10.5 Segment Lengths in Circles

What you should learn

GOAL 1 Find the lengths of segments of chords.

GOAL 2 Find the lengths of segments of tangents and secants.

Why you should learn it

▼ To find **real-life** measures, such as the radius of an aquarium tank in **Example 3**.

REAL LIFE

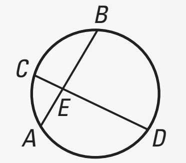
GOAL 1 FINDING LENGTHS OF SEGMENTS OF CHORDS

When two chords intersect in the interior of a circle, each chord is divided into two segments which are called *segments of a chord*. The following theorem gives a relationship between the lengths of the four segments that are formed.

THEOREM

THEOREM 10.15

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

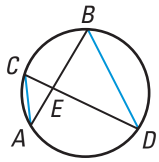


$EA \cdot EB = EC \cdot ED$

You can use similar triangles to prove Theorem 10.15.

GIVEN ▶ $\overline{AB}$, $\overline{CD}$ are chords that intersect at E.

PROVE ▶ $EA \cdot EB = EC \cdot ED$



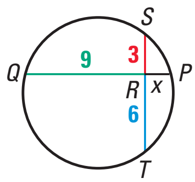
Paragraph Proof Draw $\overline{DB}$ and $\overline{AC}$. Because $\angle C$ and $\angle B$ intercept the same arc, $\angle C \cong \angle B$. Likewise, $\angle A \cong \angle D$. By the AA Similarity Postulate, $\triangle AEC \sim \triangle DEB$. So, the lengths of corresponding sides are proportional.

$$\frac{EA}{ED} = \frac{EC}{EB}$$ The lengths of the sides are proportional.

$$EA \cdot EB = EC \cdot ED$$ Cross Product Property

EXAMPLE 1 *Finding Segment Lengths*

Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.



SOLUTION

$RQ \cdot RP = RS \cdot RT$ Use Theorem 10.15.

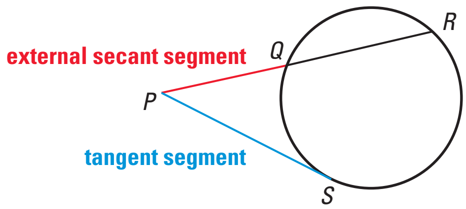
$9 \cdot x = 3 \cdot 6$ Substitute.

$9x = 18$ Simplify.

$x = 2$ Divide each side by 9.

GOAL 2 USING SEGMENTS OF TANGENTS AND SECANTS

In the figure shown below, $\overline{PS}$ is called a **tangent segment** because it is tangent to the circle at an endpoint. Similarly, $\overline{PR}$ is a **secant segment** and $\overline{PQ}$ is the **external segment** of $\overline{PR}$.

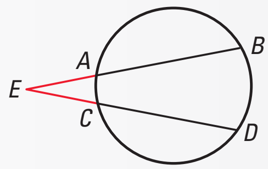


You are asked to prove the following theorems in Exercises 31 and 32.

THEOREMS

THEOREM 10.16

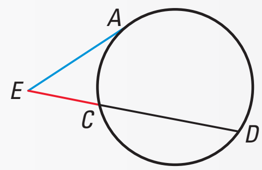
If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



$EA \cdot EB = EC \cdot ED$

THEOREM 10.17

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.

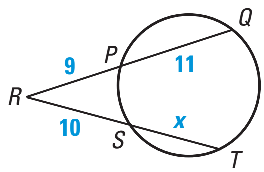


$(EA)^2 = EC \cdot ED$

EXAMPLE 2 *Finding Segment Lengths*

Using Algebra

Find the value of x.



SOLUTION

$RP \cdot RQ = RS \cdot RT$	Use Theorem 10.16.
$9 \cdot (11 + 9) = 10 \cdot (x + 10)$	Substitute.
$180 = 10x + 100$	Simplify.
$80 = 10x$	Subtract 100 from each side.
$8 = x$	Divide each side by 10.

