1.4

Angles and Their Measures

GOAL 1 USING ANGLE POSTULATES

An **angle** consists of two different rays that have the same initial point. The rays are the **sides** of the angle. The initial point is the **vertex** of the angle.

The angle that has sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is denoted by $\angle BAC$, $\angle CAB$, or $\angle A$. The point A is the vertex of the angle.
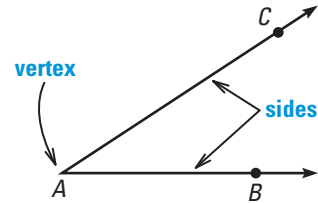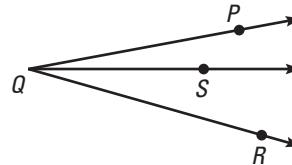
EXAMPLE 1 *Naming Angles*

Name the angles in the figure.

**SOLUTION**

There are three different angles.

- $\angle PQS$ or $\angle SQP$
- $\angle SQR$ or $\angle RQS$
- $\angle PQR$ or $\angle RQP$

You should not name any of these angles as $\angle Q$ because all three angles have Q as their vertex. The name $\angle Q$ would not distinguish one angle from the others.

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The *measure* of $\angle A$ is denoted by $m\angle A$. The measure of an angle can be approximated with a protractor, using units called *degrees* (°). For instance, $\angle BAC$ has a measure of 50°, which can be written as

$$m\angle BAC = 50°.$$
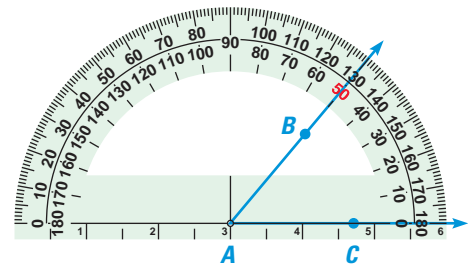
Angles that have the same measure are called **congruent angles**. For instance, $\angle BAC$ and $\angle DEF$ each have a measure of 50°, so they are congruent.
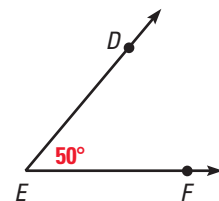
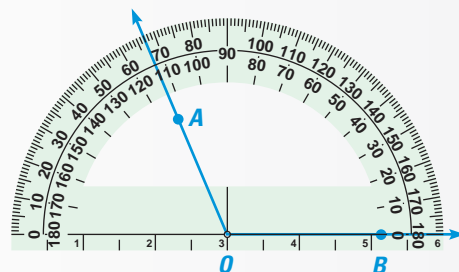| MEASURES ARE EQUAL.	ANGLES ARE CONGRUENT.
$m\angle BAC = m\angle DEF$	$\angle BAC \cong \angle DEF$
"is equal to"	"is congruent to"

POSTULATE

**POSTULATE 3** *Protractor Postulate*

Consider a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

A point is in the **interior** of an angle if it is between points that lie on each side of the angle.

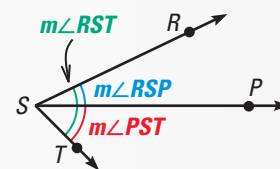A point is in the **exterior** of an angle if it is not on the angle or in its interior.
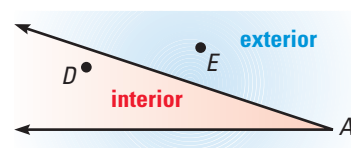
POSTULATE

POSTULATE 4 *Angle Addition Postulate*

If P is in the interior of $\angle RST$, then

$$m\angle RSP + m\angle PST = m\angle RST.$$
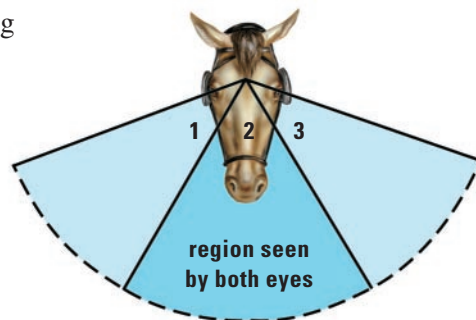
EXAMPLE 2 *Calculating Angle Measures*

VISION Each eye of a horse wearing blinkers has an angle of vision that measures 100°. The angle of vision that is seen by both eyes measures 60°.

