11.3 Perimeters and Areas of Similar Figures

What *you should learn*

GOAL 1 Compare perimeters and areas of similar figures.

GOAL 2 Use perimeters and areas of similar figures to solve **real-life** problems, as applied in **Example 2**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the area of the walkway around a polygonal pool in **Exs. 25–27**.

REAL LIFE

Frank Lloyd Wright included this triangular pool and walkway in his design of *Taliesin West* in Scottsdale, Arizona.

GOAL 1 COMPARING PERIMETER AND AREA

For any polygon, the *perimeter of the polygon* is the sum of the lengths of its sides and the *area of the polygon* is the number of square units contained in its interior.

In Lesson 8.3, you learned that if two polygons are *similar*, then the ratio of their perimeters is the same as the ratio of the lengths of their corresponding sides. In Activity 11.3 on page 676, you may have discovered that the ratio of the areas of two similar polygons is *not* this same ratio, as shown in Theorem 11.5. Exercise 22 asks you to write a proof of this theorem for rectangles.

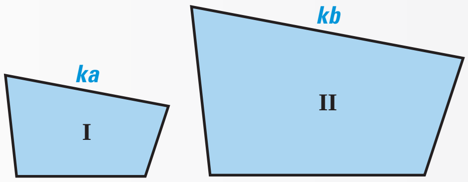
THEOREM

THEOREM 11.5 ***Areas of Similar Polygons***

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Quad. I}}{\text{Side length of Quad. II}} = \frac{a}{b}$$

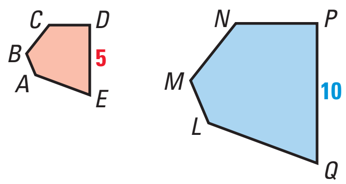
$$\frac{\text{Area of Quad. I}}{\text{Area of Quad. II}} = \frac{a^2}{b^2}$$



Quad. I ~ Quad. II

EXAMPLE 1 *Finding Ratios of Similar Polygons*

Pentagons *ABCDE* and *LMNPQ* are similar.



a. Find the ratio (red to blue) of the perimeters of the pentagons.

b. Find the ratio (red to blue) of the areas of the pentagons.

SOLUTION

The ratio of the lengths of corresponding sides in the pentagons is $\frac{5}{10} = \frac{1}{2}$, or $1:2$.

a. The ratio of the perimeters is also $1:2$. So, the perimeter of pentagon *ABCDE* is half the perimeter of pentagon *LMNPQ*.

b. Using Theorem 11.5, the ratio of the areas is $1^2:2^2$, or $1:4$. So, the area of pentagon *ABCDE* is one fourth the area of pentagon *LMNPQ*.

STUDENT HELP

Study Tip

The ratio "*a* to *b*," for example, can be written using a fraction bar $\left(\frac{a}{b}\right)$ or a colon ($a:b$).

GOAL 2 Using Perimeter and Area in Real Life

EXAMPLE 2 *Using Areas of Similar Figures*

Comparing Costs You are buying photographic paper to print a photo in different sizes. An 8 inch by 10 inch sheet of the paper costs \$.42. What is a reasonable cost for a 16 inch by 20 inch sheet?

Solution

Because the ratio of the lengths of the sides of the two rectangular pieces of paper is 1:2, the ratio of the areas of the pieces of paper is $1^2:2^2$, or 1:4. Because the cost of the paper should be a function of its area, the larger piece of paper should cost about four times as much, or \$1.68.

EXAMPLE 3 *Finding Perimeters and Areas of Similar Polygons*

Focus on Applications

Chicago Board of Trade
Commodities such as grains, coffee, and financial securities are exchanged at this marketplace. Associated traders stand on the descending steps in the same "pie-slice" section of an octagonal pit. The different levels allow buyers and sellers to see each other as orders are yelled out.

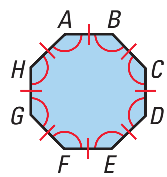
Application Link
www.mcdougallittell.com

Octagonal Floors A trading pit at the Chicago Board of Trade is in the shape of a series of regular octagons. One octagon has a side length of about 14.25 feet and an area of about 980.4 square feet. Find the area of a smaller octagon that has a perimeter of about 76 feet.

Solution

All regular octagons are similar because all corresponding angles are congruent and the corresponding side lengths are proportional.

Draw and label a sketch.



