

11.1

Angle Measures in Polygons

What you should learn

GOAL 1 Find the measures of interior and exterior angles of polygons.

GOAL 2 Use measures of angles of polygons to solve real-life problems.

Why you should learn it

▼ To solve real-life problems, such as finding the measures of the interior angles of a home plate marker of a softball field in Example 4.



GOAL 1 MEASURES OF INTERIOR AND EXTERIOR ANGLES

You have already learned that the name of a polygon depends on the number of sides in the polygon: triangle, quadrilateral, pentagon, hexagon, and so forth. The sum of the measures of the interior angles of a polygon also depends on the number of sides.

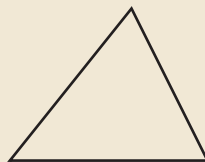
In Lesson 6.1, you found the sum of the measures of the interior angles of a quadrilateral by dividing the quadrilateral into two triangles. You can use this triangle method to find the sum of the measures of the interior angles of any convex polygon with n sides, called an n -gon.

ACTIVITY

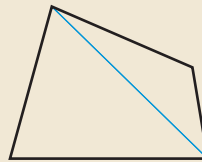
Developing Concepts

Investigating the Sum of Polygon Angle Measures

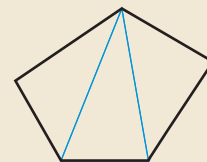
Draw examples of 3-sided, 4-sided, 5-sided, and 6-sided convex polygons. In each polygon, draw all the diagonals from one vertex. Notice that this divides each polygon into triangular regions.



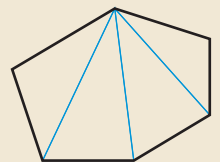
Triangle



Quadrilateral



Pentagon



Hexagon

Complete the table below. What is the pattern in the sum of the measures of the interior angles of any convex n -gon?

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180^\circ = 180^\circ$
Quadrilateral	?	?	$2 \cdot 180^\circ = 360^\circ$
Pentagon	?	?	?
Hexagon	?	?	?
⋮	⋮	⋮	⋮
n -gon	n	?	?

STUDENT HELP**Look Back**

For help with *regular* polygons, see p. 323.

You may have found in the activity that the sum of the measures of the interior angles of a convex n -gon is $(n - 2) \cdot 180^\circ$. This relationship can be used to find the measure of each interior angle in a *regular* n -gon, because the angles are all congruent. Exercises 43 and 44 ask you to write proofs of the following results.

THEOREMS ABOUT INTERIOR ANGLES**THEOREM 11.1 Polygon Interior Angles Theorem**

The sum of the measures of the interior angles of a convex n -gon is $(n - 2) \cdot 180^\circ$.

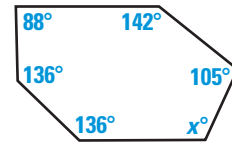
COROLLARY TO THEOREM 11.1

The measure of each interior angle of a regular n -gon is

$$\frac{1}{n} \cdot (n - 2) \cdot 180^\circ, \text{ or } \frac{(n - 2) \cdot 180^\circ}{n}.$$

**EXAMPLE 1 Finding Measures of Interior Angles of Polygons**

Find the value of x in the diagram shown.

**SOLUTION**

The sum of the measures of the interior angles of any hexagon is $(6 - 2) \cdot 180^\circ = 4 \cdot 180^\circ = 720^\circ$.

Add the measures of the interior angles of the hexagon.

$$136^\circ + 136^\circ + 88^\circ + 142^\circ + 105^\circ + x^\circ = 720^\circ \quad \text{The sum is } 720^\circ.$$

$$607 + x = 720 \quad \text{Simplify.}$$

$$x = 113 \quad \text{Subtract } 607 \text{ from each side.}$$

▶ The measure of the sixth interior angle of the hexagon is 113° .

EXAMPLE 2 Finding the Number of Sides of a Polygon

The measure of each interior angle of a regular polygon is 140° . How many sides does the polygon have?

