10.3 Inscribed Angles

What *you should learn*

GOAL 1 Use inscribed angles to solve problems.

GOAL 2 Use properties of inscribed polygons.

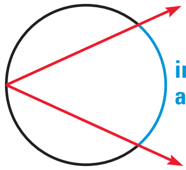
Why *you should learn it*

▼ To solve **real-life** problems, such as finding the different seats in a theater that will give you the same viewing angle, as in **Example 4**.

REAL LIFE

GOAL 1 USING INSCRIBED ANGLES

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

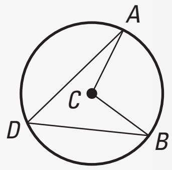


THEOREM

THEOREM 10.8 ***Measure of an Inscribed Angle***

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

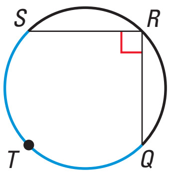
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$



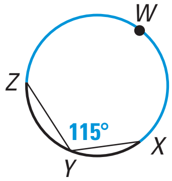
EXAMPLE 1 *Finding Measures of Arcs and Inscribed Angles*

Find the measure of the blue arc or angle.

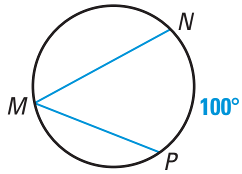
a.



b.



c.



SOLUTION

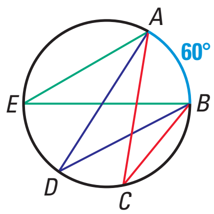
a. $m\overset{\frown}{QTS} = 2m\angle QRS = 2(90°) = 180°$

b. $m\overset{\frown}{ZWX} = 2m\angle ZYX = 2(115°) = 230°$

c. $m\angle NMP = \frac{1}{2}m\overset{\frown}{NP} = \frac{1}{2}(100°) = 50°$

EXAMPLE 2 *Comparing Measures of Inscribed Angles*

Find $m\angle ACB$, $m\angle ADB$, and $m\angle AEB$.



SOLUTION

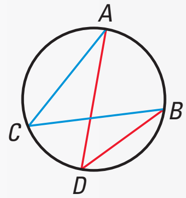
The measure of each angle is half the measure of $\overset{\frown}{AB}$. $m\overset{\frown}{AB} = 60°$, so the measure of each angle is 30°.

Example 2 suggests the following theorem. You are asked to prove Theorem 10.8 and Theorem 10.9 in Exercises 35–38.

THEOREM

THEOREM 10.9

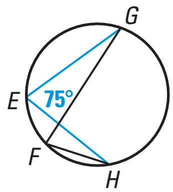
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle C \cong \angle D$

EXAMPLE 3 *Finding the Measure of an Angle*

It is given that $m\angle E = 75°$. What is $m\angle F$?



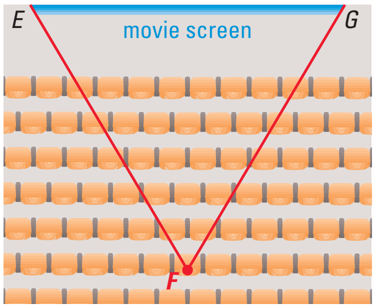
SOLUTION

$\angle E$ and $\angle F$ both intercept $\overset{\frown}{GH}$, so $\angle E \cong \angle F$.

▶ So, $m\angle F = m\angle E = 75°$.

EXAMPLE 4 *Using the Measure of an Inscribed Angle*

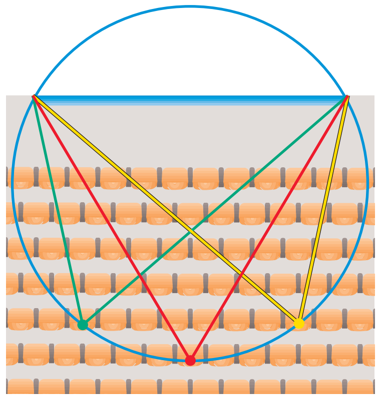
THEATER DESIGN When you go to the movies, you want to be close to the movie screen, but you don't want to have to move your eyes too much to see the edges of the picture. If E and G are the ends of the screen and you are at F, $m\angle EFG$ is called your *viewing angle*.



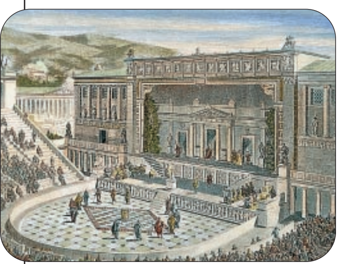
You decide that the middle of the sixth row has the best viewing angle. If someone is sitting there, where else can you sit to have the same viewing angle?

