1.3 Segments and Their Measures

What you should learn

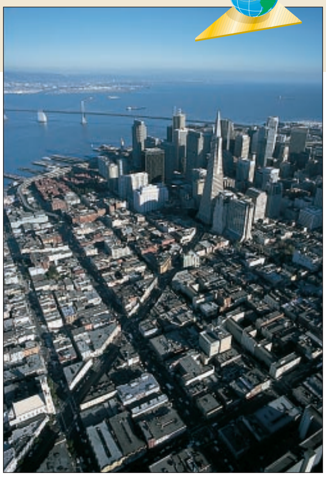
GOAL 1 Use segment postulates.

GOAL 2 Use the Distance Formula to measure distances, as applied in **Exs. 45–54**.

Why you should learn it

▼ To solve **real-life** problems, such as finding distances along a diagonal city street in **Example 4**.

REAL LIFE



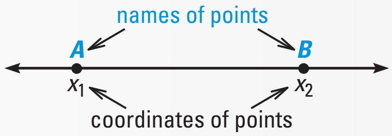
GOAL 1 USING SEGMENT POSTULATES

In geometry, rules that are accepted without proof are called **postulates** or **axioms**. Rules that are proved are called *theorems*. In this lesson, you will study two postulates about the lengths of segments.

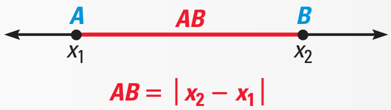
POSTULATE

POSTULATE 1 ***Ruler Postulate***

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B.



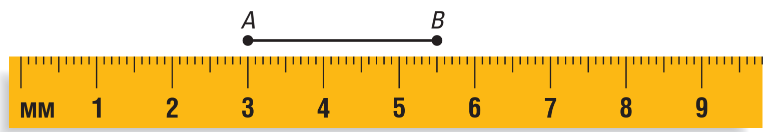
AB is also called the **length** of $\overline{AB}$.

EXAMPLE 1 ***Finding the Distance Between Two Points***

Measure the length of the segment to the nearest millimeter.

SOLUTION

Use a metric ruler. Align one mark of the ruler with A. Then estimate the coordinate of B. For example, if you align A with 3, B appears to align with 5.5.



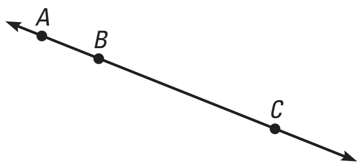
$$AB = |5.5 - 3| = |2.5| = 2.5$$

▶ The distance between A and B is about 2.5 cm.

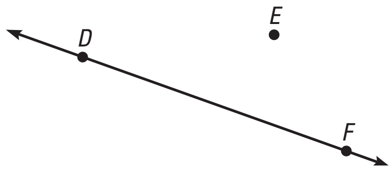
. . . .

It doesn't matter how you place the ruler. For example, if the ruler in Example 1 is placed so that A is aligned with 4, then B aligns with 6.5. The difference in the coordinates is the same.

When three points lie on a line, you can say that one of them is **between** the other two. This concept applies to collinear points only. For instance, in the figures below, point B is between points A and C, but point E is not between points D and F.



Point *B* is between points *A* and *C*.



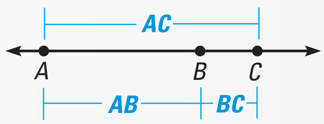
Point *E* is not between points *D* and *F*.

POSTULATE

POSTULATE 2 ***Segment Addition Postulate***

If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 2 *Finding Distances on a Map*

MAP READING Use the map to find the distances between the three cities that lie on a line.

SOLUTION

Using the scale on the map, you can estimate that the distance between Athens and Macon is

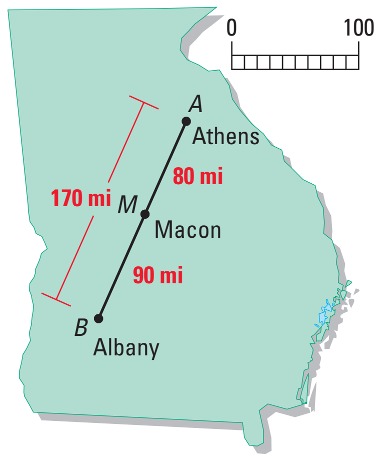
$AM = 80$ miles.

The distance between Macon and Albany is

$MB = 90$ miles.

