

# 10.4

## Other Angle Relationships in Circles

### What you should learn

**GOAL 1** Use angles formed by tangents and chords to solve problems in geometry.

**GOAL 2** Use angles formed by lines that intersect a circle to solve problems.

### Why you should learn it

▼ To solve **real-life** problems, such as finding from how far away you can see fireworks, as in **Ex. 35**.



### GOAL 1 USING TANGENTS AND CHORDS

You know that the measure of an angle inscribed in a circle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle. You will be asked to prove Theorem 10.12 in Exercises 37–39.

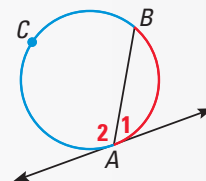
#### THEOREM

##### THEOREM 10.12

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

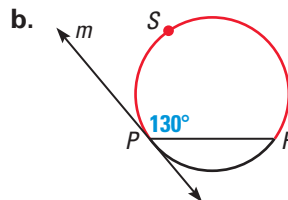
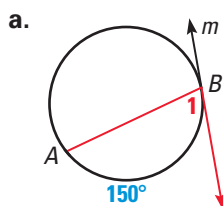
$$m\angle 1 = \frac{1}{2}m\widehat{AB}$$

$$m\angle 2 = \frac{1}{2}m\widehat{BCA}$$



### EXAMPLE 1 Finding Angle and Arc Measures

Line  $m$  is tangent to the circle. Find the measure of the red angle or arc.



#### SOLUTION

a.  $m\angle 1 = \frac{1}{2}(150^\circ) = 75^\circ$

b.  $m\widehat{RSP} = 2(130^\circ) = 260^\circ$

### EXAMPLE 2 Finding an Angle Measure

In the diagram below,  $\overleftrightarrow{BC}$  is tangent to the circle. Find  $m\angle CBD$ .

#### SOLUTION

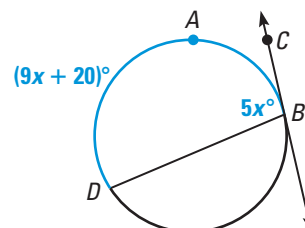
$$m\angle CBD = \frac{1}{2}m\widehat{DAB}$$

$$5x = \frac{1}{2}(9x + 20)$$

$$10x = 9x + 20$$

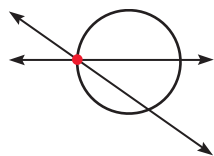
$$x = 20$$

►  $m\angle CBD = 5(20^\circ) = 100^\circ$

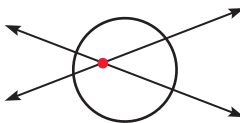


## GOAL 2 LINES INTERSECTING INSIDE OR OUTSIDE A CIRCLE

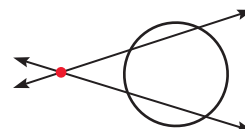
If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle



outside the circle

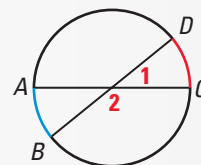
You know how to find angle and arc measures when lines intersect *on* the circle. You can use Theorems 10.13 and 10.14 to find measures when the lines intersect *inside* or *outside* the circle. You will prove these theorems in Exercises 40 and 41.

### THEOREMS

#### THEOREM 10.13

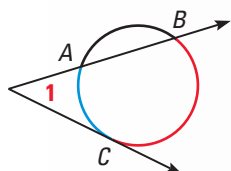
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

$$m\angle 1 = \frac{1}{2}(m\widehat{CD} + m\widehat{AB}), m\angle 2 = \frac{1}{2}(m\widehat{BC} + m\widehat{AD})$$

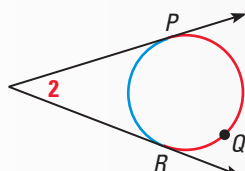


#### THEOREM 10.14

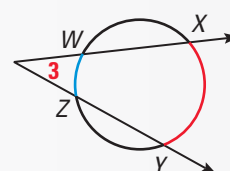
If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$$



$$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$$



$$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$$

### EXAMPLE 3 Finding the Measure of an Angle Formed by Two Chords

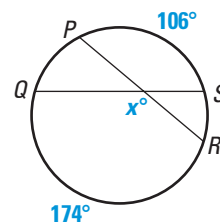
Find the value of  $x$ .