SOLUTION

Draw the circle that is determined by the endpoints of the screen and the sixth row center seat. Any other location on the circle will have the same viewing angle.



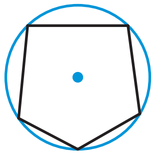
FOCUS ON APPLICATIONS



REAL LIFE **THEATER DESIGN** In Ancient Greece, stages were often part of a circle and the seats were on concentric circles.

GOAL 2 Using Properties of Inscribed Polygons

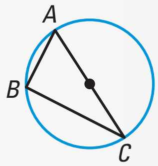
If all of the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an *inscribed polygon* and the circle is a *circumscribed circle*. You are asked to justify Theorem 10.10 and part of Theorem 10.11 in Exercises 39 and 40. A complete proof of Theorem 10.11 appears on page 840.



THEOREMS ABOUT INSCRIBED POLYGONS

THEOREM 10.10

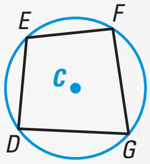
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$\angle B$ is a right angle if and only if $\overline{AC}$ is a diameter of the circle.

THEOREM 10.11

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



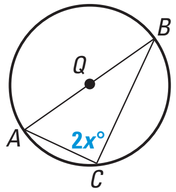
D, E, F, and G lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$.

EXAMPLE 5 *Using Theorems 10.10 and 10.11*

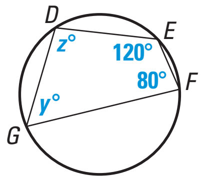
Using Algebra

Find the value of each variable.

a.



b.



Solution

a. $\overline{AB}$ is a diameter. So, $\angle C$ is a right angle and $m\angle C = 90°$.

$2x° = 90°$

$x = 45$

b. *DEFG* is inscribed in a circle, so opposite angles are supplementary.

$m\angle D + m\angle F = 180°$ | $m\angle E + m\angle G = 180°$

$z + 80 = 180$ | $120 + y = 180$

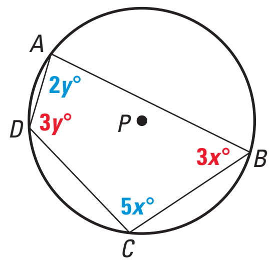
$z = 100$ | $y = 60$

STUDENT HELP

Skills Review
For help with solving systems of equations, see p. 796.

EXAMPLE 6 *Using an Inscribed Quadrilateral*

In the diagram, *ABCD* is inscribed in $\odot P$. Find the measure of each angle.



SOLUTION

ABCD is inscribed in a circle, so opposite angles are supplementary.

$3x + 3y = 180$ $\qquad$ $5x + 2y = 180$

To solve this system of linear equations, you can solve the first equation for y to get $y = 60 - x$. Substitute this expression into the second equation.

$5x + 2y = 180$	**Write second equation.**
$5x + 2(60 - x) = 180$	**Substitute $60 - x$ for y.**
$5x + 120 - 2x = 180$	**Distributive property**
$3x = 60$	**Subtract 120 from each side.**
$x = 20$	**Divide each side by 3.**
$y = 60 - 20 = 40$	**Substitute and solve for y.**

▶ $x = 20$ and $y = 40$, so $m\angle A = 80°$, $m\angle B = 60°$, $m\angle C = 100°$, and $m\angle D = 120°$.

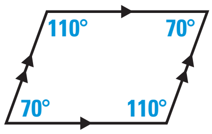
GUIDED PRACTICE

Vocabulary Check ✓

1. Draw a circle and an inscribed angle, $\angle ABC$. Name the intercepted arc of $\angle ABC$. Label additional points on your sketch if you need to.

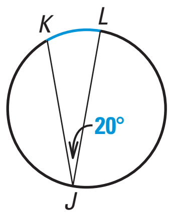
Concept Check ✓

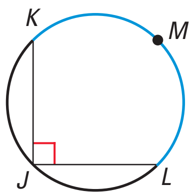
2. Determine whether the quadrilateral can be inscribed in a circle. Explain your reasoning.

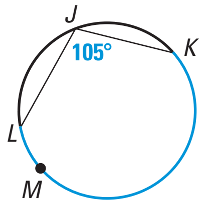


Skill Check ✓

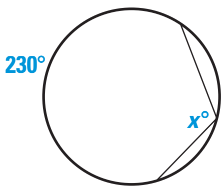
Find the measure of the blue arc.

