10.2 Arcs and Chords

What *you should learn*

GOAL 1 Use properties of arcs of circles, as applied in **Exs. 49–51**.

GOAL 2 Use properties of chords of circles, as applied in **Ex. 52**.

Why *you should learn it*

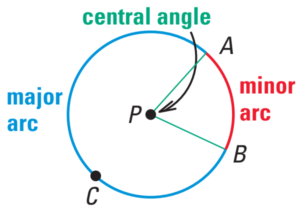
▼ To find the centers of **real-life** arcs, such as the arc of an ax swing in **Example 6**.

REAL LIFE

GOAL 1 Using Arcs of Circles

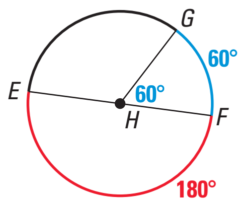
In a plane, an angle whose vertex is the center of a circle is a **central angle** of the circle.

If the measure of a central angle, $\angle APB$, is less than 180°, then A and B and the points of $\odot P$ in the interior of $\angle APB$ form a **minor arc** of the circle. The points A and B and the points of $\odot P$ in the *exterior* of $\angle APB$ form a **major arc** of the circle. If the endpoints of an arc are the endpoints of a diameter, then the arc is a **semicircle**.



Naming Arcs Arcs are named by their endpoints. For example, the minor arc associated with $\angle APB$ above is $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and by a point on the arc. For example, the major arc associated with $\angle APB$ above is $\overset{\frown}{ACB}$. $\overset{\frown}{EGF}$ below is a semicircle.

Measuring Arcs The **measure of a minor arc** is defined to be the measure of its central angle. For instance, $m\overset{\frown}{GF} = m\angle GHF = 60°$. "$m\overset{\frown}{GF}$" is read "the measure of arc GF." You can write the measure of an arc next to the arc. The measure of a semicircle is 180°.



The **measure of a major arc** is defined as the difference between 360° and the measure of its associated minor arc. For example, $m\overset{\frown}{GEF} = 360° - 60° = 300°$. The measure of a whole circle is 360°.

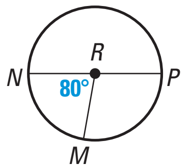
EXAMPLE 1 *Finding Measures of Arcs*

Find the measure of each arc of $\odot R$.

a. $\overset{\frown}{MN}$

b. $\overset{\frown}{MPN}$

c. $\overset{\frown}{PMN}$



Solution

a. $\overset{\frown}{MN}$ is a minor arc, so $m\overset{\frown}{MN} = m\angle MRN = 80°$

b. $\overset{\frown}{MPN}$ is a major arc, so $m\overset{\frown}{MPN} = 360° - 80° = 280°$

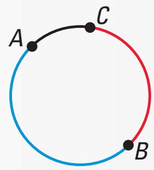
c. $\overset{\frown}{PMN}$ is a semicircle, so $m\overset{\frown}{PMN} = 180°$

Two arcs of the same circle are *adjacent* if they intersect at exactly one point. You can add the measures of adjacent arcs.

POSTULATE

POSTULATE 26 ***Arc Addition Postulate***

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

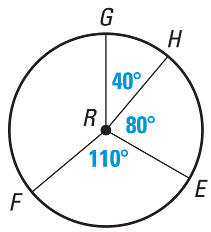


$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

EXAMPLE 2 *Finding Measures of Arcs*

Find the measure of each arc.

a. $\overset{\frown}{GE}$ **b.** $\overset{\frown}{GEF}$ **c.** $\overset{\frown}{GF}$



Solution

a. $m\overset{\frown}{GE} = m\overset{\frown}{GH} + m\overset{\frown}{HE} = 40° + 80° = 120°$

b. $m\overset{\frown}{GEF} = m\overset{\frown}{GE} + m\overset{\frown}{EF} = 120° + 110° = 230°$