FOCUS ON APPLICATIONS

AQUARIUM TANK The Caribbean Coral Reef Tank at the New England Aquarium is a circular tank 24 feet deep. The 200,000 gallon tank contains an elaborate coral reef and many exotic fishes.

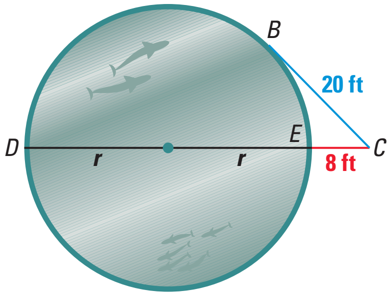
REAL LIFE

INTERNET APPLICATION LINK
www.mcdougallittell.com

In Lesson 10.1, you learned how to use the Pythagorean Theorem to estimate the radius of a grain silo. Example 3 shows you another way to estimate the radius of a circular object.

EXAMPLE 3 *Estimating the Radius of a Circle*

AQUARIUM TANK You are standing at point C, about 8 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is about 20 feet. Estimate the radius of the tank.



SOLUTION

You can use Theorem 10.17 to find the radius.

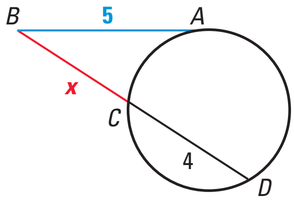
$(CB)^2 = CE \cdot CD$	**Use Theorem 10.17.**
$20^2 \approx 8 \cdot (2r + 8)$	**Substitute.**
$400 \approx 16r + 64$	**Simplify.**
$336 \approx 16r$	**Subtract 64 from each side.**
$21 \approx r$	**Divide each side by 16.**

▶ So, the radius of the tank is about 21 feet.

EXAMPLE 4 *Finding Segment Lengths*

Using Algebra

Use the figure at the right to find the value of x.



SOLUTION

$(BA)^2 = BC \cdot BD$	**Use Theorem 10.17.**
$5^2 = x \cdot (x + 4)$	**Substitute.**
$25 = x^2 + 4x$	**Simplify.**
$0 = x^2 + 4x - 25$	**Write in standard form.**
$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-25)}}{2}$	**Use Quadratic Formula.**
$x = -2 \pm \sqrt{29}$	**Simplify.**

Use the positive solution, because lengths cannot be negative.

▶ So, $x = -2 + \sqrt{29} \approx 3.39$.

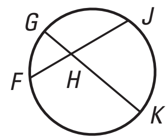
GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch a circle with a secant segment. Label each endpoint and point of intersection. Then name the external segment.

Concept Check ✓

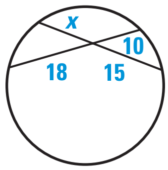
2. How are the lengths of the segments in the figure at the right related to each other?



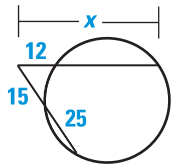
Skill Check ✓

Fill in the blanks. Then find the value of *x*.

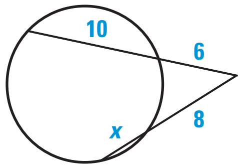
3. $x \cdot \underline{\ ?\ } = 10 \cdot \underline{\ ?\ }$



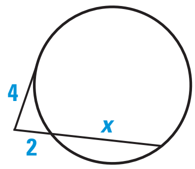
4. $\underline{\ ?\ } \cdot x = \underline{\ ?\ } \cdot 40$



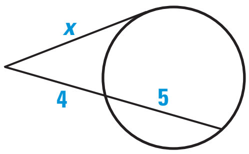
5. $6 \cdot \underline{\ ?\ } = 8 \cdot \underline{\ ?\ }$



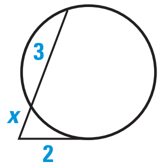
6. $4^2 = 2 \cdot (\underline{\ ?\ } + x)$



7. $x^2 = 4 \cdot \underline{\ ?\ }$



8. $x \cdot \underline{\ ?\ } = \underline{\ ?\ }$



9. **ZOO HABITAT** A zoo has a large circular aviary, a habitat for birds. You are standing about 40 feet from the aviary. The distance from you to a point of tangency on the aviary is about 60 feet. Describe how to estimate the radius of the aviary.