Find the angle of vision seen by the left eye alone.

region seen by both eyes

SOLUTION

You can use the Angle Addition Postulate.

$m\angle 2 + m\angle 3 = 100°$ **Total vision for left eye is 100°.**

$m\angle 3 = 100° - m\angle 2$ **Subtract $m\angle 2$ from each side.**

$m\angle 3 = 100° - 60°$ **Substitute 60° for $m\angle 2$.**

$m\angle 3 = 40°$ **Subtract.**
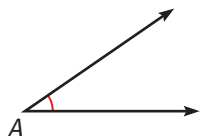
▶ So, the vision for the left eye alone measures 40°.

Angles are classified as **acute**, **right**, **obtuse**, and **straight**, according to their measures. Angles have measures greater than 0° and less than or equal to 180°.
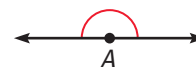

Acute angle	**Right angle**	**Obtuse angle**	**Straight angle**
$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

EXAMPLE 3 *Classifying Angles in a Coordinate Plane*

Plot the points $L(-4, 2)$, $M(-1, -1)$, $N(2, 2)$, $Q(4, -1)$, and $P(2, -4)$. Then measure and classify the following angles as acute, right, obtuse, or straight.
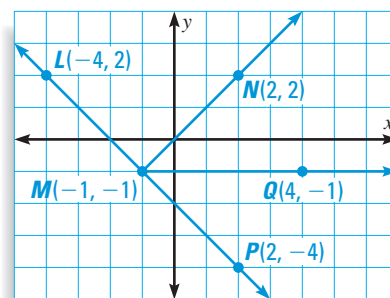
a. $\angle LMN$ **b.** $\angle LMP$ **c.** $\angle NMQ$ **d.** $\angle LMQ$

SOLUTION

Begin by plotting the points. Then use a protractor to measure each angle.

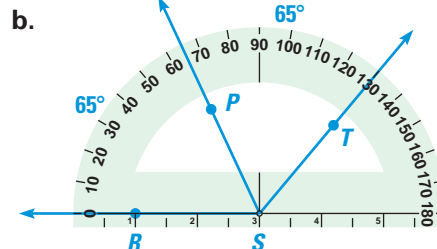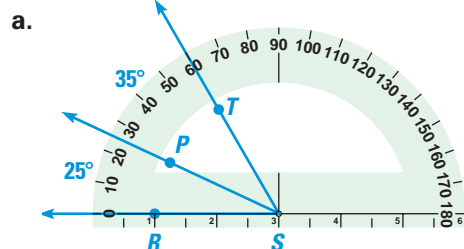| MEASURE	CLASSIFICATION
a. $m\angle LMN = 90°$	right angle
b. $m\angle LMP = 180°$	straight angle
c. $m\angle NMQ = 45°$	acute angle
d. $m\angle LMQ = 135°$	obtuse angle

Two angles are **adjacent angles** if they share a common vertex and side, but have no common interior points.

EXAMPLE 4 *Drawing Adjacent Angles*

Use a protractor to draw two adjacent acute angles $\angle RSP$ and $\angle PST$ so that $\angle RST$ is (**a**) acute and (**b**) obtuse.

SOLUTION
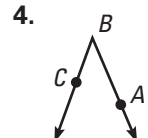
a.

b.

GUIDED PRACTICE

Vocabulary Check ✓ Match the angle with its classification.

A. acute   **B.** obtuse   **C.** right   **D.** straight

1.

2.
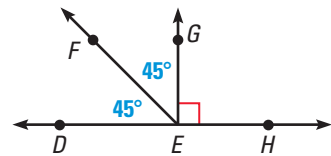

3.

4.
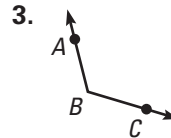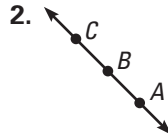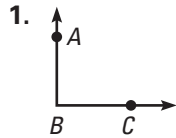
Concept Check ✓ Use the diagram at the right to answer the questions. Explain your answers.