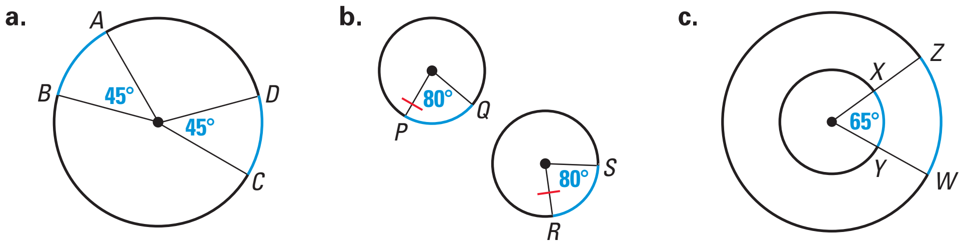
c. $m\overset{\frown}{GF} = 360° - m\overset{\frown}{GEF} = 360° - 230° = 130°$

Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are congruent.

EXAMPLE 3 *Identifying Congruent Arcs*

Logical Reasoning

Find the measures of the blue arcs. Are the arcs congruent?



Solution

a. $\overset{\frown}{AB}$ and $\overset{\frown}{DC}$ are in the same circle and $m\overset{\frown}{AB} = m\overset{\frown}{DC} = 45°$. So, $\overset{\frown}{AB} \cong \overset{\frown}{DC}$.

b. $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are in congruent circles and $m\overset{\frown}{PQ} = m\overset{\frown}{RS} = 80°$. So, $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$.

c. $m\overset{\frown}{XY} = m\overset{\frown}{ZW} = 65°$, but $\overset{\frown}{XY}$ and $\overset{\frown}{ZW}$ are not arcs of the same circle or of congruent circles, so $\overset{\frown}{XY}$ and $\overset{\frown}{ZW}$ are *not* congruent.

GOAL 2 USING CHORDS OF CIRCLES

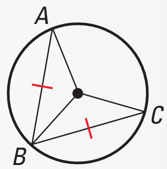
A point Y is called the *midpoint* of $\overset{\frown}{XYZ}$ if $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$. Any line, segment, or ray that contains Y *bisects* $\overset{\frown}{XYZ}$. You will prove Theorems 10.4–10.6 in the exercises.

THEOREMS ABOUT CHORDS OF CIRCLES

THEOREM 10.4

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

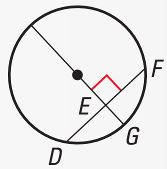
$\overset{\frown}{AB} \cong \overset{\frown}{BC}$ if and only if $\overline{AB} \cong \overline{BC}$.



THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

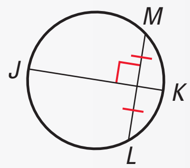
$\overline{DE} \cong \overline{EF}$, $\overset{\frown}{DG} \cong \overset{\frown}{GF}$



THEOREM 10.6

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

$\overline{JK}$ is a diameter of the circle.



EXAMPLE 4 *Using Theorem 10.4*

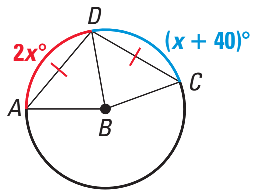
Using Algebra

You can use Theorem 10.4 to find $m\overset{\frown}{AD}$.

Because $\overline{AD} \cong \overline{DC}$, $\overset{\frown}{AD} \cong \overset{\frown}{DC}$. So, $m\overset{\frown}{AD} = m\overset{\frown}{DC}$.

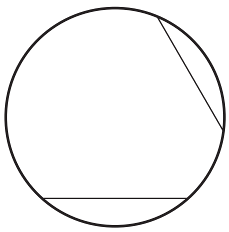
$2x = x + 40$ **Substitute.**

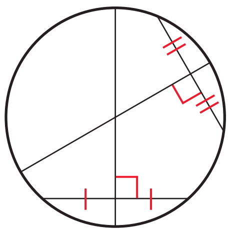
$x = 40$ **Subtract x from each side.**

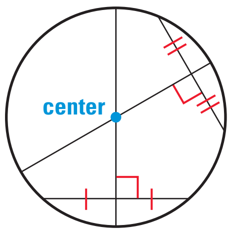


EXAMPLE 5 *Finding the Center of a Circle*

Theorem 10.6 can be used to locate a circle's center, as shown below.