#### SOLUTION

$$x^\circ = \frac{1}{2}(m\widehat{PS} + m\widehat{RQ}) \quad \text{Apply Theorem 10.13.}$$

$$x^\circ = \frac{1}{2}(106^\circ + 174^\circ) \quad \text{Substitute.}$$

$$x = 140 \quad \text{Simplify.}$$



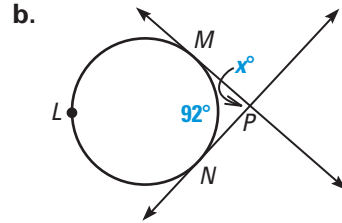
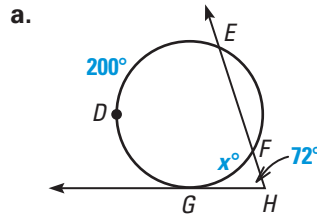
#### STUDENT HELP



**HOMEWORK HELP**  
Visit our Web site  
[www.mcdougallittell.com](http://www.mcdougallittell.com)  
for extra examples.

**EXAMPLE 4** Using Theorem 10.14

Find the value of  $x$ .



**SOLUTION**

a.  $m\angle GHF = \frac{1}{2}(m\widehat{EDG} - m\widehat{GF})$  **Apply Theorem 10.14.**

$$72^\circ = \frac{1}{2}(200^\circ - x^\circ) \quad \text{Substitute.}$$

$$144 = 200 - x \quad \text{Multiply each side by 2.}$$

$$x = 56 \quad \text{Solve for } x.$$

b. Because  $\widehat{MN}$  and  $\widehat{MLN}$  make a whole circle,  $m\widehat{MLN} = 360^\circ - 92^\circ = 268^\circ$ .

$$x = \frac{1}{2}(m\widehat{MLN} - m\widehat{MN}) \quad \text{Apply Theorem 10.14.}$$

$$= \frac{1}{2}(268 - 92) \quad \text{Substitute.}$$

$$= \frac{1}{2}(176) \quad \text{Subtract.}$$

$$= 88 \quad \text{Multiply.}$$



Mount Rainier, Washington

**EXAMPLE 5** Describing the View from Mount Rainier



**VIEWS** You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level. Find the measure of the arc  $\widehat{CD}$  that represents the part of Earth that you can see.

**SOLUTION**

$\overrightarrow{BC}$  and  $\overrightarrow{BD}$  are tangent to Earth. You can solve right  $\triangle BCA$  to see that  $m\angle CBA \approx 87.9^\circ$ . So,  $m\angle CBD \approx 175.8^\circ$ . Let  $m\widehat{CD} = x^\circ$ .

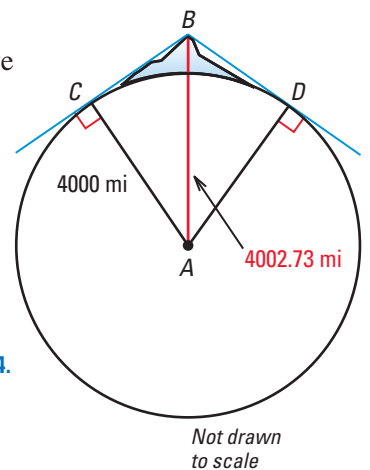
$$175.8 \approx \frac{1}{2}[(360 - x) - x] \quad \text{Apply Theorem 10.14.}$$

$$175.8 \approx \frac{1}{2}(360 - 2x) \quad \text{Simplify.}$$

$$175.8 \approx 180 - x \quad \text{Distributive property}$$

$$x \approx 4.2 \quad \text{Solve for } x.$$

► From the peak, you can see an arc of about  $4^\circ$ .



**STUDENT HELP**

**Look Back**  
For help with solving a right triangle, see pp. 567–569.

# GUIDED PRACTICE

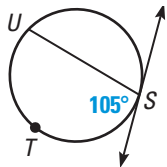
## Concept Check ✓

1. If a chord of a circle intersects a tangent to the circle at the point of tangency, what is the relationship between the angles formed and the intercepted arcs?

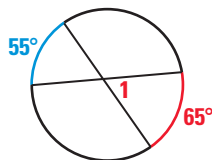
## Skill Check ✓

Find the indicated measure or value.