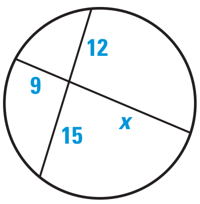
PRACTICE AND APPLICATIONS

STUDENT HELP

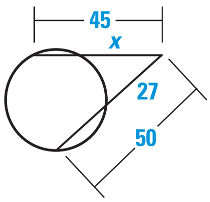
Extra Practice to help you master skills is on p. 822.

FINDING SEGMENT LENGTHS **Fill in the blanks. Then find the value of *x*.**

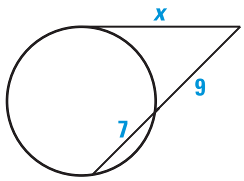
10. $x \cdot \underline{\ ?\ } = 12 \cdot \underline{\ ?\ }$



11. $x \cdot \underline{\ ?\ } = \underline{\ ?\ } \cdot 50$



12. $x^2 = 9 \cdot \underline{\ ?\ }$

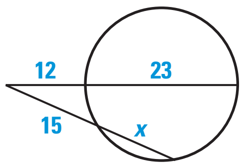


STUDENT HELP

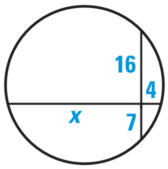
HOMEWORK HELP
Example 1: Exs. 10, 14–17, 26–29
Example 2: Exs. 11, 13, 18, 19, 24, 25

FINDING SEGMENT LENGTHS **Find the value of *x*.**

13.



14.



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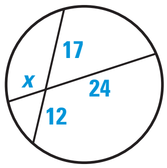
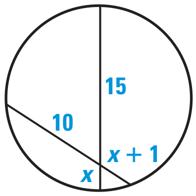


STUDENT HELP

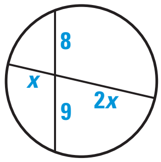
HOMEWORK HELP
Example 3: Exs. 12, 20–23, 25–27
Example 4: Exs. 12, 20–23, 25–27

FINDING SEGMENT LENGTHS Find the value of *x*.

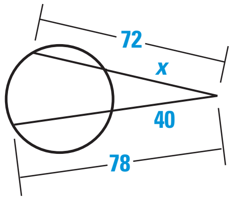
16.



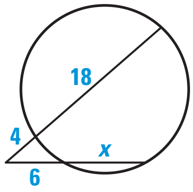
17.



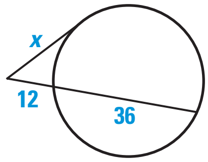
18.



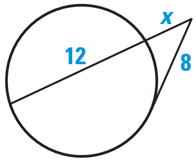
19.



20.



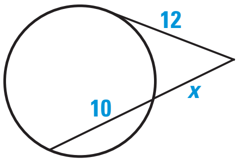
21.



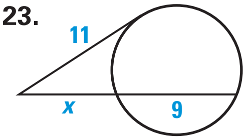
STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with using the Quadratic Formula in Exs. 21–27.

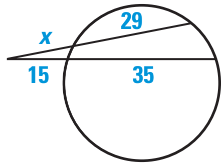
22.



23.

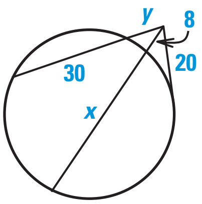


24.

