11.4 Circumference and Arc Length

What you should learn

GOAL 1 Find the circumference of a circle and the length of a circular arc.

GOAL 2 Use circumference and arc length to solve **real-life** problems such as finding the distance around a track in **Example 5**.

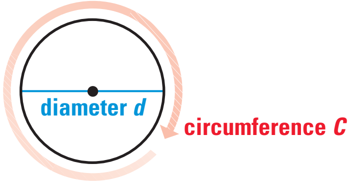
Why you should learn it

▼ To solve **real-life** problems, such as finding the number of revolutions a tire needs to make to travel a given distance in **Example 4** and **Exs. 39–41**.

REAL LIFE

GOAL 1 FINDING CIRCUMFERENCE AND ARC LENGTH

The **circumference** of a circle is the distance around the circle. For all circles, the ratio of the circumference to the diameter is the same. This ratio is known as π, or *pi*.



THEOREM

THEOREM 11.6 *Circumference of a Circle*

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

EXAMPLE 1 *Using Circumference*

Find (**a**) the circumference of a circle with radius 6 centimeters and (**b**) the radius of a circle with circumference 31 meters. Round decimal answers to two decimal places.

SOLUTION

a. $C = 2\pi r$

$= 2 \cdot \pi \cdot 6$

$= 12\pi$ **Use a calculator.**

≈ 37.70

▶ So, the circumference is about 37.70 centimeters.

b. $C = 2\pi r$

$31 = 2\pi r$

$\frac{31}{2\pi} = r$ **Use a calculator.**

$4.93 \approx r$

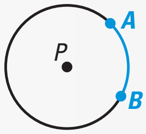
▶ So, the radius is about 4.93 meters.

An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

COROLLARY

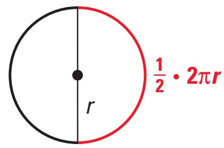
ARC LENGTH COROLLARY

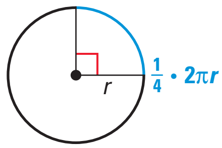
In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

The length of a **semicircle** is one half the circumference, and the length of a **90° arc** is one quarter of the circumference.

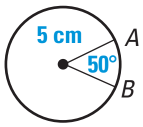




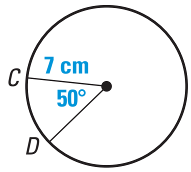
EXAMPLE 2 *Finding Arc Lengths*

Find the length of each arc.

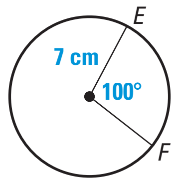
a.



b.



c.



STUDENT HELP

Study Tip
Throughout this chapter, you should use the π key on a calculator, then round decimal answers to two decimal places unless instructed otherwise.

SOLUTION

a. Arc length of $\widehat{AB} = \dfrac{50°}{360°} \cdot 2\pi(5) \approx 4.36$ centimeters

b. Arc length of $\widehat{CD} = \dfrac{50°}{360°} \cdot 2\pi(7) \approx 6.11$ centimeters

c. Arc length of $\widehat{EF} = \dfrac{100°}{360°} \cdot 2\pi(7) \approx 12.22$ centimeters

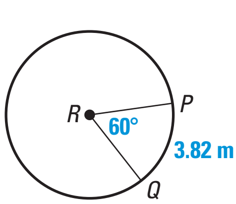
....

In parts (a) and (b) in Example 2, note that the arcs have the same measure, but different lengths because the circumferences of the circles are not equal.

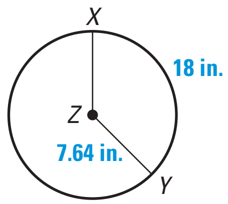
EXAMPLE 3 *Using Arc Lengths*

Find the indicated measure.

a. Circumference



b. $m\widehat{XY}$



SOLUTION

a.
$$\frac{\text{Arc length of } \widehat{PQ}}{2\pi r} = \frac{m\widehat{PQ}}{360°}$$
$$\frac{3.82}{2\pi r} = \frac{60°}{360°}$$
$$\frac{3.82}{2\pi r} = \frac{1}{6}$$
$$3.82(6) = 2\pi r$$
$$22.92 = 2\pi r$$

▶ So, $C = 2\pi r \approx 22.92$ meters.

b.
$$\frac{\text{Arc length of } \widehat{XY}}{2\pi r} = \frac{m\widehat{XY}}{360°}$$
$$\frac{18}{2\pi(7.64)} = \frac{m\widehat{XY}}{360°}$$
$$360° \cdot \frac{18}{2\pi(7.64)} = m\widehat{XY}$$
$$135° \approx m\widehat{XY}$$

▶ So, $m\widehat{XY} \approx 135°$.