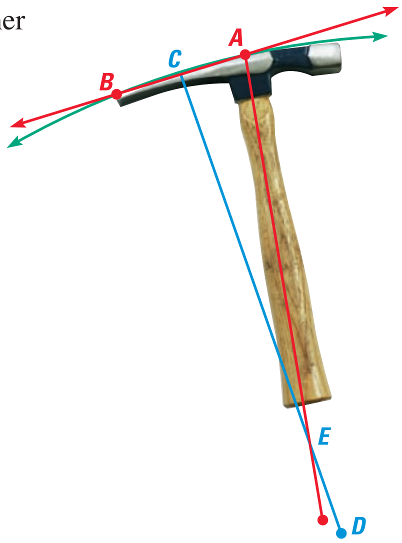


1. Draw any two chords that are not parallel to each other.
2. Draw the perpendicular bisector of each chord. These are diameters.
3. The perpendicular bisectors intersect at the circle's center.

EXAMPLE 6 *Using Properties of Chords*

REAL LIFE

MASONRY HAMMER A masonry hammer has a hammer on one end and a curved pick on the other. The pick works best if you swing it along a circular curve that matches the shape of the pick. Find the center of the circular swing.



SOLUTION

Draw a segment $\overline{AB}$, from the top of the masonry hammer to the end of the pick. Find the midpoint C, and draw a perpendicular bisector $\overline{CD}$. Find the intersection of $\overline{CD}$ with the line formed by the handle.

▶ So, the center of the swing lies at E.

You are asked to prove Theorem 10.7 in Exercises 61 and 62.

STUDENT HELP

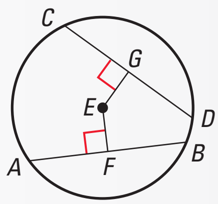
Look Back
Remember that the distance from a point to a line is the length of the perpendicular segment from the point to the line. (p. 266)

THEOREM

THEOREM 10.7

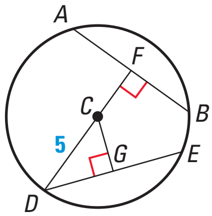
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ if and only if $\overline{EF} \cong \overline{EG}$.



EXAMPLE 7 *Using Theorem 10.7*

$AB = 8$, $DE = 8$, and $CD = 5$. Find CF.



SOLUTION

Because $\overline{AB}$ and $\overline{DE}$ are congruent chords, they are equidistant from the center. So, $\overline{CF} \cong \overline{CG}$. To find CG, first find DG.

$\overline{CG} \perp \overline{DE}$, so $\overline{CG}$ bisects $\overline{DE}$. Because $DE = 8$, $DG = \frac{8}{2} = 4$.

Then use DG to find CG.

$DG = 4$ and $CD = 5$, so $\triangle CGD$ is a 3-4-5 right triangle. So, $CG = 3$.

Finally, use CG to find CF.

▶ Because $\overline{CF} \cong \overline{CG}$, $CF = CG = 3$.

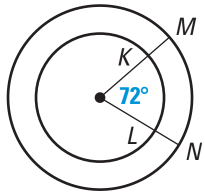
GUIDED PRACTICE

Vocabulary Check ✓

1. The measure of an arc is 170°. Is the arc a *major arc*, a *minor arc*, or a *semicircle*?

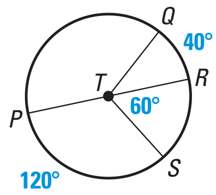
Concept Check ✓

2. In the figure at the right, what is $m\overset{\frown}{KL}$? What is $m\overset{\frown}{MN}$? Are $\overset{\frown}{KL}$ and $\overset{\frown}{MN}$ congruent? Explain.



Skill Check ✓

Find the measure in ⊙*T*.