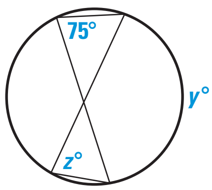
3.


4.


5.


Find the value of each variable.

6.


7.


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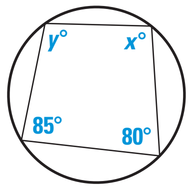
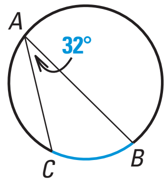


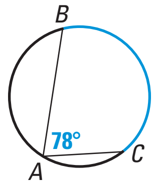
PRACTICE AND APPLICATIONS

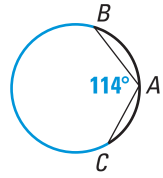
STUDENT HELP

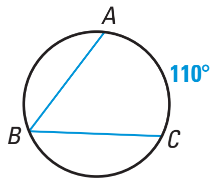
Extra Practice to help you master skills is on p. 821.

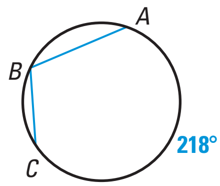
ARC AND ANGLE MEASURES **Find the measure of the blue arc or angle.**

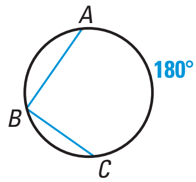
9.


10.


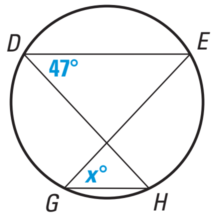
11.


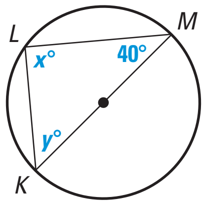
12.


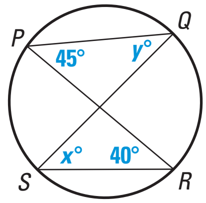
13.


14.


USING ALGEBRA **Find the value of each variable. Explain.**

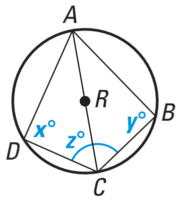
15.


16.


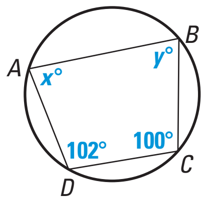
17.


USING ALGEBRA **Find the values of *x*, *y*, and *z*.**

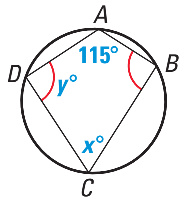
18. $m\overparen{BCD} = 136°$



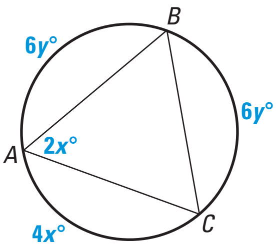
19. $m\overparen{BCD} = z°$

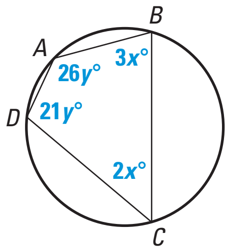


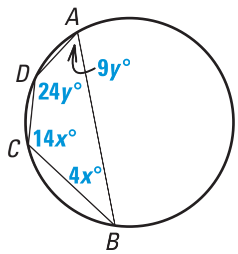
20. $m\overparen{ABC} = z°$



USING ALGEBRA **Find the values of *x* and *y*. Then find the measures of the interior angles of the polygon.**

21.


22.


23.


LOGICAL REASONING **Can the quadrilateral always be inscribed in a circle? Explain your reasoning.**

24. square
25. rectangle
26. parallelogram
27. kite
28. rhombus
29. isosceles trapezoid

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–14, 19–21
Example 2: Exs. 15, 17
Example 3: Exs. 15, 17
Example 4: Exs. 15, 17
Example 5: Exs. 15–20, 24–29, 31–34
Example 6: Exs. 21–23

30. **CONSTRUCTION** Construct a $\odot C$ and a point A on $\odot C$. Construct the tangent to $\odot C$ at A. Explain why your construction works.

CONSTRUCTION In Exercises 31–33, you will construct a tangent to a circle from a point outside the circle.

31. Construct a $\odot C$ and a point outside the circle, A. Draw $\overline{AC}$ and construct its midpoint M. Construct $\odot M$ with radius MC. What kind of chord is $\overline{AC}$?

