10.6

Equations of Circles

What you should learn

GOAL 1 Write the equation of a circle.

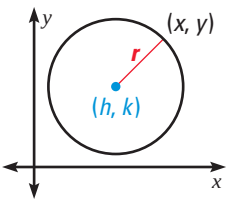
GOAL 2 Use the equation of a circle and its graph to solve problems.

Why you should learn it

▼ To solve **real-life** problems, such as determining cellular phone coverage, as in **Exs. 41 and 42.**

GOAL 1 FINDING EQUATIONS OF CIRCLES

You can write an equation of a circle in a coordinate plane if you know its radius and the coordinates of its center. Suppose the radius of a circle is r and the center is (h, k). Let (x, y) be any point on the circle. The distance between (x, y) and (h, k) is r, so you can use the Distance Formula.



$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

Square both sides to find the **standard equation of a circle** with radius r and center (h, k).

Standard equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

If the center is the origin, then the standard equation is $x^2 + y^2 = r^2$.

EXAMPLE 1 *Writing a Standard Equation of a Circle*

Write the standard equation of the circle with center $(-4, 0)$ and radius 7.1.

SOLUTION

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$[x - (-4)]^2 + (y - 0)^2 = 7.1^2$ — Substitute.

$(x + 4)^2 + y^2 = 50.41$ — Simplify.

EXAMPLE 2 *Writing a Standard Equation of a Circle*

The point $(1, 2)$ is on a circle whose center is $(5, -1)$. Write the standard equation of the circle.

SOLUTION

Find the radius. The radius is the distance from the point $(1, 2)$ to the center $(5, -1)$.

$r = \sqrt{(5 - 1)^2 + (-1 - 2)^2}$ — Use the Distance Formula.

$r = \sqrt{4^2 + (-3)^2}$ — Simplify.

$r = 5$ — Simplify.

Substitute $(h, k) = (5, -1)$ and $r = 5$ into the standard equation of a circle.

$(x - 5)^2 + (y - (-1))^2 = 5^2$ — Standard equation of a circle

$(x - 5)^2 + (y + 1)^2 = 25$ — Simplify.

GOAL 2 GRAPHING CIRCLES

If you know the equation of a circle, you can graph the circle by identifying its center and radius.

STUDENT HELP

Study Tip

You can sketch the graph of the circle in Example 3 without a compass by first plotting the four points shown in red. Then sketch a circle through the points.

EXAMPLE 3 *Graphing a Circle*

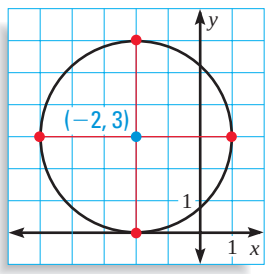
The equation of a circle is $(x + 2)^2 + (y - 3)^2 = 9$. Graph the circle.

Rewrite the equation to find the center and radius:

$$(x + 2)^2 + (y - 3)^2 = 9$$

$$[x - (-2)]^2 + (y - 3)^2 = 3^2$$

The center is $(-2, 3)$ and the radius is 3. To graph the circle, place the point of a compass at $(-2, 3)$, set the radius at 3 units, and swing the compass to draw a full circle.



EXAMPLE 4 *Applying Graphs of Circles*

THEATER LIGHTING A bank of lights is arranged over a stage. Each light illuminates a circular area on the stage. A coordinate plane is used to arrange the lights, using the corner of the stage as the origin. The equation $(x - 13)^2 + (y - 4)^2 = 16$ represents one of the disks of light.

a. Graph the disk of light.

b. Three actors are located as follows: Henry is at (11, 4), Jolene is at (8, 5), and Martin is at (15, 5). Which actors are in the disk of light?

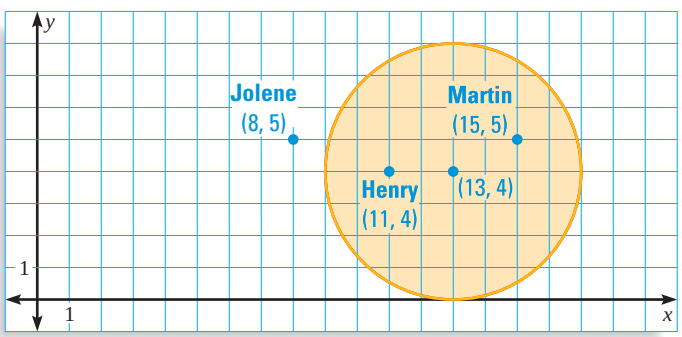
SOLUTION

a. Rewrite the equation to find the center and radius:

$$(x - 13)^2 + (y - 4)^2 = 16$$

$$(x - 13)^2 + (y - 4)^2 = 4^2$$

The center is $(13, 4)$ and the radius is 4. The circle is shown below.



b. The graph shows that Henry and Martin are both in the disk of light.