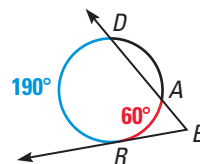
2.  $m\widehat{STU}$



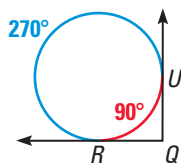
3.  $m\angle 1$



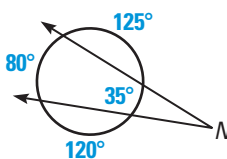
4.  $m\angle DBR$



5.  $m\angle RQU$



6.  $m\angle N$



7.  $m\angle 1$



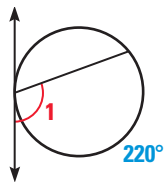
# PRACTICE AND APPLICATIONS

## STUDENT HELP

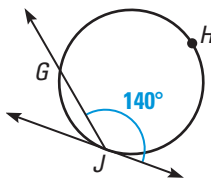
**Extra Practice** to help you master skills is on p. 822.

**FINDING MEASURES** Find the indicated measure.

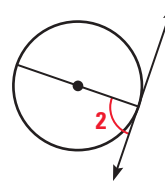
8.  $m\angle 1$



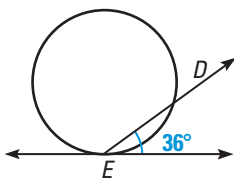
9.  $m\widehat{GHJ}$



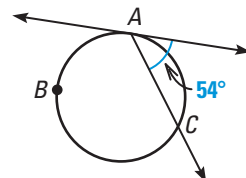
10.  $m\angle 2$



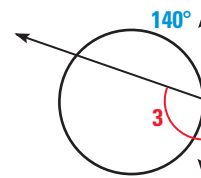
11.  $m\widehat{DE}$



12.  $m\widehat{ABC}$

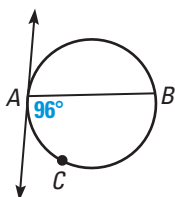


13.  $m\angle 3$

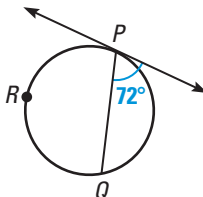


**USING ALGEBRA** Find the value of  $x$ .

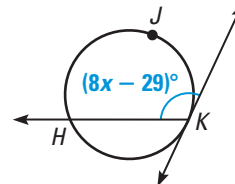
14.  $m\widehat{AB} = x^\circ$



15.  $m\widehat{PQ} = (5x + 17)^\circ$



16.  $m\widehat{HJK} = (10x + 50)^\circ$

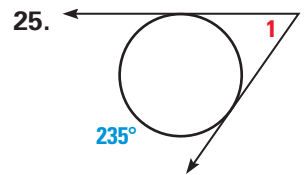
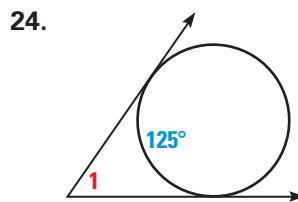
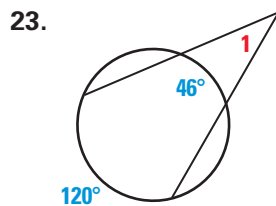
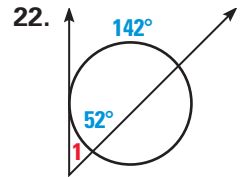
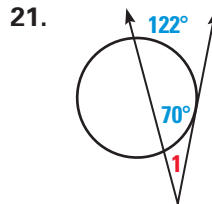
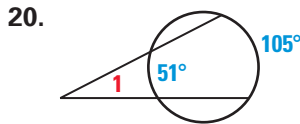
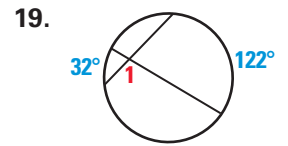
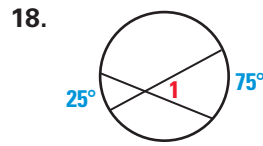
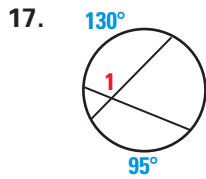


## STUDENT HELP

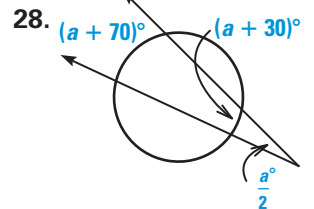
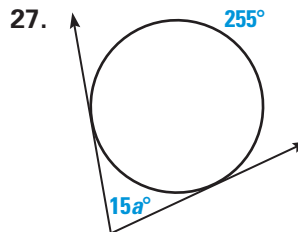
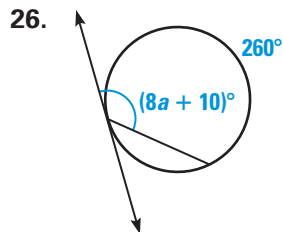
### HOMEWORK HELP