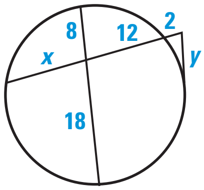


USING ALGEBRA Find the values of *x* and *y*.

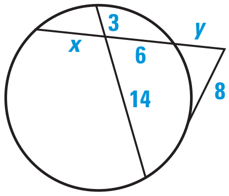
25.



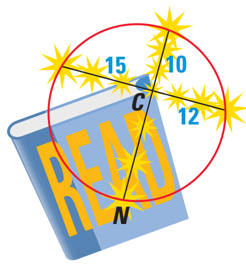
26.



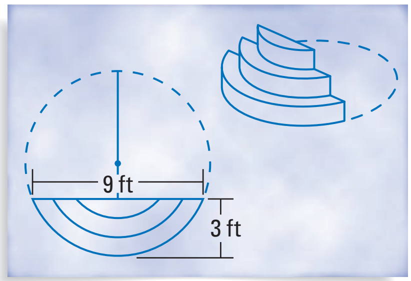
27.



28. **DESIGNING A LOGO** Suppose you are designing an animated logo for a television commercial. You want sparkles to leave point C and move to the circle along the segments shown. You want each of the sparkles to reach the circle at the same time. To calculate the speed for each sparkle, you need to know the distance from point C to the circle along each segment. What is the distance from C to N?



29. **BUILDING STAIRS** You are making curved stairs for students to stand on for photographs at a homecoming dance. The diagram shows a top view of the stairs. What is the radius of the circle shown? Explain how you can use Theorem 10.15 to find the answer.



FOCUS ON APPLICATIONS

REAL LIFE

GPS Some cars have navigation systems that use GPS to tell you where you are and how to get where you want to go.

INTERNET APPLICATION LINK
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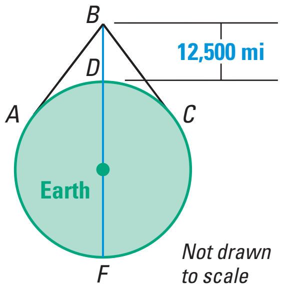
30. GLOBAL POSITIONING SYSTEM Satellites in the Global Positioning System (GPS) orbit 12,500 miles above Earth. GPS signals can't travel through Earth, so a satellite at point B can transmit signals only to points on $\widehat{AC}$. How far must the satellite be able to transmit to reach points A and C? Find BA and BC. The diameter of Earth is about 8000 miles. Give your answer to the nearest thousand miles.

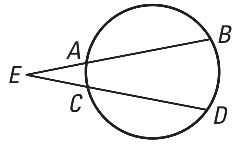


31. PROVING THEOREM 10.16 Use the plan to write a paragraph proof.

GIVEN ▶ $\overline{EB}$ and $\overline{ED}$ are secant segments.

PROVE ▶ $EA \cdot EB = EC \cdot ED$

Plan for Proof Draw $\overline{AD}$ and $\overline{BC}$, and show that $\triangle BCE$ and $\triangle DAE$ are similar. Use the fact that corresponding sides of similar triangles are proportional.

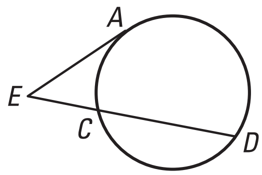


32. PROVING THEOREM 10.17 Use the plan to write a paragraph proof.

GIVEN ▶ $\overline{EA}$ is a tangent segment and $\overline{ED}$ is a secant segment.

PROVE ▶ $(EA)^2 = EC \cdot ED$

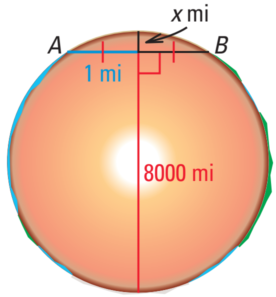
Plan for Proof Draw $\overline{AD}$ and $\overline{AC}$. Use the fact that corresponding sides of similar triangles are proportional.