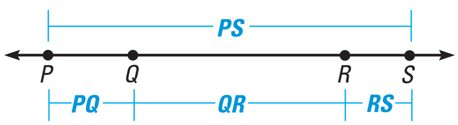
Knowing that Athens, Macon, and Albany lie on the same line, you can use the Segment Addition Postulate to conclude that the distance between Athens and Albany is

$AB = AM + MB = 80 + 90 = 170$ miles.



The Segment Addition Postulate can be generalized to three or more segments, as long as the segments lie on a line. If P, Q, R, and S lie on a line as shown, then

$PS = PQ + QR + RS$.



GOAL 2 Using the Distance Formula

STUDENT HELP

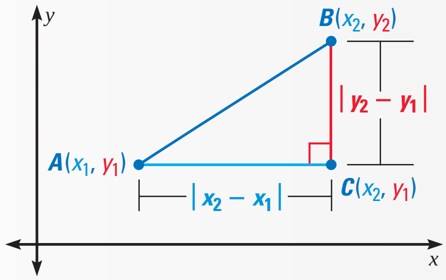
Study Tip
The small numbers in x_1 and x_2 are called *subscripts*. You read them as "x sub 1" and "x sub 2."

The **Distance Formula** is a formula for computing the distance between two points in a *coordinate* plane.

THE DISTANCE FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

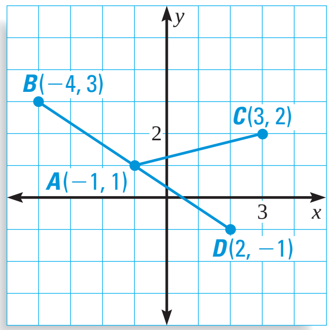
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



EXAMPLE 3 *Using the Distance Formula*

Using Algebra

Find the lengths of the segments. Tell whether any of the segments have the same length.



SOLUTION

Use the Distance Formula.

$$AB = \sqrt{[(-4) - (-1)]^2 + (3 - 1)^2}$$
$$= \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$AC = \sqrt{[3 - (-1)]^2 + (2 - 1)^2}$$
$$= \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$AD = \sqrt{[2 - (-1)]^2 + (-1 - 1)^2}$$
$$= \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

▶ So, $\overline{AB}$ and $\overline{AD}$ have the same length, but $\overline{AC}$ has a different length.

Segments that have the same length are called **congruent segments**. For instance, in Example 3, $\overline{AB}$ and $\overline{AD}$ are congruent because each has a length of $\sqrt{13}$. There is a special symbol, $\cong$, for indicating *congruence*.

LENGTHS ARE EQUAL.	SEGMENTS ARE CONGRUENT.
$AB = AD$	$\overline{AB} \cong \overline{AD}$
"is equal to"	"is congruent to"

The Distance Formula is based on the *Pythagorean Theorem,* which you will see again when you work with right triangles in Chapter 9.

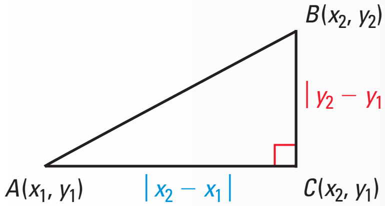
STUDENT HELP

Study Tip
The red mark at one corner of each triangle indicates a right angle.

CONCEPT SUMMARY — **DISTANCE FORMULA AND PYTHAGOREAN THEOREM**

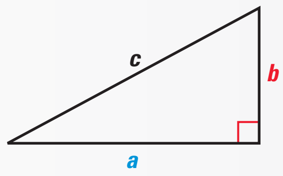
DISTANCE FORMULA

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



PYTHAGOREAN THEOREM

$$c^2 = a^2 + b^2$$

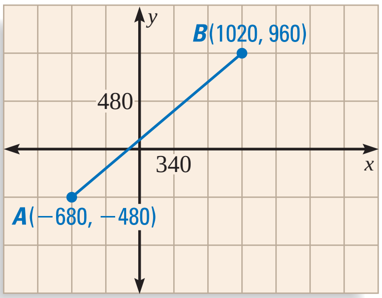


EXAMPLE 4 *Finding Distances on a City Map*

MAP READING On the map, the city blocks are 340 feet apart east-west and 480 feet apart north-south.

a. Find the walking distance between A and B.

b. What would the distance be if a diagonal street existed between the two points?