- Example 1: Exs. 8–13
- Example 2: Exs. 14–16
- Example 3: Exs. 17–25
- Example 4: Exs. 26–28
- Example 5: Ex. 35

**FINDING ANGLE MEASURES** Find  $m\angle 1$ .



**xy USING ALGEBRA** Find the value of  $a$ .



**FINDING ANGLE MEASURES** Use the diagram at the right to find the measure of the angle.

29.  $m\angle 1$

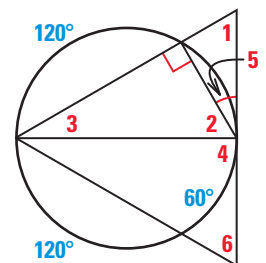
30.  $m\angle 2$

31.  $m\angle 3$

32.  $m\angle 4$

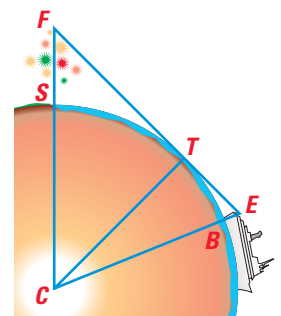
33.  $m\angle 5$

34.  $m\angle 6$



**STUDENT HELP**  
**APPLICATION LINK**  
 Visit our Web site  
[www.mcdougallittell.com](http://www.mcdougallittell.com)

35. **FIREWORKS** You are watching fireworks over San Diego Bay  $S$  as you sail away in a boat. The highest point the fireworks reach  $F$  is about 0.2 mile above the bay and your eyes  $E$  are about 0.01 mile above the water. At point  $B$  you can no longer see the fireworks because of the curvature of Earth. The radius of Earth is about 4000 miles and  $\overline{FE}$  is tangent to Earth at  $T$ . Find  $m\widehat{SB}$ . Give your answer to the nearest tenth of a degree.




Not drawn to scale

**STUDENT HELP**

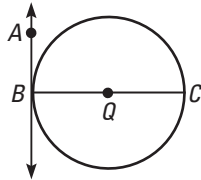


**SOFTWARE HELP**

Visit our Web site [www.mcdougallittell.com](http://www.mcdougallittell.com) to see instructions for several software applications.

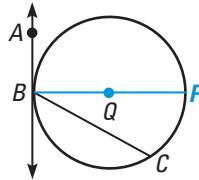
36.  **TECHNOLOGY** Use geometry software to construct and label circle  $O$ ,  $\overline{AB}$  which is tangent to  $\odot O$  at point  $A$ , and any point  $C$  on  $\odot O$ . Then construct secant  $\overline{AC}$ . Measure  $\angle BAC$  and  $\widehat{AC}$ . Compare the measures of  $\angle BAC$  and its intercepted arc as you drag point  $C$  on the circle. What do you notice? What theorem from this lesson have you illustrated?

 **PROVING THEOREM 10.12** The proof of Theorem 10.12 can be split into three cases, as shown in the diagrams.



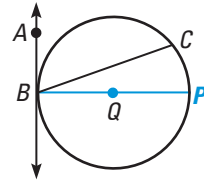
**Case 1**

The center of the circle is on one side of  $\angle ABC$ .




**Case 2**

The center of the circle is in the interior of  $\angle ABC$ .



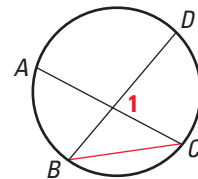
**Case 3**

The center of the circle is in the exterior of  $\angle ABC$ .


37. In Case 1, what type of chord is  $\overline{BC}$ ? What is the measure of  $\angle ABC$ ? What theorem earlier in this chapter supports your conclusion?
38. Write a plan for a proof of Case 2 of Theorem 10.12. (*Hint:* Use the auxiliary line and the Angle Addition Postulate.)
39. Describe how the proof of Case 3 of Theorem 10.12 is different from the proof of Case 2.
40.  **PROVING THEOREM 10.13** Fill in the blanks to complete the proof of Theorem 10.13.

