

Reteaching with Practice

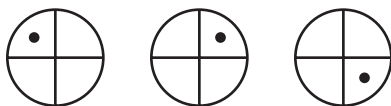
For use with pages 3–9

GOAL

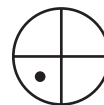
Find and describe patterns and use inductive reasoning

VOCABULARYA **conjecture** is an unproven statement that is based on observations.**Inductive reasoning** is a process that involves looking for patterns and making conjectures.A **counterexample** is an example that shows a conjecture is false.**EXAMPLE 1*****Describing a Visual Pattern***

Sketch the next figure in the pattern.

**SOLUTION**

Each figure looks like the one before it except that it has rotated 90° . The next figure will have the smaller circle in the lower-left quadrant of the bigger circle.

***Exercise for Example 1***

1. Sketch the next figure in the pattern.

**EXAMPLE 2*****Describing a Number Pattern***

Describe a pattern in the sequence of numbers. Predict the next number.

a. 5, 3, 1, -1 , ... b. 1, -4 , 9, -16 , ... c. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...

SOLUTION

- a. These are consecutive odd numbers, but listed backwards starting with 5. The next number is -3 .
- b. These numbers look like consecutive perfect squares, except that every other one is negative. The next number is 25.
- c. Each number is $\frac{1}{2}$ times the previous number. The next number is $\frac{1}{16}$.

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For use with pages 3–9

Exercises for Example 2

Describe a pattern in the sequence of numbers. Predict the next number.

2. 1, 2, 6, 24, ...

3. 0, 3, 8, 15, 24, ...

EXAMPLE 3 Making a Conjecture

Complete the conjecture.

Conjecture: The product of two consecutive even integers is divisible by ____?

SOLUTION

List some specific examples and look for a pattern.

Examples:

$$2 \times 4 = 8 = 8 \times 1$$

$$6 \times 8 = 48 = 8 \times 6$$

$$10 \times 12 = 120 = 8 \times 15$$

$$4 \times 6 = 24 = 8 \times 3$$

$$8 \times 10 = 80 = 8 \times 10$$

$$12 \times 14 = 168 = 8 \times 21$$

Conjecture: The product of two consecutive even integers is divisible by 8.

Exercise for Example 3

Complete the conjecture based on the pattern you observe in the specific cases.

4. **Conjecture:** For any two numbers a and b , the product of $(a + b)$ and $(a - b)$ is always equal to ____?

$$(2 + 1) \times (2 - 1) = 3 = 2^2 - 1^2$$

$$(4 + 2) \times (4 - 2) = 12 = 4^2 - 2^2$$

$$(3 + 2) \times (3 - 2) = 5 = 3^2 - 2^2$$

$$(6 + 3) \times (6 - 3) = 27 = 6^2 - 3^2$$

EXAMPLE 4 Finding a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: All odd numbers are prime.

SOLUTION

The conjecture is false. Here is a counterexample: The number 9 is odd and is a composite number, not a prime number.

Exercise for Example 4

Show the conjecture is false by finding a counterexample.

5. The square of the sum of two numbers is equal to the sum of the squares of the two numbers. That is, $(a + b)^2 = a^2 + b^2$.