SOLUTION

$$\frac{1}{n} \cdot (n - 2) \cdot 180^\circ = 140^\circ \quad \text{Corollary to Theorem 11.1}$$

$$(n - 2) \cdot 180 = 140n \quad \text{Multiply each side by } n.$$

$$180n - 360 = 140n \quad \text{Distributive property}$$

$$40n = 360 \quad \text{Addition and subtraction properties of equality}$$

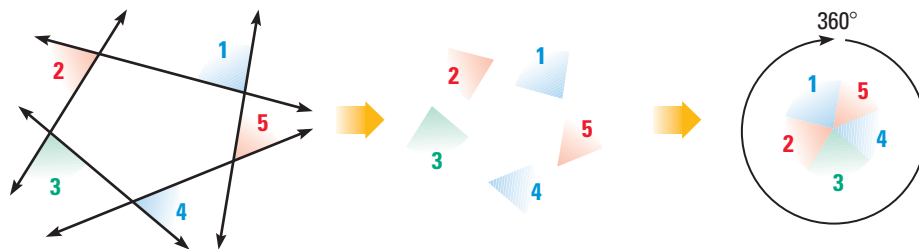
$$n = 9 \quad \text{Divide each side by } 40.$$

▶ The polygon has 9 sides. It is a regular nonagon.

STUDENT HELP**INTERNET HOMEWORK HELP**

Visit our Web site www.mcdougallittell.com for extra examples.

The diagrams below show that the sum of the measures of the *exterior* angles of any convex polygon is 360° . You can also find the measure of each exterior angle of a *regular* polygon. Exercises 45 and 46 ask for proofs of these results.



- 1 Shade one exterior angle at each vertex.
- 2 Cut out the exterior angles.
- 3 Arrange the exterior angles to form 360° .

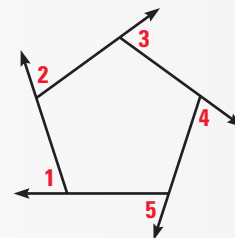
THEOREMS ABOUT EXTERIOR ANGLES

THEOREM 11.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360° .

COROLLARY TO THEOREM 11.2

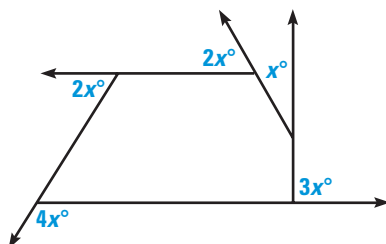
The measure of each exterior angle of a regular n -gon is $\frac{1}{n} \cdot 360^\circ$, or $\frac{360^\circ}{n}$.



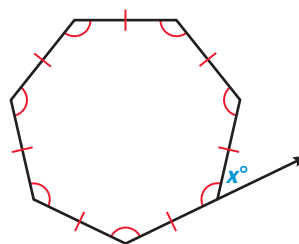
EXAMPLE 3 Finding the Measure of an Exterior Angle

Find the value of x in each diagram.

a.



b.



SOLUTION

$$\text{a. } 2x^\circ + x^\circ + 3x^\circ + 4x^\circ + 2x^\circ = 360^\circ$$

$$12x = 360$$

$$x = 30$$

Use Theorem 11.2.

Combine like terms.

Divide each side by 12.

$$\text{b. } x^\circ = \frac{1}{7} \cdot 360^\circ$$

$$\approx 51.4$$

Use $n = 7$ in the Corollary to Theorem 11.2.

Use a calculator.

- The measure of each exterior angle of a regular heptagon is about 51.4° .

GOAL 2

USING ANGLE MEASURES IN REAL LIFE

You can use Theorems 11.1 and 11.2 and their corollaries to find angle measures.

EXAMPLE 4

Finding Angle Measures of a Polygon

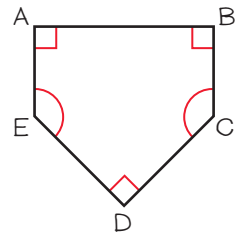
SOFTBALL A home plate marker for a softball field is a pentagon. Three of the interior angles of the pentagon are right angles. The remaining two interior angles are congruent. What is the measure of each angle?

PROBLEM SOLVING STRATEGY

SOLUTION

DRAW A SKETCH

Sketch and label a diagram for the home plate marker. It is a nonregular pentagon. The right angles are $\angle A$, $\angle B$, and $\angle D$. The remaining angles are congruent. So $\angle C \cong \angle E$. The sum of the measures of the interior angles of the pentagon is 540° .