5. Is $\angle DEF \cong \angle FEG$?
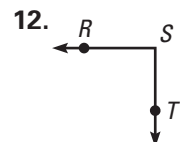
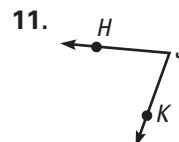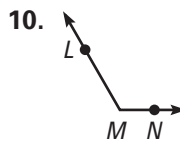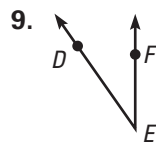**6.** Is $\angle DEG \cong \angle HEG$?

7. Are $\angle DEF$ and $\angle FEH$ adjacent?

8. Are $\angle GED$ and $\angle DEF$ adjacent?

Skill Check ✓ Name the vertex and sides of the angle. Then estimate its measure.

9.

10.

11.

12.

Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

13. $m\angle A = 180°$
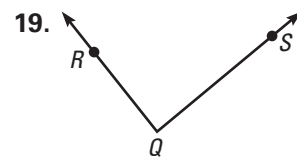
14. $m\angle B = 90°$

15. $m\angle C = 100°$
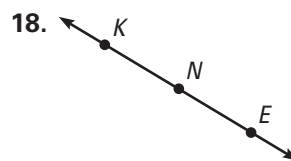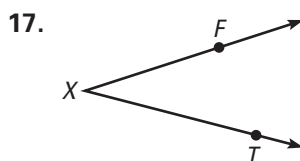
16. $m\angle D = 45°$

PRACTICE AND APPLICATIONS

NAMING PARTS Name the vertex and sides of the angle.

17.

18.

19.

NAMING ANGLES Write two names for the angle.
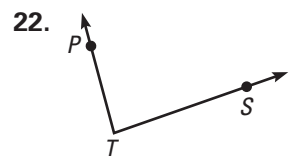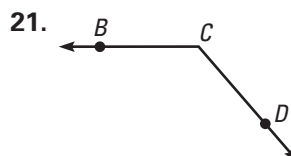
20.

21.

22.

MEASURING ANGLES Copy the angle, extend its sides, and use a protractor to measure it to the nearest degree.

23.
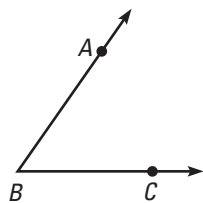
24.

25.

ANGLE ADDITION Use the Angle Addition Postulate to find the measure of the unknown angle.

26. $m\angle ABC = \underline{\ ?\ }$

27. $m\angle DEF = \underline{\ ?\ }$

28. $m\angle PQR = \underline{\ ?\ }$

LOGICAL REASONING Draw a sketch that uses all of the following information.

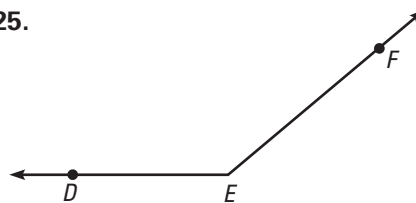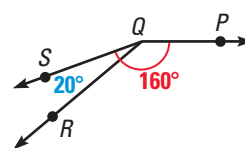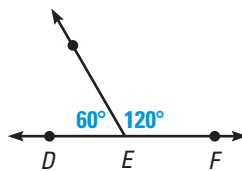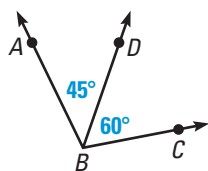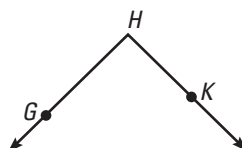|	
D is in the interior of $\angle BAE$.	$m\angle BAC = 130°$
E is in the interior of $\angle DAF$.	$m\angle EAC = 100°$
F is in the interior of $\angle EAC$.	$m\angle BAD = m\angle EAF = m\angle FAC$

29. Find $m\angle FAC$.　　　**30.** Find $m\angle BAD$.　　　**31.** Find $m\angle FAB$.

32. Find $m\angle DAE$.　　　**33.** Find $m\angle FAD$.　　　**34.** Find $m\angle BAE$.

CLASSIFYING ANGLES State whether the angle appears to be *acute*, *right*, *obtuse*, or *straight*. Then estimate its measure.

35.

36.