STUDENT HELP

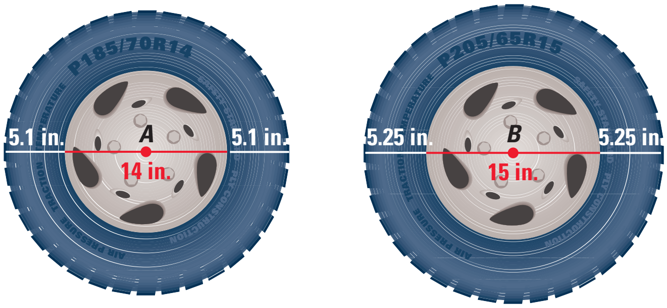
INTERNET

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 CIRCUMFERENCE CIRCUMFERENCES

EXAMPLE 4 *Comparing Circumferences*

REAL LIFE **TIRE REVOLUTIONS** Tires from two different automobiles are shown below. How many revolutions does each tire make while traveling 100 feet? Round decimal answers to one decimal place.



SOLUTION

Tire A has a diameter of $14 + 2(5.1)$, or 24.2 inches. Its circumference is $\pi(24.2)$, or about 76.03 inches.

Tire B has a diameter of $15 + 2(5.25)$, or 25.5 inches. Its circumference is $\pi(25.5)$, or about 80.11 inches.

Divide the distance traveled by the tire circumference to find the number of revolutions made. First convert 100 feet to 1200 inches.

Tire A: $\frac{100 \text{ ft}}{76.03 \text{ in.}} = \frac{1200 \text{ in.}}{76.03 \text{ in.}}$

≈ 15.8 revolutions

Tire B: $\frac{100 \text{ ft}}{80.11 \text{ in.}} = \frac{1200 \text{ in.}}{80.11 \text{ in.}}$

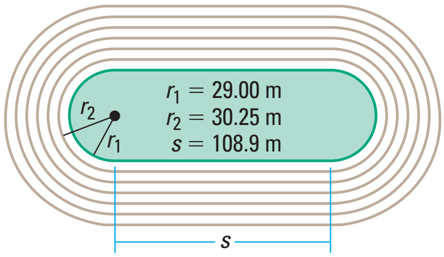
≈ 15.0 revolutions

EXAMPLE 5 *Finding Arc Length*

TRACK The track shown has six lanes. Each lane is 1.25 meters wide. There is a 180° arc at each end of the track. The radii for the arcs in the first two lanes are given.

a. Find the distance around Lane 1.

b. Find the distance around Lane 2.



SOLUTION

The track is made up of two semicircles and two straight sections with length s. To find the total distance around each lane, find the sum of the lengths of each part. Round decimal answers to one decimal place.

a. Distance $= 2s + 2\pi r_1$

$= 2(108.9) + 2\pi(29.00)$

≈ 400.0 meters

b. Distance $= 2s + 2\pi r_2$

$= 2(108.9) + 2\pi(30.25)$

≈ 407.9 meters

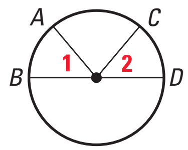
FOCUS ON PEOPLE

REAL LIFE **JACOB HEILVEIL** was born in Korea and now lives in the United States. He was the top American finisher in the 10,000 meter race at the 1996 Paralympics held in Atlanta, Georgia.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the difference between *arc measure* and *arc length*?