VERBAL MODEL

$$\text{Sum of measures of interior angles} = 3 \cdot \text{Measure of each right angle} + 2 \cdot \text{Measure of } \angle C \text{ and } \angle E$$

LABELS

Sum of measures of interior angles = **540** (degrees)
 Measure of each right angle = **90** (degrees)
 Measure of $\angle C$ and $\angle E$ = **x** (degrees)

REASONING

$$\begin{aligned} 540 &= 3 \cdot 90 + 2x && \text{Write the equation.} \\ 540 &= 270 + 2x && \text{Simplify.} \\ 270 &= 2x && \text{Subtract 270 from each side.} \\ 135 &= x && \text{Divide each side by 2.} \end{aligned}$$

▶ So, the measure of each of the two congruent angles is 135° .

EXAMPLE 5

Using Angle Measures of a Regular Polygon

SPORTS EQUIPMENT If you were designing the home plate marker for some new type of ball game, would it be possible to make a home plate marker that is a regular polygon with each interior angle having a measure of (a) 135° ? (b) 145° ?

SOLUTION

- Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180^\circ = 135^\circ$ for n . You get $n = 8$.
 - ▶ Yes, it would be possible. A polygon can have 8 sides.
- Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180^\circ = 145^\circ$ for n . You get $n \approx 10.3$.
 - ▶ No, it would not be possible. A polygon cannot have 10.3 sides.

FOCUS ON PEOPLE

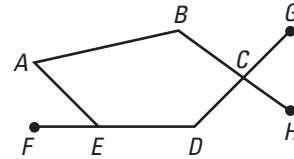


REAL LIFE **JOAN JOYCE** set a number of softball pitching records from 1956–1975. She delivered 40 pitches to slugger Ted Williams during an exhibition game in 1962. Williams only connected twice, for one foul ball and one base hit.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name an *interior angle* and an *exterior angle* of the polygon shown at the right.



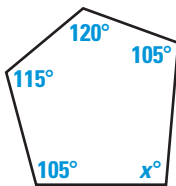
Concept Check ✓

2. How many exterior angles are there in an n -gon? Are they all considered when using the Polygon Exterior Angles Theorem? Explain.

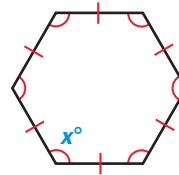
Skill Check ✓

Find the value of x .

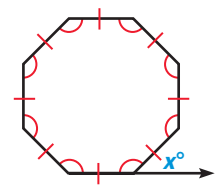
3.



4.



5.



PRACTICE AND APPLICATIONS

STUDENT HELP

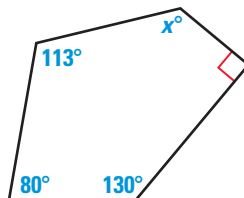
► **Extra Practice**
to help you master
skills is on p. 823.

SUMS OF ANGLE MEASURES Find the sum of the measures of the interior angles of the convex polygon.

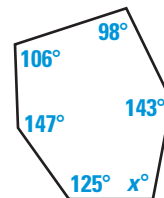
- | | | | |
|------------|------------|------------|-------------|
| 6. 10-gon | 7. 12-gon | 8. 15-gon | 9. 18-gon |
| 10. 20-gon | 11. 30-gon | 12. 40-gon | 13. 100-gon |

ANGLE MEASURES In Exercises 14–19, find the value of x .

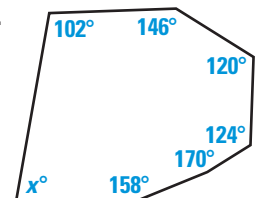
14.



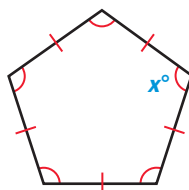
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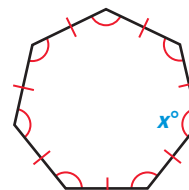
16.



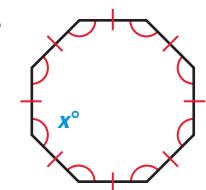
17.



18.



19.



STUDENT HELP

► HOMEWORK HELP

- Example 1:** Exs. 6–16, 20, 21
Example 2: Exs. 17–19, 22–28
Example 3: Exs. 29–38
Example 4: Exs. 39, 40, 49, 50
Example 5: Exs. 51–54

20. A convex quadrilateral has interior angles that measure 80° , 110° , and 80° . What is the measure of the fourth interior angle?
21. A convex pentagon has interior angles that measure 60° , 80° , 120° , and 140° . What is the measure of the fifth interior angle?

DETERMINING NUMBER OF SIDES In Exercises 22–25, you are given the measure of each interior angle of a regular n -gon. Find the value of n .

