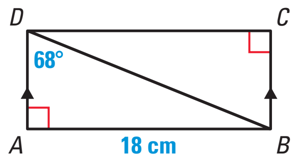
MIXED REVIEW

SIMPLIFYING RATIOS In Exercises 46–49, simplify the ratio. (Review 8.1 for 11.6)

46. $\frac{8 \text{ cats}}{20 \text{ cats}}$ **47.** $\frac{6 \text{ teachers}}{32 \text{ teachers}}$ **48.** $\frac{12 \text{ inches}}{63 \text{ inches}}$ **49.** $\frac{52 \text{ weeks}}{143 \text{ weeks}}$

50. The length of the diagonal of a square is 30. What is the length of each side? (Review 9.4)

FINDING MEASURES Use the diagram to find the indicated measure. Round decimals to the nearest tenth. (Review 9.6)



51. BD **52.** DC

53. $m\angle DBC$ **54.** BC

WRITING EQUATIONS Write the standard equation of the circle with the given center and radius. (Review 10.6)

55. center $(-2, -7)$, radius 6 **56.** center $(0, -9)$, radius 10

57. center $(-4, 5)$, radius 3.2 **58.** center $(8, 2)$, radius $\sqrt{11}$

FINDING MEASURES Find the indicated measure. (Review 11.4)

59. Circumference **60.** Length of $\widehat{AB}$ **61.** Radius