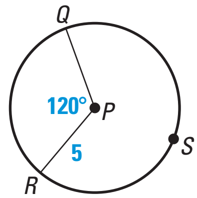


Concept Check ✓

2. In the diagram, $\overline{BD}$ is a diameter and $\angle 1 \cong \angle 2$. Explain why $\overparen{AB}$ and $\overparen{CD}$ have the same length.

Skill Check ✓

In Exercises 3–8, match the measure with its value.



A. $\frac{10}{3}\pi$ **B.** 10π **C.** $\frac{20}{3}\pi$

D. 10 **E.** 5π **F.** $120°$

3. $m\overparen{QR}$ **4.** Diameter of $\odot P$

5. Length of $\overparen{QSR}$ **6.** Circumference of $\odot P$

7. Length of $\overparen{QR}$ **8.** Length of semicircle of $\odot P$

Is the statement *true* or *false*? If it is false, provide a counterexample.

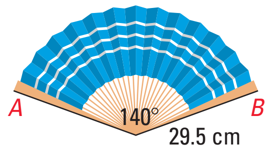
9. Two arcs with the same measure have the same length.

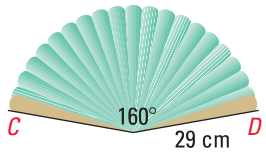
10. If the radius of a circle is doubled, its circumference is multiplied by 4.

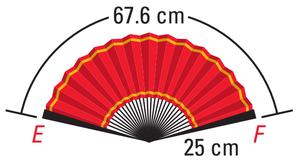
11. Two arcs with the same length have the same measure.

FANS **Find the indicated measure.**

12. Length of $\overparen{AB}$ **13.** Length of $\overparen{CD}$ **14.** $m\overparen{EF}$







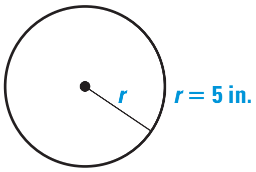
PRACTICE AND APPLICATIONS

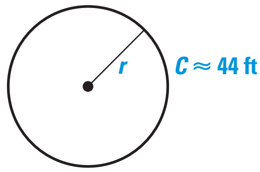
STUDENT HELP

Extra Practice to help you master skills is on p. 824.

USING CIRCUMFERENCE **In Exercises 15 and 16, find the indicated measure.**

15. Circumference **16.** Radius





17. Find the circumference of a circle with diameter 8 meters.

18. Find the circumference of a circle with radius 15 inches. (Leave your answer in terms of π.)

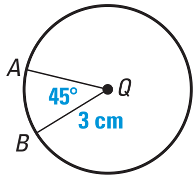
19. Find the radius of a circle with circumference 32 yards.

STUDENT HELP

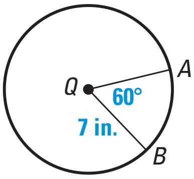
HOMEWORK HELP
Example 1: Exs. 15–19
Example 2: Exs. 20–23
Example 3: Exs. 24–29
Example 4: Exs. 39–41
Example 5: Exs. 42–46

FINDING ARC LENGTHS In Exercises 20–22, find the length of $\widehat{AB}$.

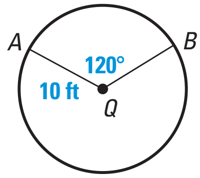
20.



21.



22.

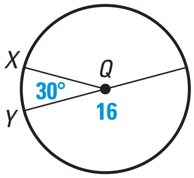


23. FINDING VALUES Complete the table.

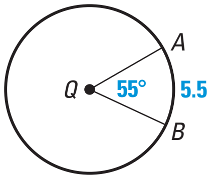
Radius	?	3	0.6	3.5	?	$3\sqrt{3}$
$m\widehat{AB}$	45°	30°	?	192°	90°	?
Length of $\widehat{AB}$	3π	?	0.4π	?	2.55π	3.09π

FINDING MEASURES Find the indicated measure.