**GIVEN**  $\blacktriangleright$  Chords  $\overline{AC}$  and  $\overline{BD}$  intersect.

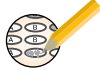
**PROVE**  $\blacktriangleright m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$



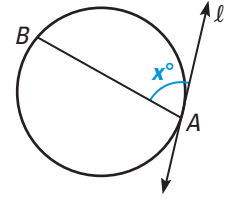
Statements	Reasons
1. Chords $\overline{AC}$ and $\overline{BD}$ intersect.	1. <u>?</u>
2. Draw $\overline{BC}$ .	2. <u>?</u>
3. $m\angle 1 = m\angle DBC + m\angle$ <u>?</u>	3. <u>?</u>
4. $m\angle DBC = \frac{1}{2}m\widehat{DC}$	4. <u>?</u>
5. $m\angle ACB = \frac{1}{2}m\widehat{AB}$	5. <u>?</u>
6. $m\angle 1 = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{AB}$	6. <u>?</u>
7. $m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$	7. <u>?</u>

41.  **JUSTIFYING THEOREM 10.14** Look back at the diagrams for Theorem 10.14 on page 622. Copy the diagram for the case of a tangent and a secant and draw  $\overline{BC}$ . Explain how to use the Exterior Angle Theorem in the proof of this case. Then copy the diagrams for the other two cases, draw appropriate auxiliary segments, and write plans for the proofs of the cases.

## Test Preparation



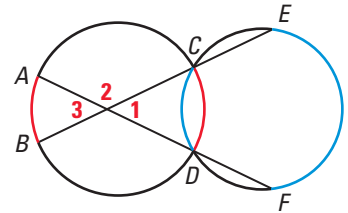
42. **MULTIPLE CHOICE** The diagram at the right is not drawn to scale.  $\overline{AB}$  is any chord of the circle. The line is tangent to the circle at point A. Which of the following must be true?



- (A)  $x < 90$     (B)  $x \leq 90$     (C)  $x = 90$   
 (D)  $x > 90$     (E) Cannot be determined from given information

43. **MULTIPLE CHOICE** In the figure at the right, which relationship is *not* true?

- (A)  $m\angle 1 = \frac{1}{2}(m\widehat{CD} + m\widehat{AB})$   
 (B)  $m\angle 1 = \frac{1}{2}(m\widehat{EF} - m\widehat{CD})$   
 (C)  $m\angle 2 = \frac{1}{2}(m\widehat{BD} - m\widehat{AC})$   
 (D)  $m\angle 3 = \frac{1}{2}(m\widehat{EF} - m\widehat{CD})$



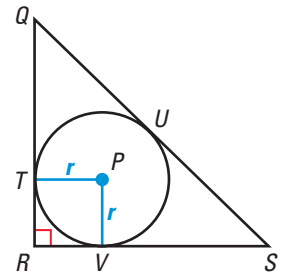
## ★ Challenge

44. **PROOF** Use the plan to write a paragraph proof.

**GIVEN**  $\triangle R$  is a right angle. Circle  $P$  is inscribed in  $\triangle QRS$ .  $T$ ,  $U$ , and  $V$  are points of tangency.

**PROVE**  $r = \frac{1}{2}(QR + RS - QS)$

**Plan for Proof** Prove that  $TPVR$  is a square. Then show that  $\overline{QT} \cong \overline{QU}$  and  $\overline{SU} \cong \overline{SV}$ . Finally, use the Segment Addition Postulate and substitution.



### EXTRA CHALLENGE

[www.mcdougallittell.com](http://www.mcdougallittell.com)

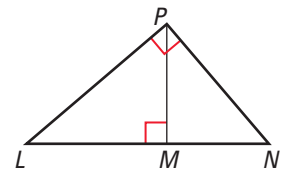
45. **FINDING A RADIUS** Use the result from Exercise 44 to find the radius of an inscribed circle of a right triangle with side lengths of 3, 4, and 5.

## MIXED REVIEW

**USING SIMILAR TRIANGLES** Use the diagram at the right and the given information. (Review 9.1)

46.  $MN = 9$ ,  $PM = 12$ ,  $LP = ?$

47.  $LM = 4$ ,  $LN = 9$ ,  $LP = ?$



48. **FINDING A RADIUS** You are 10 feet from a circular storage tank. You are 22 feet from a point of tangency on the tank. Find the tank's radius. (Review 10.1)

**xy USING ALGEBRA**  $\overline{AB}$  and  $\overline{AD}$  are tangent to  $\odot L$ . Find the value of  $x$ . (Review 10.1)