- | | | | |
|-----------------|-----------------|-----------------|-------------------|
| 22. 144° | 23. 120° | 24. 140° | 25. 157.5° |
|-----------------|-----------------|-----------------|-------------------|

 **CONSTRUCTION** Use a compass, protractor, and ruler to check the results of Example 2 on page 662.

26. Draw a large angle that measures 140° . Mark congruent lengths on the sides of the angle.
27. From the end of one of the congruent lengths in Exercise 26, draw the second side of another angle that measures 140° . Mark another congruent length along this new side.
28. Continue to draw angles that measure 140° until a polygon is formed. Verify that the polygon is regular and has 9 sides.

DETERMINING ANGLE MEASURES In Exercises 29–32, you are given the number of sides of a regular polygon. Find the measure of each exterior angle.

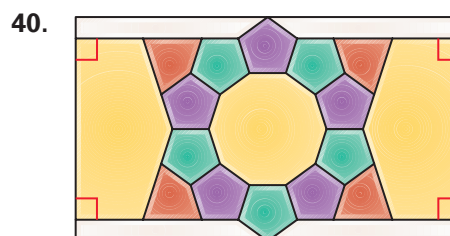
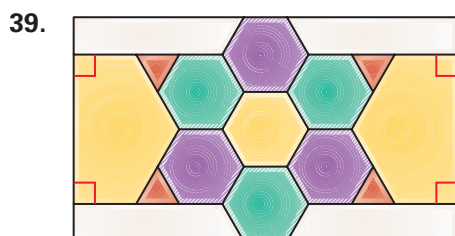
29. 12 30. 11 31. 21 32. 15

DETERMINING NUMBER OF SIDES In Exercises 33–36, you are given the measure of each exterior angle of a regular n -gon. Find the value of n .

33. 60° 34. 20° 35. 72° 36. 10°

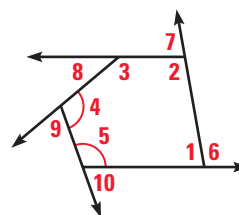
37. A convex hexagon has exterior angles that measure 48° , 52° , 55° , 62° , and 68° . What is the measure of the exterior angle of the sixth vertex?
38. What is the measure of each exterior angle of a regular decagon?

 **STAINED GLASS WINDOWS** In Exercises 39 and 40, the purple and green pieces of glass are in the shape of regular polygons. Find the measure of each interior angle of the red and yellow pieces of glass.




41. FINDING MEASURES OF ANGLES

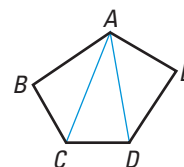
In the diagram at the right, $m\angle 2 = 100^\circ$, $m\angle 8 = 40^\circ$, $m\angle 4 = m\angle 5 = 110^\circ$. Find the measures of the other labeled angles and explain your reasoning.



42. **Writing** Explain why the sum of the measures of the interior angles of any two n -gons with the same number of sides (two octagons, for example) is the same. Do the n -gons need to be regular? Do they need to be similar?

43.  **PROOF** Use $ABCDE$ to write a paragraph proof to prove Theorem 11.1 for pentagons.

44.  **PROOF** Use a paragraph proof to prove the Corollary to Theorem 11.1.



FOCUS ON APPLICATIONS



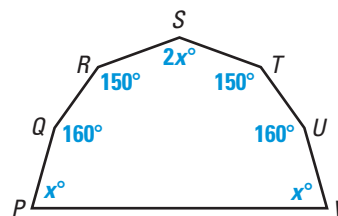
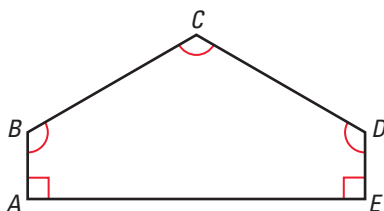
 **REAL LIFE STAINED GLASS**

is tinted glass that has been cut into shapes and arranged to form a picture or design. The pieces of glass are held in place by strips of lead.

45. **PROOF** Use this plan to write a paragraph proof of Theorem 11.2.
Plan for Proof In a convex n -gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180° . Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using Theorem 11.1.
46. **PROOF** Use a paragraph proof to prove the Corollary to Theorem 11.2.