Test Preparation

MULTI-STEP PROBLEM In Exercises 33–35, use the following information.

A person is standing at point A on a beach and looking 2 miles down the beach to point B, as shown at the right. The beach is very flat but, because of Earth's curvature, the ground between A and B is x mi higher than $\overline{AB}$.



Not drawn to scale

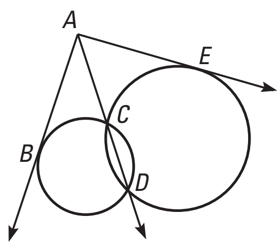
33. Find the value of x.

34. Convert your answer to inches. Round to the nearest inch.

35. *Writing* Why do you think people historically thought that Earth was flat?

★ Challenge

In the diagram at the right, $\overrightarrow{AB}$ and $\overrightarrow{AE}$ are tangents.



36. Write an equation that shows how AB is related to AC and AD.

37. Write an equation that shows how AE is related to AC and AD.

38. How is AB related to AE? Explain.

39. Make a conjecture about tangents to intersecting circles. Then test your conjecture by looking for a counterexample.

EXTRA CHALLENGE
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MIXED REVIEW

FINDING DISTANCE AND MIDPOINT **Find AB to the nearest hundredth. Then find the coordinates of the midpoint of $\overline{AB}$.** (Review 1.3, 1.5 for 10.6)

40. $A(2, 5), B(-3, 3)$

41. $A(6, -4), B(0, 4)$

42. $A(-8, -6), B(1, 9)$

43. $A(-1, -5), B(-10, 7)$

44. $A(0, -11), B(8, 2)$

45. $A(5, -2), B(-9, -2)$

WRITING EQUATIONS **Write an equation of a line perpendicular to the given line at the given point.** (Review 3.7 for 10.6)

46. $y = -2x - 5, (-2, -1)$

47. $y = \frac{2}{3}x + 4, (6, 8)$

48. $y = -x + 9, (0, 9)$

49. $y = 3x - 10, (2, -4)$

50. $y = \frac{1}{5}x + 1, (-10, -1)$

51. $y = -\frac{7}{3}x - 5, (-6, 9)$

DRAWING TRANSLATIONS **Quadrilateral $ABCD$ has vertices $A(-6, 8)$, $B(-1, 4)$, $C(-2, 2)$, and $D(-7, 3)$. Draw its image after the translation.** (Review 7.4 for 10.6)

52. $(x, y) \to (x + 7, y)$

53. $(x, y) \to (x - 2, y + 3)$

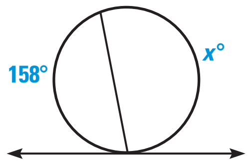
54. $(x, y) \to \left(x, y - \frac{11}{2}\right)$

QUIZ 2

Self-Test for Lessons 10.4 and 10.5

Find the value of x. (Lesson 10.4)

1.



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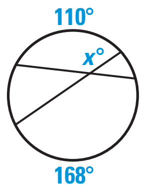
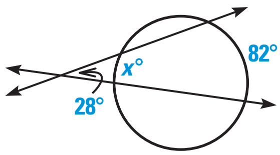
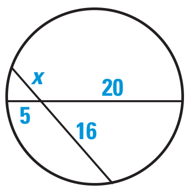


3.

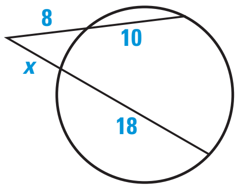


Find the value of x. (Lesson 10.5)

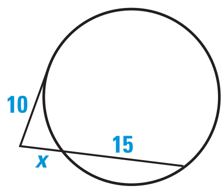
4.



5.



6.



7. **SWIMMING POOL** You are standing 20 feet from the circular wall of an above ground swimming pool and 49 feet from a point of tangency. Describe two different methods you could use to find the radius of the pool. What is the radius? (Lesson 10.5)