Find the ratio of the side lengths of the two octagons, which is the same as the ratio of their perimeters.

$$\frac{\text{perimeter of } ABCDEFGH}{\text{perimeter of } JKLMNPQR} = \frac{a}{b} \approx \frac{76}{8(14.25)} = \frac{76}{114} = \frac{2}{3}$$

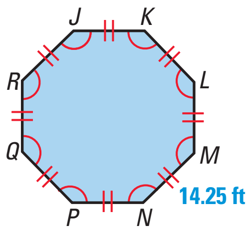
Calculate the area of the smaller octagon. Let A represent the area of the smaller octagon. The ratio of the areas of the smaller octagon to the larger is $a^2:b^2 = 2^2:3^2$, or 4:9.

$\frac{A}{980.4} = \frac{4}{9}$ **Write proportion.**

$9A = 980.4 \cdot 4$ **Cross product property**

$A = \frac{3921.6}{9}$ **Divide each side by 9.**

$A \approx 435.7$ **Use a calculator.**



▶ The area of the smaller octagon is about 435.7 square feet.

GUIDED PRACTICE

Vocabulary Check ✓

1. If two polygons are *similar* with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their perimeters is __?__ and the ratio of their areas is __?__.

Concept Check ✓

Tell whether the statement is *true* or *false*. Explain.

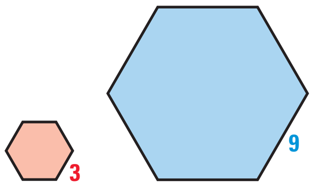
2. Any two regular polygons with the same number of sides are similar.

3. Doubling the side length of a square doubles the area.

Skill Check ✓

In Exercises 4 and 5, the red and blue figures are similar. Find the ratio (red to blue) of their perimeters and of their areas.

4.



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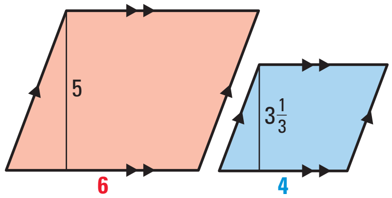


6. **PHOTOGRAPHY** Use the information from Example 2 on page 678 to find a reasonable cost for a sheet of 4 inch by 5 inch photographic paper.

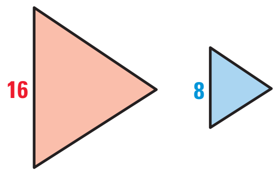
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 823.

FINDING RATIOS In Exercises 7–10, the polygons are similar. Find the ratio (red to blue) of their perimeters and of their areas.

7.



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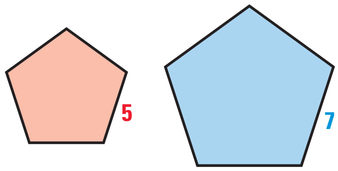

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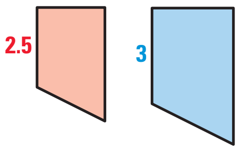

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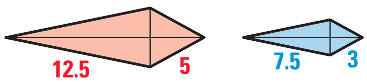


LOGICAL REASONING In Exercises 11–13, complete the statement using *always*, *sometimes*, or *never*.

11. Two similar hexagons __?__ have the same perimeter.

12. Two rectangles with the same area are __?__ similar.

13. Two regular pentagons are __?__ similar.

14. **HEXAGONS** The ratio of the lengths of corresponding sides of two similar hexagons is 2:5. What is the ratio of their areas?

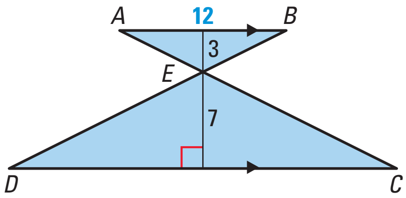
15. **OCTAGONS** A regular octagon has an area of 49 m^2. Find the scale factor of this octagon to a similar octagon that has an area of 100 m^2.

STUDENT HELP

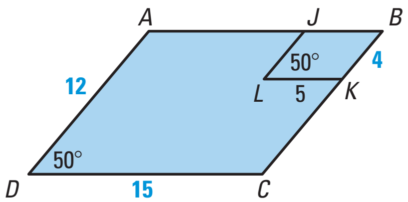
HOMEWORK HELP
Example 1: Exs. 7–10, 14–18
Example 2: Exs. 23, 24
Example 3: Exs. 25–28

16. **RIGHT TRIANGLES** $\triangle ABC$ is a right triangle whose hypotenuse $\overline{AC}$ is 8 inches long. Given that the area of $\triangle ABC$ is 13.9 square inches, find the area of similar triangle $\triangle DEF$ whose hypotenuse $\overline{DF}$ is 20 inches long.