3. $m\overset{\frown}{RS}$

4. $m\overset{\frown}{RPS}$

5. $m\overset{\frown}{PQR}$

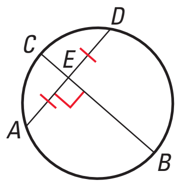
6. $m\overset{\frown}{QS}$

7. $m\overset{\frown}{QSP}$

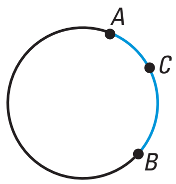
8. $m\angle QTR$

What can you conclude about the diagram? State a postulate or theorem that justifies your answer.

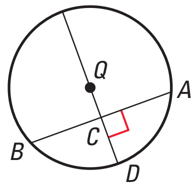
9.



10.



11.

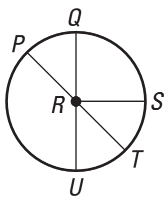


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 821.

UNDERSTANDING THE CONCEPT **Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of ⊙*R*.**



12. $\overset{\frown}{PQ}$

13. $\overset{\frown}{SU}$

14. $\overset{\frown}{PQT}$

15. $\overset{\frown}{QT}$

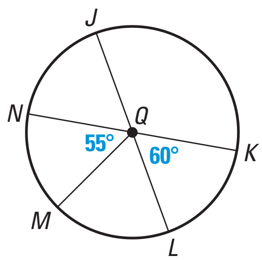
16. $\overset{\frown}{TUQ}$

17. $\overset{\frown}{TUP}$

18. $\overset{\frown}{QUT}$

19. $\overset{\frown}{PUQ}$

MEASURING ARCS AND CENTRAL ANGLES **$\overline{KN}$ and $\overline{JL}$ are diameters. Copy the diagram. Find the indicated measure.**



20. $m\overset{\frown}{KL}$

21. $m\overset{\frown}{MN}$

22. $m\overset{\frown}{LNK}$

23. $m\overset{\frown}{MKN}$

24. $m\overset{\frown}{NJK}$

25. $m\overset{\frown}{JML}$

26. $m\angle JQN$

27. $m\angle MQL$

28. $m\overset{\frown}{JN}$

29. $m\overset{\frown}{ML}$

30. $m\overset{\frown}{JM}$

31. $m\overset{\frown}{LN}$

STUDENT HELP

HOMEWORK HELP

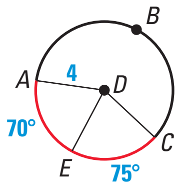
Example 1: Exs. 12–29
Example 2: Exs. 30–34, 49, 50
Example 3: Ex. 35

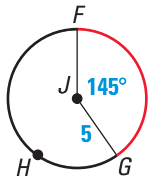
continued on p. 608

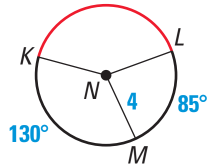
STUDENT HELP

HOMEWORK HELP
continued from p. 607
Example 4: Exs. 36–38
Example 5: Exs. 52, 54
Example 6: Exs. 52, 54
Example 7: Exs. 39–47

FINDING ARC MEASURES **Find the measure of the red arc.**

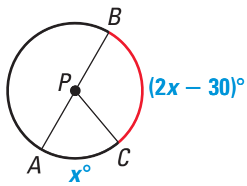
32.


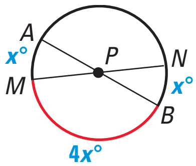
33.


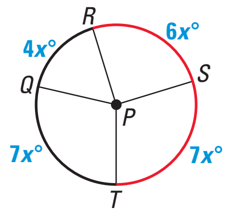
34.


35. Name two pairs of congruent arcs in Exercises 32–34. Explain your reasoning.

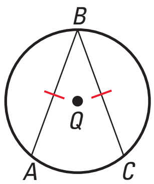
USING ALGEBRA **Use $\odot P$ to find the value of x. Then find the measure of the red arc.**

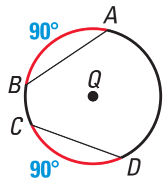
36.