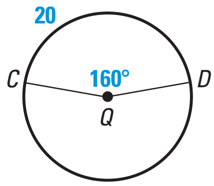
24. Length of $\widehat{XY}$



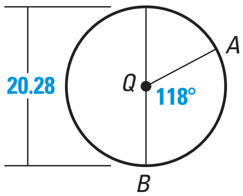
25. Circumference



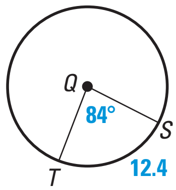
26. Radius



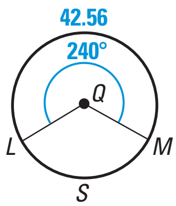
27. Length of $\widehat{AB}$



28. Circumference

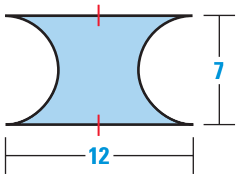


29. Radius

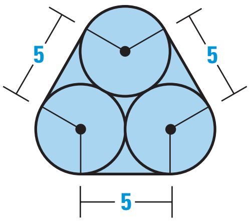


CALCULATING PERIMETERS In Exercises 30–32, the region is bounded by circular arcs and line segments. Find the perimeter of the region.

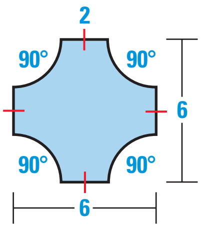
30.



31.

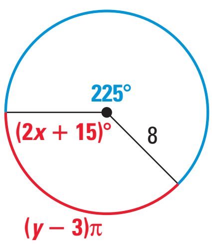


32.

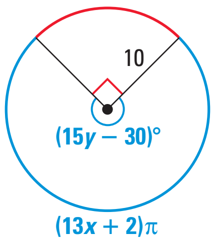


USING ALGEBRA Find the values of *x* and *y*.

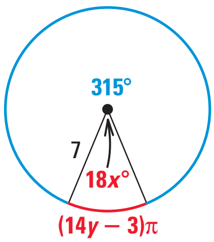
33.



34.



35.



USING ALGEBRA **Find the circumference of the circle whose equation is given. (Leave your answer in terms of π.)**

36. $x^2 + y^2 = 9$

37. $x^2 + y^2 = 28$

38. $(x + 1)^2 + (y - 5)^2 = 4$

AUTOMOBILE TIRES **In Exercises 39–41, use the table below. The table gives the rim diameters and sidewall widths of three automobile tires.**

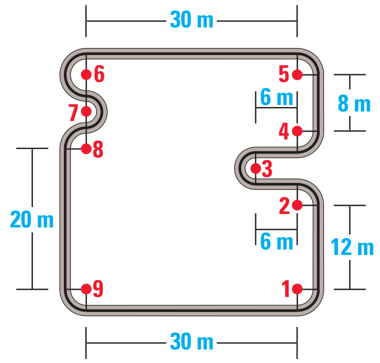
	Rim diameter	Sidewall width
Tire A	15 in.	4.60 in.
Tire B	16 in.	4.43 in.
Tire C	17 in.	4.33 in.

39. Find the diameter of each automobile tire.

40. How many revolutions does each tire make while traveling 500 feet?

41. A student determines that the circumference of a tire with a rim diameter of 15 inches and a sidewall width of 5.5 inches is 64.40 inches. Explain the error.

GO-CART TRACK **Use the diagram of the go-cart track for Exercises 42 and 43. Turns 1, 2, 4, 5, 6, 8, and 9 all have a radius of 3 meters. Turns 3 and 7 each have a radius of 2.25 meters.**



42. Calculate the length of the track.

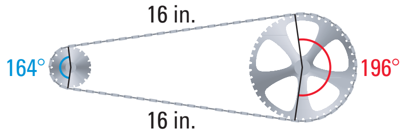
43. How many laps do you need to make to travel 1609 meters (about 1 mile)?

44. **MOUNT RAINIER** In Example 5 on page 623 of Lesson 10.4, you calculated the measure of the arc of Earth's surface seen from the top of Mount Rainier. Use that information to calculate the distance in miles that can be seen looking in one direction from the top of Mount Rainier.

FOCUS ON APPLICATIONS

REAL LIFE **MT. RAINIER**, at 14,410 ft high, is the tallest mountain in Washington State.