17. **FINDING AREA** Explain why $\triangle CDE$ is similar to $\triangle ABE$. Find the area of $\triangle CDE$.



18. **FINDING AREA** Explain why $\square JBKL \sim \square ABCD$. The area of $\square JBKL$ is 15.3 square inches. Find the area of $\square ABCD$.



STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with scale factors in Exs. 19–21.

19. **SCALE FACTOR** Regular pentagon $ABCDE$ has a side length of $6\sqrt{5}$ centimeters. Regular pentagon $QRSTU$ has a perimeter of 40 centimeters. Find the ratio of the perimeters of $ABCDE$ to $QRSTU$.

20. **SCALE FACTOR** A square has a perimeter of 36 centimeters. A smaller square has a side length of 4 centimeters. What is the ratio of the areas of the larger square to the smaller one?

21. **SCALE FACTOR** A regular nonagon has an area of 90 square feet. A similar nonagon has an area of 25 square feet. What is the ratio of the perimeters of the first nonagon to the second?

22. **PROOF** Prove Theorem 11.5 for rectangles.

RUG COSTS Suppose you want to be sure that a large rug is priced fairly. The price of a small rug (29 inches by 47 inches) is $79 and the price of the large rug (4 feet 10 inches by 7 feet 10 inches) is $299.

23. What are the areas of the two rugs? What is the ratio of the areas?

24. Compare the rug costs. Do you think the large rug is a good buy? Explain.

FOCUS ON APPLICATIONS

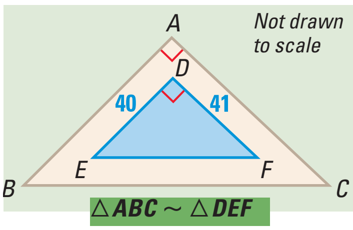
REAL LIFE FORT JEFFERSON is in the Dry Tortugas National Park 70 miles west of Key West, Florida. The fort has been used as a prison, a navy base, a seaplane port, and an observation post.

TRIANGULAR POOL In Exercises 25–27, use the following information. The pool at *Taliesin West* (see page 677) is a right triangle with legs of length 40 feet and 41 feet.

25. Find the area of the triangular pool, $\triangle DEF$.

26. The walkway bordering the pool is 40 inches wide, so the scale factor of the similar triangles is about 1.3 : 1. Find AB.

27. Find the area of $\triangle ABC$. What is the area of the walkway?



28. **FORT JEFFERSON** The outer wall of Fort Jefferson, which was originally constructed in the mid-1800s, is in the shape of a hexagon with an area of about 466,170 square feet. The length of one side is about 477 feet. The inner courtyard is a similar hexagon with an area of about 446,400 square feet. Calculate the length of a corresponding side in the inner courtyard to the nearest foot.

Test Preparation

29. MULTI-STEP PROBLEM Use the following information about similar triangles $\triangle ABC$ and $\triangle DEF$.

The scale factor of $\triangle ABC$ to $\triangle DEF$ is 15 : 2.

The area of $\triangle ABC$ is $25x$. The area of $\triangle DEF$ is $x - 5$.

The perimeter of $\triangle ABC$ is $8 + y$. The perimeter of $\triangle DEF$ is $3y - 19$.

a. Use the scale factor to find the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$.

b. Write and solve a proportion to find the value of x.

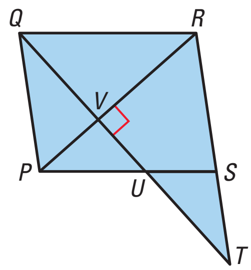
c. Use the scale factor to find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$.

d. Write and solve a proportion to find the value of y.

e. *Writing* Explain how you could find the value of z if $AB = 22.5$ and the length of the corresponding side $\overline{DE}$ is $13z - 10$.

★ Challenge

Use the figure shown at the right. *PQRS* is a parallelogram.



30. Name three pairs of similar triangles and explain how you know that they are similar.

31. The ratio of the area of $\triangle PVQ$ to the area of $\triangle RVT$ is 9 : 25, and the length RV is 10. Find PV.

32. If VT is 15, find VQ, VU, and UT.

33. Find the ratio of the areas of each pair of similar triangles that you found in Exercise 30.

EXTRA CHALLENGE
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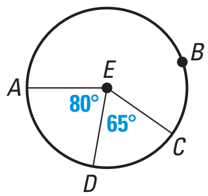
MIXED REVIEW

FINDING MEASURES In Exercises 34–37, use the diagram shown at the right. (Review 10.2 for 11.4)



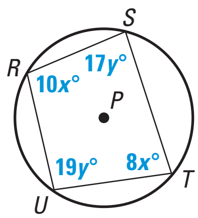
34. Find $m\overset{\frown}{AD}$.