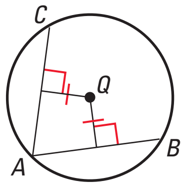
37.


38.


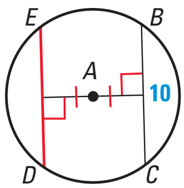
LOGICAL REASONING **What can you conclude about the diagram? State a postulate or theorem that justifies your answer.**

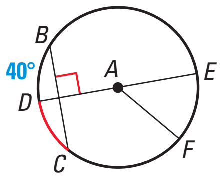
39.


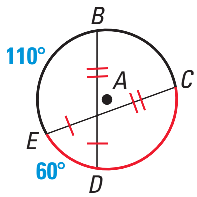
40.


41.


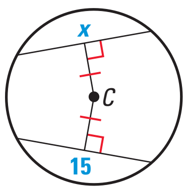
MEASURING ARCS AND CHORDS **Find the measure of the red arc or chord in $\odot A$. Explain your reasoning.**

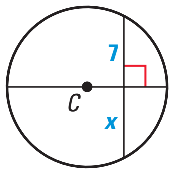
42.


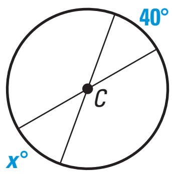
43.


44.


MEASURING ARCS AND CHORDS **Find the value of x in $\odot C$. Explain your reasoning.**

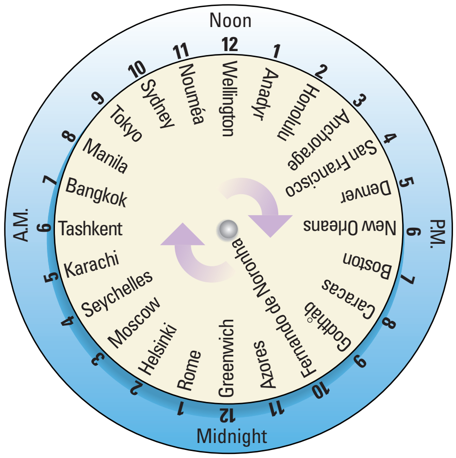
45.


46.


47.


48. SKETCHING Draw a circle with two noncongruent chords. Is the shorter chord's midpoint farther from the center or closer to the center than the longer chord's midpoint?

TIME ZONE WHEEL **In Exercises 49–51, use the following information.** The time zone wheel shown at the right consists of two concentric circular pieces of cardboard fastened at the center so the smaller wheel can rotate. To find the time in Tashkent when it is 4 P.M. in San Francisco, you rotate the small wheel until 4 P.M. and San Francisco line up as shown. Then look at Tashkent to see that it is 6 A.M. there. The arcs between cities are congruent.

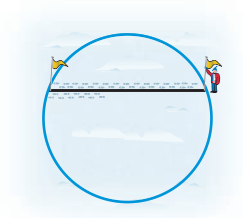


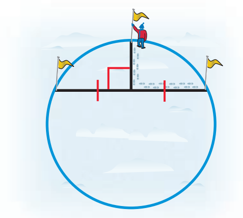
49. What is the arc measure for each time zone on the wheel?

50. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

51. If two cities differ by 180° on the wheel, then it is 3:00 P.M. in one city if and only if it is _?_ in the other city.

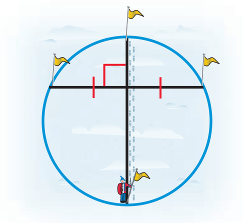
52. **AVALANCHE RESCUE BEACON** An avalanche rescue beacon is a small device carried by backcountry skiers that gives off a signal that can be picked up only within a circle of a certain radius. During a practice drill, a ski patrol uses steps similar to the following to locate a beacon buried in the snow. Write a paragraph explaining why this procedure works. ▶ Source: The Mountaineers

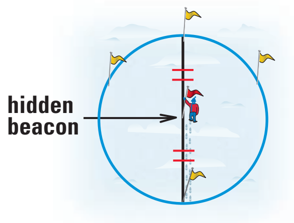




1. Walk until the signal disappears, turn around, and pace the distance in a straight line until the signal disappears again.