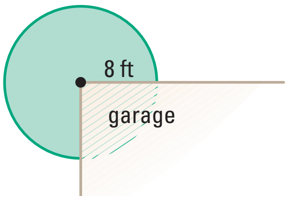
BICYCLES **Use the diagram of a bicycle chain for a fixed gear bicycle in Exercises 45 and 46.**



45. The chain travels along the front and rear sprockets. The circumference of each sprocket is given. About how long is the chain?

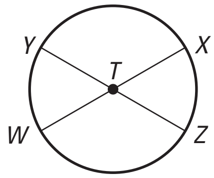
46. On a chain, the teeth are spaced in $\frac{1}{2}$ inch intervals. How many teeth are there on this chain?

47. **ENCLOSING A GARDEN** Suppose you have planted a circular garden adjacent to one of the corners of your garage, as shown at the right. If you want to fence in your garden, how much fencing do you need?



Test Preparation

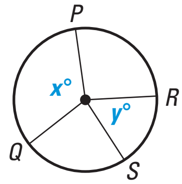
48. MULTIPLE CHOICE In the diagram shown, $\overline{YZ}$ and $\overline{WX}$ each measure 8 units and are diameters of $\odot T$. If $\overparen{YX}$ measures $120°$, what is the length of $\overparen{XZ}$?



Ⓐ $\frac{2}{3}\pi$ Ⓑ $\frac{4}{3}\pi$ Ⓒ $\frac{8}{3}\pi$

Ⓓ 4π Ⓔ 8π

49. MULTIPLE CHOICE In the diagram shown, the ratio of the length of $\overparen{PQ}$ to the length of $\overparen{RS}$ is 2 to 1. What is the ratio of x to y?

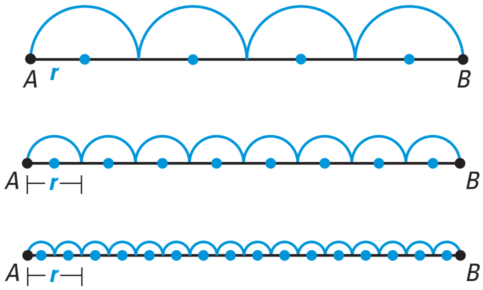


Ⓐ 4 to 1 Ⓑ 2 to 1 Ⓒ 1 to 1

Ⓓ 1 to 2 Ⓔ 1 to 4

★ Challenge

CALCULATING ARC LENGTHS Suppose $\overline{AB}$ is divided into four congruent segments and semicircles with radius r are drawn.



50. What is the sum of the four arc lengths if the radius of each arc is r?

51. Imagine that $\overline{AB}$ is divided into n congruent segments and that semicircles are drawn. What would the sum of the arc lengths be for 8 segments? 16 segments? n segments? Does the number of segments matter?

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

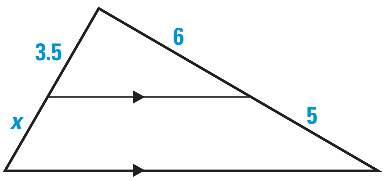
FINDING AREA In Exercises 52–55, the radius of a circle is given. Use the formula $A = \pi r^2$ to calculate the area of the circle. (Review 1.7 for 11.5)

52. $r = 9$ ft **53.** $r = 3.3$ in. **54.** $r = \frac{27}{5}$ cm **55.** $r = 4\sqrt{11}$ m

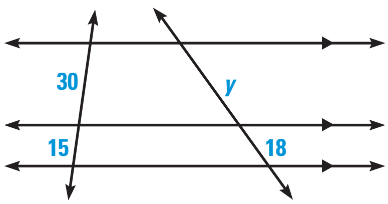
56. **USING ALGEBRA** Line n_1 has the equation $y = \frac{2}{3}x + 8$. Line n_2 is parallel to n_1 and passes through the point $(9, -2)$. Write an equation for n_2. (Review 3.6)

USING PROPORTIONALITY THEOREMS In Exercises 57 and 58, find the value of the variable. (Review 8.6)

57.



58.



CALCULATING ARC MEASURES You are given the measure of an inscribed angle of a circle. Find the measure of its intercepted arc. (Review 10.3)

59. $48°$ **60.** $88°$ **61.** $129°$ **62.** $15.5°$