37.
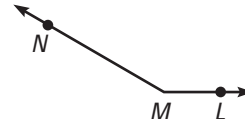
LOGICAL REASONING Draw five points, *A, B, C, D*, and *E* so that all three statements are true.

38. $\angle DBE$ is a straight angle.　　**39.** C is in the interior of $\angle ADE$.
　　$\angle DBA$ is a right angle.　　　　　$m\angle ADC + m\angle CDE = 120°$.
　　$\angle ABC$ is a straight angle.　　　　$\angle CDB$ is a straight angle.

USING ALGEBRA In a coordinate plane, plot the points and sketch $\angle ABC$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle.

40. $A(3, -2)$
　　$B(5, -1)$
　　$C(4, -4)$

41. $A(5, -1)$
　　$B(3, -2)$
　　$C(4, -4)$

42. $A(5, -1)$
　　$B(3, -2)$
　　$C(0, -1)$

43. $A(-3, 1)$
　　$B(-2, 2)$
　　$C(-1, 4)$

🌎 **GEOGRAPHY** For each city on the polar map, estimate the measure of ∠*BOA*, where *B* is on the Prime Meridian (0° longitude), *O* is the North Pole, and *A* is the city.

**44.** Clyde River, Canada   **45.** Fairbanks, Alaska   **46.** Angmagssalik, Greenland

47. Old Crow, Canada   **48.** Reykjavik, Iceland   **49.** Tuktoyaktuk, Canada
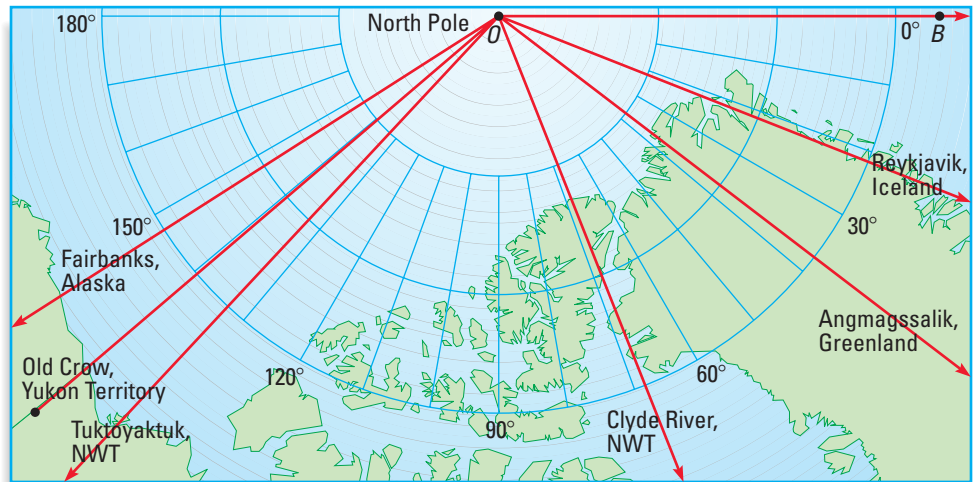

🌎 **PLAYING DARTS** In Exercises 50–53, use the following information to find the score for the indicated dart toss landing at point *A*.

A dartboard is 18 inches across. It is divided into twenty wedges of equal size. The score of a toss is indicated by numbers around the board. The score is doubled if a dart lands in the *double ring* and tripled if it lands in the *triple ring*. Only the top half of the dart board is shown.
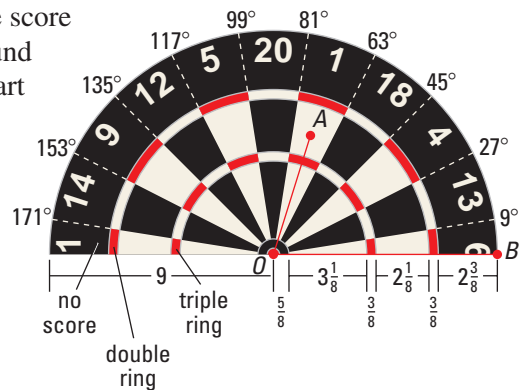
50. $m\angle BOA = 160°$; $AO = 3$ in.

51. $m\angle BOA = 35°$; $AO = 4$ in.

52. $m\angle BOA = 60°$; $AO = 5$ in.