35. Find $m\angle AEC$.

36. Find $m\overset{\frown}{AC}$.

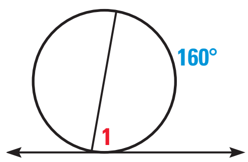
37. Find $m\overset{\frown}{ABC}$.

38. USING AN INSCRIBED QUADRILATERAL In the diagram shown at the right, quadrilateral $RSTU$ is inscribed in circle P. Find the values of x and y, and use them to find the measures of the angles of $RSTU$. (Review 10.3)



FINDING ANGLE MEASURES Find the measure of ∠1. (Review 10.4 for 11.4)

39.



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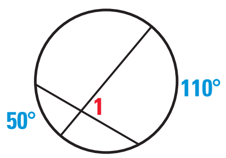

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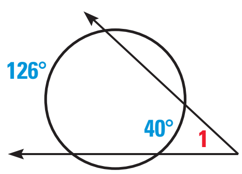


QUIZ 1

Self-Test for Lessons 11.1–11.3

1. Find the sum of the measures of the interior angles of a convex 20-gon. **(Lesson 11.1)**

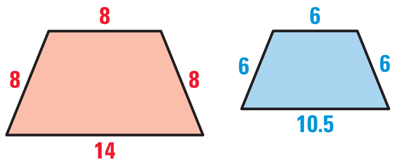
2. What is the measure of each exterior angle of a regular 25-gon? **(Lesson 11.1)**

3. Find the area of an equilateral triangle with a side length of 17 inches. **(Lesson 11.2)**

4. Find the area of a regular nonagon with an apothem of 9 centimeters. **(Lesson 11.2)**

In Exercises 5 and 6, the polygons are similar. Find the ratio (red to blue) of their perimeters and of their areas. (Lesson 11.3)

5.



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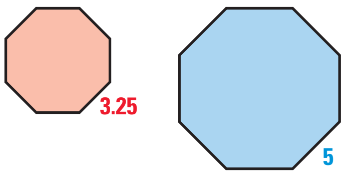

7. **CARPET** You just carpeted a 9 foot by 12 foot room for $480. The carpet is priced by the square foot. About how much would you expect to pay for the same carpet in another room that is 21 feet by 28 feet? **(Lesson 11.3)**

MATH & *History*

History of Approximating Pi

APPLICATION LINK
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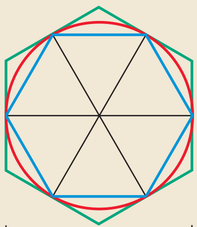
THEN

THOUSANDS OF YEARS AGO, people first noticed that the circumference of a circle is the product of its diameter and a value that is a little more than three. Over time, various methods have been used to find better approximations of this value, called π (*pi*).

1. In the third century B.C., Archimedes approximated the value of π by calculating the perimeters of inscribed and circumscribed regular polygons of a circle with diameter 1 unit. Copy the diagram and follow the steps below to use his method.

- Find the perimeter of the **inscribed** hexagon in terms of the length of the diameter of the circle.
- Draw a radius of the **circumscribed** hexagon. Find the length of one side of the hexagon. Then find its perimeter.
- Write an inequality that approximates the value of π:

$$\frac{\text{perimeter of}}{\text{inscribed hexagon}} < \pi < \frac{\text{perimeter of}}{\text{circumscribed hexagon}}$$



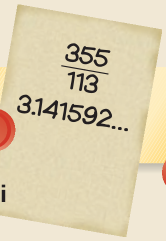
NOW

MATHEMATICIANS use computers to calculate the value of π to billions of decimal places.

200s B.C.

Archimedes uses perimeters of polygons.

A.D. 400s

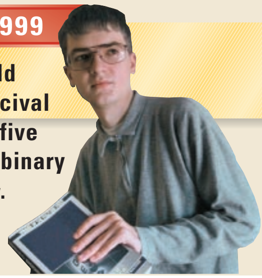


Tsu Chung Chi finds π to six decimal places.

1949

ENIAC computer finds π to 2037 decimal places.

1999



17 year old Colin Percival finds the five trillionth binary digit of π.