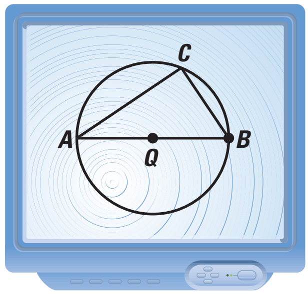
32. $\odot C$ and $\odot M$ have two points of intersection. Label one of the points B. Draw $\overline{AB}$ and $\overline{CB}$. What is $m\angle CBA$? How do you know?

33. Which segment is tangent to $\odot C$ from A? Explain.

STUDENT HELP

INTERNET **SOFTWARE HELP**
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

34. **USING TECHNOLOGY** Use geometry software to construct $\odot Q$, diameter $\overline{AB}$, and point C on $\odot Q$. Construct $\overline{AC}$ and $\overline{CB}$. Measure the angles of $\triangle ABC$. Drag point C along $\odot Q$. Record and explain your observations.

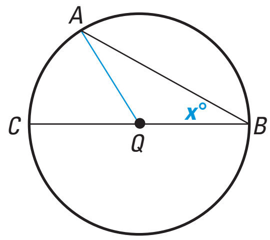


PROVING THEOREM 10.8 If an angle is inscribed in $\odot Q$, the center Q can be on a side of the angle, in the interior of the angle, or in the exterior of the angle. To prove Theorem 10.8, you must prove each of these cases.

35. Fill in the blanks to complete the proof.

GIVEN ▶ $\angle ABC$ is inscribed in $\odot Q$.
Point Q lies on $\overline{BC}$.

PROVE ▶ $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



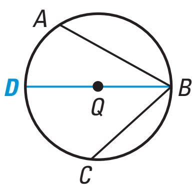
Paragraph Proof Let $m\angle ABC = x°$. Because $\overline{QA}$ and $\overline{QB}$ are both radii of $\odot Q$, $\overline{QA} \cong$ __?__ and $\triangle AQB$ is __?__. Because $\angle A$ and $\angle B$ are __?__ of an isosceles triangle, __?__. So, by substitution, $m\angle A = x°$.

By the __?__ Theorem, $m\angle AQC = m\angle A + m\angle B =$ __?__. So, by the definition of the measure of a minor arc, $m\overset{\frown}{AC} =$ __?__. Divide each side by __?__ to show that $x° =$ __?__. Then, by substitution, $m\angle ABC =$ __?__.

36. Write a plan for a proof.

GIVEN ▶ $\angle ABC$ is inscribed in $\odot Q$.
Point Q is in the interior of $\angle ABC$.

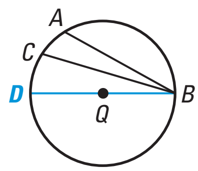
PROVE ▶ $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



37. Write a plan for a proof.

GIVEN ▶ $\angle ABC$ is inscribed in $\odot Q$.
Point Q is in the exterior of $\angle ABC$.

PROVE ▶ $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



38. **PROVING THEOREM 10.9** Write a proof of Theorem 10.9. First draw a diagram and write GIVEN and PROVE statements.

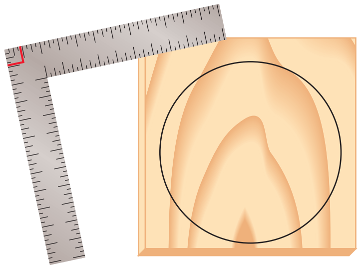
39. **PROVING THEOREM 10.10** Theorem 10.10 is written as a conditional statement and its converse. Write a plan for a proof of each statement.

40. **PROVING THEOREM 10.11** Draw a diagram and write a proof of part of Theorem 10.11.

GIVEN ▶ *DEFG* is inscribed in a circle.

PROVE ▶ $m\angle D + m\angle F = 180°$, $m\angle E + m\angle G = 180°$

41. **CARPENTER'S SQUARE** A carpenter's square is an L-shaped tool used to draw right angles. Suppose you are making a copy of a wooden plate. You trace the plate on a piece of wood. How could you use a carpenter's square to find the center of the circle?

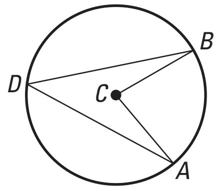


Test Preparation

42. **MULTIPLE CHOICE** In the diagram at the right, if $\angle ACB$ is a central angle and $m\angle ACB = 80°$, what is $m\angle ADB$?

Ⓐ 20° Ⓑ 40° Ⓒ 80°

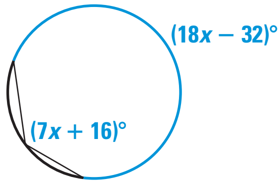
Ⓓ 100° Ⓔ 160°



43. **MULTIPLE CHOICE** In the diagram at the right, what is the value of x?