**53.** $m\angle BOA = 90°$; $AO = 6.5$ in.

Test Preparation

54. MULTI-STEP PROBLEM Use a piece of paper folded in half three times and labeled as shown.
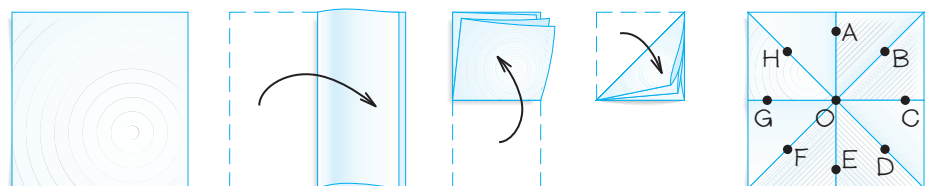
a. Name eight congruent acute angles.

b. Name eight right angles.

c. Name eight congruent obtuse angles.

d. Name two adjacent angles that combine to form a straight angle.

🌐 **AIRPORT RUNWAYS In Exercises 55–60,
use the diagram of Ronald Reagan
Washington National Airport and the
information about runway numbering
on page 1.**
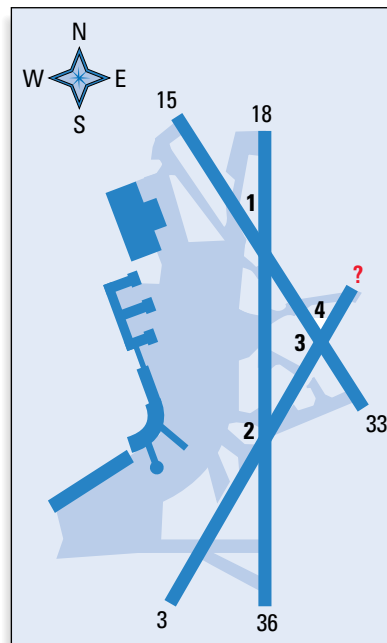
An airport runway is named by dividing its
bearing (the angle measured clockwise from
due north) by 10. Because a full circle contains
360°, runway numbers range from 1 to 36.

55. Find the measure of ∠1.

56. Find the measure of ∠2.

57. Find the measure of ∠3.

58. Find the measure of ∠4.

59. What is the number of the unlabeled
runway in the diagram?

60. *Writing* Explain why the difference
between the numbers at the opposite ends
of a runway is always 18.

MIXED REVIEW

🆇🆈 **USING ALGEBRA Solve for *x*. (Skills Review, p. 790, for 1.5)**

61. $\dfrac{x + 3}{2} = 3$ **62.** $\dfrac{5 + x}{2} = 5$ **63.** $\dfrac{x + 4}{2} = -4$

64. $\dfrac{-8 + x}{2} = 12$ **65.** $\dfrac{x + 7}{2} = -10$ **66.** $\dfrac{-9 + x}{2} = -7$

67. $\dfrac{x + (-1)}{2} = 7$ **68.** $\dfrac{8 + x}{2} = -1$ **69.** $\dfrac{x + (-3)}{2} = -4$

**EVALUATING STATEMENTS Decide whether the statement is *true* or *false*.
(Review 1.2)**

70. *U, S,* and *Q* are collinear.

71. *T, Q, S,* and *P* are coplanar.

72. $\overleftrightarrow{UQ}$ and $\overleftrightarrow{PT}$ intersect.

73. $\overrightarrow{SR}$ and $\overrightarrow{TS}$ are opposite rays.
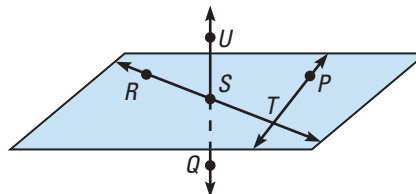
**DISTANCE FORMULA Find the distance between the two points.
(Review 1.3 for 1.5)**

74. $A(3, 10), B(-2, -2)$ **75.** $C(0, 8), D(-8, 3)$

76. $E(-3, 11), F(4, 4)$ **77.** $G(10, -2), H(0, 9)$

78. $J(5, 7), K(7, 5)$ **79.** $L(0, -3), M(-3, 0)$