GUIDED PRACTICE

Vocabulary Check ✓

1. The standard form of an equation of a circle is ___?___.

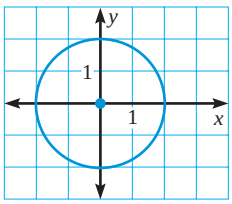
Concept Check ✓

2. Describe how to graph the circle $(x - 3)^2 + (y - 4)^2 = 9$.

Skill Check ✓

Give the coordinates of the center and the radius. Write an equation of the circle in standard form.

3.



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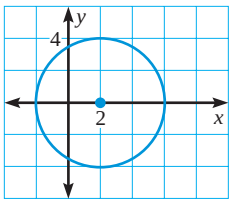

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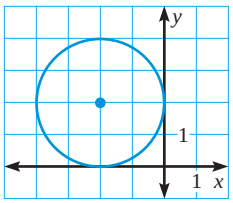


6. $P(-1, 3)$ is on a circle whose center is $C(0, 0)$. Write an equation of $\odot C$.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 822.

USING STANDARD EQUATIONS **Give the center and radius of the circle.**

7. $(x - 4)^2 + (y - 3)^2 = 16$

8. $(x - 5)^2 + (y - 1)^2 = 25$

9. $x^2 + y^2 = 4$

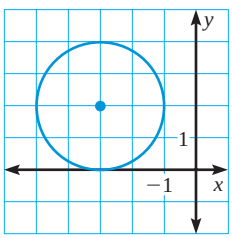
10. $(x + 2)^2 + (y - 3)^2 = 36$

11. $(x + 5)^2 + (y + 3)^2 = 1$

12. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \frac{1}{4}$

USING GRAPHS **Give the coordinates of the center, the radius, and the equation of the circle.**

13.



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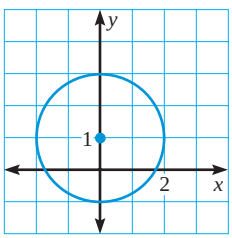

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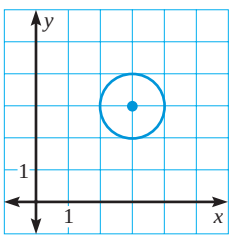

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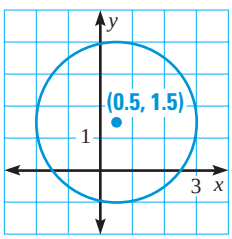

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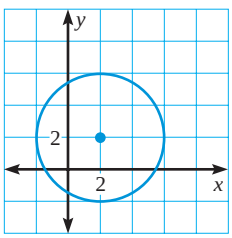

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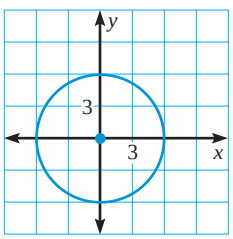


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–22
Example 2: Exs. 23–26
Example 3: Exs. 27–40
Example 4: Exs. 33–42

WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.**

19. center $(0, 0)$, radius 1

20. center $(4, 0)$, radius 4

21. center $(3, -2)$, radius 2

22. center $(-1, -3)$, radius 6

WRITING EQUATIONS **Use the given information to write the standard equation of the circle.**

23. The center is (0, 0), a point on the circle is (0, 3).

24. The center is (1, 2), a point on the circle is (4, 6).

25. The center is (3, 2), a point on the circle is (5, 2).

26. The center is (−5, 3) and the diameter is 8.

GRAPHING CIRCLES **Graph the equation.**

27. $x^2 + y^2 = 25$

28. $x^2 + (y - 4)^2 = 1$

29. $(x + 3)^2 + y^2 = 9$

30. $(x - 3)^2 + (y - 4)^2 = 16$

31. $(x + 5)^2 + (y - 1)^2 = 49$

32. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$

USING GRAPHS **The equation of a circle is $(x - 2)^2 + (y + 3)^2 = 4$. Tell whether each point is *on* the circle, in the *interior* of the circle, or in the *exterior* of the circle.**

33. (0, 0)

34. (2, −4)

35. (0, −3)

36. (3, −1)

37. (1, −4)

38. (2, −5)

39. (2, 0)

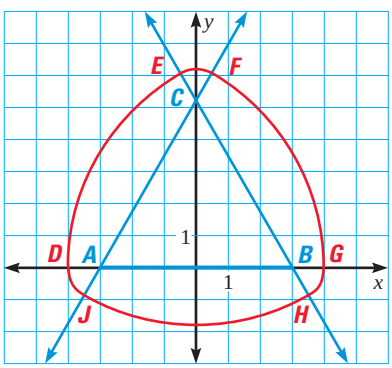
40. (2.5, −3)

CELL PHONES **In Exercises 41 and 42, use the following information.**
A cellular phone network uses towers to transmit calls. Each tower transmits to a circular area. On a grid of a city, the coordinates of the location and the radius each tower covers are as follows (integers represent miles): Tower *A* is at (0, 0) and covers a 3 mile radius, Tower *B* is at (5, 3) and covers a 2.5 mile radius, and Tower *C* is at (2, 5) and covers a 2 mile radius.