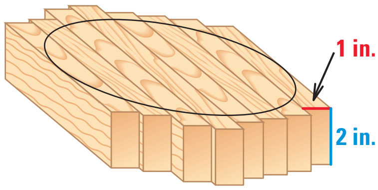
Ⓐ $\frac{48}{11}$ Ⓑ 12 Ⓒ 16

Ⓓ 18 Ⓔ 24



★ Challenge

CUTTING BOARD **In Exercises 44–47, use the following information.**
You are making a circular cutting board. To begin, you glue eight 1 inch by 2 inch boards together, as shown at the right. Then you draw and cut a circle with an 8 inch diameter from the boards.

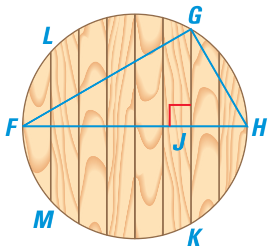


44. $\overline{FH}$ is a diameter of the circular cutting board. What kind of triangle is $\triangle FGH$?

45. How is *GJ* related to *FJ* and *JH*? State a theorem to justify your answer.

46. Find *FJ*, *JH*, and *JG*. What is the length of the seam of the cutting board that is labeled $\overline{GK}$?

47. Find the length of $\overline{LM}$.



EXTRA CHALLENGE
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MIXED REVIEW

WRITING EQUATIONS **Write an equation in slope-intercept form of the line that passes through the given point and has the given slope.** (Review 3.6)

48. $(-2, -6), m = -1$ **49.** $(5, 1), m = 2$ **50.** $(3, 3), m = 0$

51. $(0, 7), m = \frac{4}{3}$ **52.** $(-8, 4), m = -\frac{1}{2}$ **53.** $(-5, -12), m = -\frac{4}{5}$

SKETCHING IMAGES **Sketch the image of $\triangle PQR$ after a composition using the given transformations in the order in which they appear. $\triangle PQR$ has vertices $P(-5, 4)$, $Q(-2, 1)$, and $R(-1, 3)$.** (Review 7.5)

54. **translation:** $(x, y) \to (x + 6, y)$
reflection: in the x-axis

55. **translation:** $(x, y) \to (x + 8, y + 1)$
reflection: in the line $y = 1$

56. **reflection:** in the line $x = 3$
translation: $(x, y) \to (x - 1, y - 7)$

57. **reflection:** in the y-axis
rotation: 90° clockwise about the origin

58. What is the length of an altitude of an equilateral triangle whose sides have lengths of $26\sqrt{2}$? (Review 9.4)

FINDING TRIGONOMETRIC RATIOS **$\triangle ABC$ is a right triangle in which $AB = 4\sqrt{3}$, $BC = 4$, and $AC = 8$.** (Review 9.5 for 10.4)

59. $\sin A = \underline{\ ?\ }$ **60.** $\cos A = \underline{\ ?\ }$

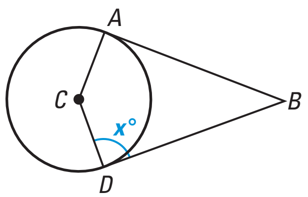
61. $\sin C = \underline{\ ?\ }$ **62.** $\tan C = \underline{\ ?\ }$

QUIZ 1

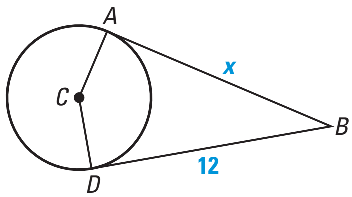
Self-Test for Lessons 10.1–10.3

$\overleftrightarrow{AB}$ is tangent to $\odot C$ at A and $\overleftrightarrow{DB}$ is tangent to $\odot C$ at D. Find the value of x. Write the postulate or theorem that justifies your answer. (Lesson 10.1)

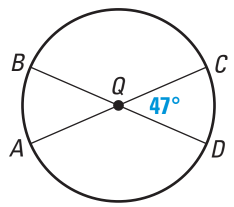
1.



2.



Find the measure of the arc of $\odot Q$. (Lesson 10.2)



3. $\overset{\frown}{AB}$ **4.** $\overset{\frown}{BC}$

5. $\overset{\frown}{ABD}$ **6.** $\overset{\frown}{BCA}$

7. $\overset{\frown}{ADC}$ **8.** $\overset{\frown}{CD}$

9. If an angle that has a measure of 42.6° is inscribed in a circle, what is the measure of its intercepted arc? (Lesson 10.3)