2. Pace back to the halfway point, and walk away from the line at a 90° angle until the signal disappears.





3. Turn around and pace the distance in a straight line until the signal disappears again.

4. Pace back to the halfway point. You will be at or near the center of the circle. The beacon is underneath you.

53. **LOGICAL REASONING** Explain why two minor arcs of the same circle or of congruent circles are congruent if and only if their central angles are congruent.

FOCUS ON CAREERS

REAL LIFE **EMTS** Some Emergency Medical Technicians (EMTs) train specifically for wilderness emergencies. These EMTs must be able to improvise with materials they have on hand.

INTERNET **CAREER LINK**
www.mcdougallittell.com

54. **CONSTRUCTION** Trace a circular object like a cup or can. Then use a compass and straightedge to find the center of the circle. Explain your steps.

55. **CONSTRUCTION** Construct a large circle with two congruent chords. Are the chords the same distance from the center? How can you tell?

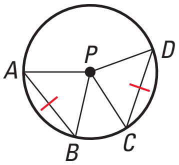
STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 56–62.

PROVING THEOREM 10.4 In Exercises 56 and 57, you will prove Theorem 10.4 for the case in which the two chords are in the same circle. Write a plan for a proof.

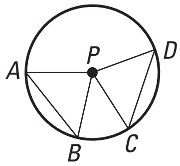
56. **GIVEN ▶** $\overline{AB}$ and $\overline{DC}$ are in $\odot P$.
$\overline{AB} \cong \overline{DC}$

PROVE ▶ $\overset{\frown}{AB} \cong \overset{\frown}{DC}$



57. **GIVEN ▶** $\overline{AB}$ and $\overline{DC}$ are in $\odot P$.
$\overset{\frown}{AB} \cong \overset{\frown}{DC}$

PROVE ▶ $\overline{AB} \cong \overline{DC}$



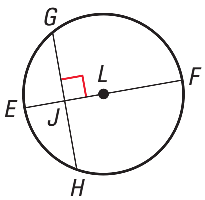
58. **JUSTIFYING THEOREM 10.4** Explain how the proofs in Exercises 56 and 57 would be different if $\overline{AB}$ and $\overline{DC}$ were in congruent circles rather than the same circle.

PROVING THEOREMS 10.5 AND 10.6 Write a proof.

59. **GIVEN ▶** $\overline{EF}$ is a diameter of $\odot L$.
$\overline{EF} \perp \overline{GH}$

PROVE ▶ $\overline{GJ} \cong \overline{JH}$, $\overset{\frown}{GE} \cong \overset{\frown}{EH}$

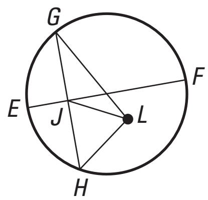
Plan for Proof Draw $\overline{LG}$ and $\overline{LH}$. Use congruent triangles to show $\overline{GJ} \cong \overline{JH}$ and $\angle GLE \cong \angle HLE$. Then show $\overset{\frown}{GE} \cong \overset{\frown}{EH}$.



60. **GIVEN ▶** $\overline{EF}$ is the $\perp$ bisector of $\overline{GH}$.

PROVE ▶ $\overline{EF}$ is a diameter of $\odot L$.

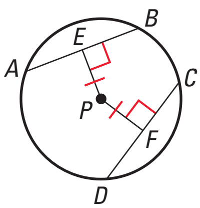
Plan for Proof Use indirect reasoning. Assume center L is not on $\overline{EF}$. Prove that $\triangle GLJ \cong \triangle HLJ$, so $\overline{JL} \perp \overline{GH}$. Then use the Perpendicular Postulate.



PROVING THEOREM 10.7 Write a proof.