41. Write the equations that represent the transmission boundaries of the towers. Graph each equation.

42. Tell which towers, if any, transmit to a phone located at *J*(1, 1), *K*(4, 2), *L*(3.5, 4.5), *M*(2, 2.8), or *N*(1, 6).

REULEAUX POLYGONS **The figure at the right is called a *Reuleaux polygon*. It is not a true polygon because its sides are not straight. $\triangle ABC$ is equilateral.**



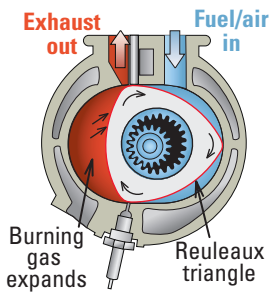
43. $\overset{\frown}{JD}$ lies on a circle with center *A* and radius *AD*. Write an equation of this circle.

44. $\overset{\frown}{DE}$ lies on a circle with center *B* and radius *BD*. Write an equation of this circle.

45. **CONSTRUCTION** The remaining arcs of the polygon are constructed in the same way as $\overset{\frown}{JD}$ and $\overset{\frown}{DE}$ in Exercises 43 and 44. Construct a Reauleaux polygon on a piece of cardboard.

46. Cut out the Reauleaux polygon from Exercise 45. Roll it on its edge like a wheel and measure its height when it is in different orientations. Explain why a Reuleaux polygon is said to have constant width.

FOCUS ON APPLICATIONS



REAL LIFE The Wankel engine is an engine with a triangular rotor that is based on a Reuleaux triangle. It has been used in sports cars, snowmobiles, and hybrid electric vehicles.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

47. TRANSLATIONS Sketch the circle whose equation is $x^2 + y^2 = 16$. Then sketch the image of the circle after the translation $(x, y) \rightarrow (x - 2, y - 4)$. What is the equation of the image?

48. WRITING AN EQUATION A circle has a center (p, q) and is tangent to the x-axis. Write the standard equation of the circle.

Test Preparation

49. MULTIPLE CHOICE What is the standard form of the equation of a circle with center $(-3, 1)$ and radius 2?

Ⓐ $(x - 3)^2 + (y - 1)^2 = 2$ Ⓑ $(x + 3)^2 + (y - 1)^2 = 2$

Ⓒ $(x - 3)^2 + (y - 1)^2 = 4$ Ⓓ $(x + 3)^2 + (y - 1)^2 = 4$

50. MULTIPLE CHOICE The center of a circle is $(-3, 0)$ and its radius is 5. Which point does *not* lie on the circle?

Ⓐ $(2, 0)$ Ⓑ $(0, 4)$ Ⓒ $(-3, 0)$ Ⓓ $(-3, -5)$ Ⓔ $(-8, 0)$

Challenge

51. CRITICAL THINKING $\odot A$ and $\odot B$ are externally tangent. Suppose you know the equation of $\odot A$, the coordinates of the single point of intersection of $\odot A$ and $\odot B$, and the radius of $\odot B$. Do you know enough to find the equation of $\odot B$? Explain.

USING ALGEBRA Find the missing coordinate of the center of the circle with the given characteristics.

EXTRA CHALLENGE
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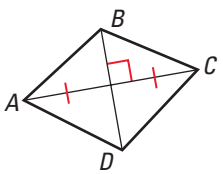
52. The center is $(1, b)$, the radius is 3, and a point on the circle is $(-2, 0)$.

53. The center is $(-3, b)$, the radius is 5, and a point on the circle is $(2, -2)$.

MIXED REVIEW

IDENTIFYING QUADRILATERALS What kind(s) of quadrilateral could *ABCD* be? *ABCD* is not drawn to scale. (Review 6.6)

54.



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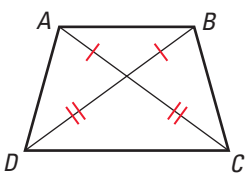

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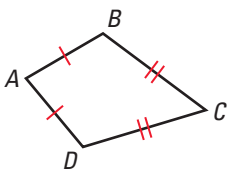


VECTORS Write the component form of vector $\overrightarrow{PQ}$. Use the component form to find the magnitude of $\overrightarrow{PQ}$ to the nearest tenth. (Review 9.7)

57. $P = (0, 0), Q = (-6, 7)$

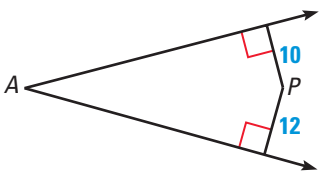
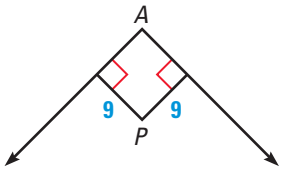
58. $P = (3, -4), Q = (-11, 2)$

59. $P = (-6, -6), Q = (9, -5)$

60. $P = (5, 6), Q = (-3, 7)$

ANGLE BISECTORS Does *P* lie on the bisector of $\angle A$? Explain your reasoning. (Review 5.1)

61.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