TECHNOLOGY In Exercises 47 and 48, use geometry software to construct a polygon. At each vertex, extend one of the sides of the polygon to form an exterior angle.

47. Measure each exterior angle and verify that the sum of the measures is 360° .
48. Move any vertex to change the shape of your polygon. What happens to the measures of the exterior angles? What happens to their sum?
49. **HOUSES** Pentagon $ABCDE$ is an outline of the front of a house. Find the measure of each angle.
50. **TENTS** Heptagon $PQRSTUV$ is an outline of a camping tent. Find the unknown angle measures.



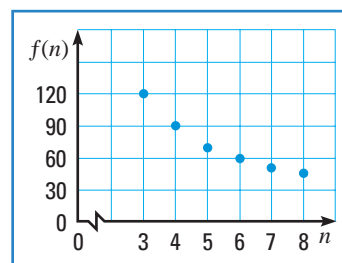
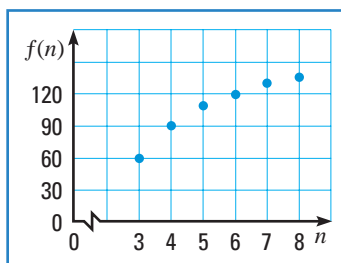
POSSIBLE POLYGONS Would it be possible for a regular polygon to have interior angles with the angle measure described? Explain.

51. 150° 52. 90° 53. 72° 54. 18°

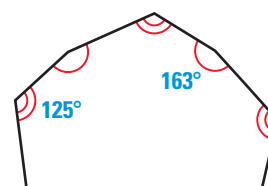
USING ALGEBRA In Exercises 55 and 56, you are given a function and its graph. In each function, n is the number of sides of a polygon and $f(n)$ is measured in degrees. How does the function relate to polygons? What happens to the value of $f(n)$ as n gets larger and larger?

55. $f(n) = \frac{180n - 360}{n}$

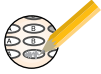
56. $f(n) = \frac{360}{n}$



57. **LOGICAL REASONING** You are shown part of a convex n -gon. The pattern of congruent angles continues around the polygon. Use the Polygon Exterior Angles Theorem to find the value of n .

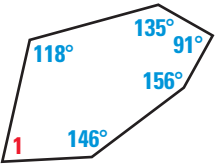
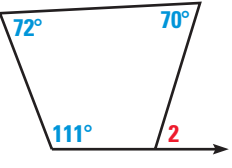


Test Preparation



QUANTITATIVE COMPARISON In Exercises 58–61, choose the statement that is true about the given quantities.

- (A) The quantity in column A is greater.
- (B) The quantity in column B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the given information.

	Column A	Column B
58.	The sum of the interior angle measures of a decagon	The sum of the interior angle measures of a 15-gon
59.	The sum of the exterior angle measures of an octagon	$8(45^\circ)$
60.	$m\angle 1$ 	$m\angle 2$ 
61.	Number of sides of a polygon with an exterior angle measuring 72°	Number of sides of a polygon with an exterior angle measuring 144°

★ Challenge

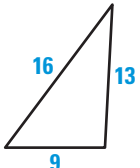
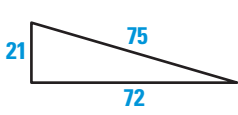
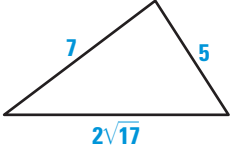
62. Polygon $STUVWXYZ$ is a regular octagon. Suppose sides \overline{ST} and \overline{UV} are extended to meet at a point R . Find the measure of $\angle TRU$.

MIXED REVIEW

FINDING AREA Find the area of the triangle described. (Review 1.7 for 11.2)

- 63. base: 11 inches; height: 5 inches
- 64. base: 43 meters; height: 11 meters
- 65. vertices: $A(2, 0)$, $B(7, 0)$, $C(5, 15)$
- 66. vertices: $D(-3, 3)$, $E(3, 3)$, $F(-7, 11)$

VERIFYING RIGHT TRIANGLES Tell whether the triangle is a right triangle. (Review 9.3)

- 67. 
- 68. 
- 69. 

FINDING MEASUREMENTS \overline{GD} and \overline{FH} are diameters of circle C . Find the indicated arc measure. (Review 10.2)

- 70. $m\widehat{DH}$
- 71. $m\widehat{ED}$
- 72. $m\widehat{EH}$
- 73. $m\widehat{EHG}$