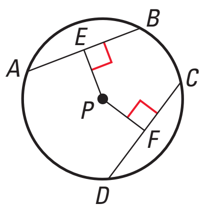
61. **GIVEN ▶** $\overline{PE} \perp \overline{AB}$, $\overline{PF} \perp \overline{DC}$,
$\overline{PE} \cong \overline{PF}$

PROVE ▶ $\overline{AB} \cong \overline{DC}$

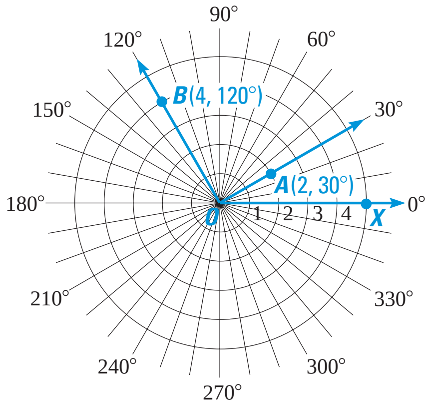


62. **GIVEN ▶** $\overline{PE} \perp \overline{AB}$, $\overline{PF} \perp \overline{DC}$,
$\overline{AB} \cong \overline{DC}$

PROVE ▶ $\overline{PE} \cong \overline{PF}$



POLAR COORDINATES **In Exercises 63–67, use the following information.**
A *polar coordinate system* locates a point in a plane by its distance from the origin O and by the measure of a central angle. For instance, the point $A(2, 30°)$ at the right is 2 units from the origin and $m\angle XOA = 30°$. Similarly, the point $B(4, 120°)$ is 4 units from the origin and $m\angle XOB = 120°$.



63. Use polar graph paper or a protractor and a ruler to graph points A and B. Also graph $C(4, 210°)$, $D(4, 330°)$, and $E(2, 150°)$.

64. Find $m\widehat{AE}$. **65.** Find $m\widehat{BC}$. **66.** Find $m\widehat{BD}$. **67.** Find $m\widehat{BCD}$.

Test Preparation

68. MULTI-STEP PROBLEM You want to find the radius of a circular object. First you trace the object on a piece of paper.

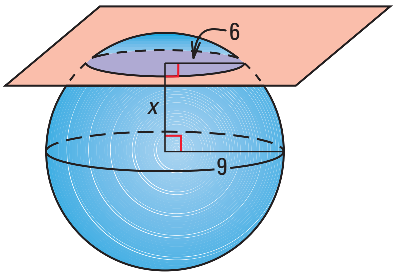
a. Explain how to use two chords that are not parallel to each other to find the radius of the circle.

b. Explain how to use two tangent lines that are not parallel to each other to find the radius of the circle.

c. *Writing* Would the methods in parts (a) and (b) work better for small objects or for large objects? Explain your reasoning.

★ Challenge

69. The plane at the right intersects the sphere in a circle that has a diameter of 12. If the diameter of the sphere is 18, what is the value of x? Give your answer in simplified radical form.



EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

INTERIOR OF AN ANGLE **Plot the points in a coordinate plane and sketch $\angle ABC$. Write the coordinates of a point that lies in the interior and a point that lies in the exterior of $\angle ABC$.** (Review 1.4 for 10.3)

70. $A(4, 2), B(0, 2), C(3, 0)$ **71.** $A(-2, 3), B(0, 0), C(4, -1)$

72. $A(-2, -3), B(0, -1), C(2, -3)$ **73.** $A(-3, 2), B(0, 0), C(3, 2)$

COORDINATE GEOMETRY **The coordinates of the vertices of parallelogram *PQRS* are given. Decide whether ▱*PQRS* is best described as a *rhombus*, a *rectangle*, or a *square*. Explain your reasoning.** (Review 6.4 for 10.3)

74. $P(-2, 1), Q(-1, 4), R(0, 1), S(-1, -2)$

75. $P(-1, 2), Q(2, 5), R(5, 2), S(2, -1)$

GEOMETRIC MEAN **Find the geometric mean of the numbers.** (Review 8.2)

76. 9, 16 **77.** 8, 32 **78.** 4, 49 **79.** 9, 36