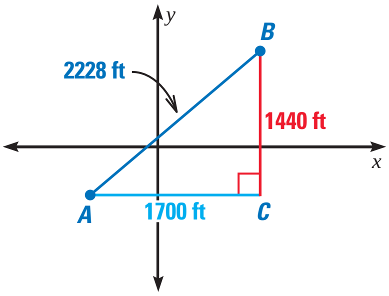
SOLUTION

a. To walk from A to B, you would have to walk five blocks east and three blocks north.

$$5 \text{ blocks} \cdot 340 \frac{\text{feet}}{\text{block}} = 1700 \text{ feet}$$

$$3 \text{ blocks} \cdot 480 \frac{\text{feet}}{\text{block}} = 1440 \text{ feet}$$

▶ So, the walking distance is 1700 + 1440, which is a total of **3140** feet.



b. To find the diagonal distance between A and B, use the Distance Formula.

$$AB = \sqrt{[1020 - (-680)]^2 + [960 - (-480)]^2}$$

$$= \sqrt{1700^2 + 1440^2}$$

$$= \sqrt{4{,}963{,}600} \approx 2228 \text{ feet}$$

▶ So, the diagonal distance would be about 2228 feet, which is 912 feet less than the walking distance.

STUDENT HELP

Study Tip
If you use a calculator to compute distances, use the parenthesis keys to group what needs to be squared.

GUIDED PRACTICE

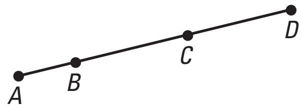
Vocabulary Check ✓

1. What is a *postulate*?

Concept Check ✓

2. Draw a sketch of three collinear points. Label them. Then write the Segment Addition Postulate for the points.

3. Use the diagram. How can you determine BD if you know BC and CD? if you know AB and AD?



Skill Check ✓

Find the distance between the two points.

4. $C(0, 0), D(5, 2)$ **5.** $G(3, 0), H(8, 10)$ **6.** $M(1, -3), N(3, 5)$

7. $P(-8, -6), Q(-3, 0)$ **8.** $S(7, 3), T(1, -5)$ **9.** $V(-2, -6), W(1, -2)$

Use the Distance Formula to decide whether $\overline{JK} \cong \overline{KL}$.

10. $J(3, -5)$
$K(-1, 2)$
$L(-5, -5)$

11. $J(0, -8)$
$K(4, 3)$
$L(-2, -7)$

12. $J(10, 2)$
$K(7, -3)$
$L(4, -8)$

PRACTICE AND APPLICATIONS

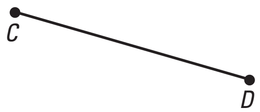
STUDENT HELP

Extra Practice
to help you master skills is on p. 803.

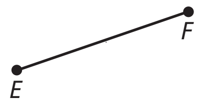
MEASUREMENT **Measure the length of the segment to the nearest millimeter.**

13.

14.



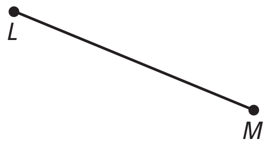
15.



16.

17.

18.



BETWEENNESS **Draw a sketch of the three collinear points. Then write the Segment Addition Postulate for the points.**

19. E is between D and F. **20.** H is between G and J.

21. M is between N and P. **22.** R is between Q and S.

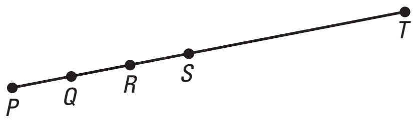
LOGICAL REASONING **In the diagram of the collinear points, $PT = 20$, $QS = 6$, and $PQ = QR = RS$. Find each length.**

23. QR **24.** RS

25. PQ **26.** ST

27. RP **28.** RT

29. SP **30.** QT



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–18
Example 2: Exs. 19–33
Example 3: Exs. 34–43
Example 4: Exs. 44–54

USING ALGEBRA **Suppose M is between L and N. Use the Segment Addition Postulate to solve for the variable. Then find the lengths of $\overline{LM}$, $\overline{MN}$, and $\overline{LN}$.**

31. $LM = 3x + 8$
$MN = 2x - 5$
$LN = 23$

32. $LM = 7y + 9$
$MN = 3y + 4$
$LN = 143$

33. $LM = \frac{1}{2}z + 2$
$MN = 3z + \frac{3}{2}$
$LN = 5z + 2$

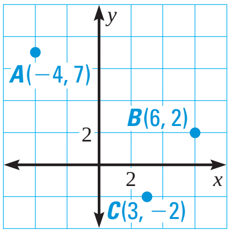
STUDENT HELP

INTERNET

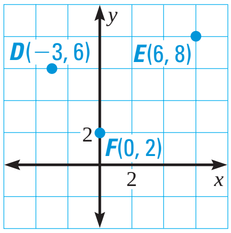
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 34–36.

DISTANCE FORMULA **Find the distance between each pair of points.**

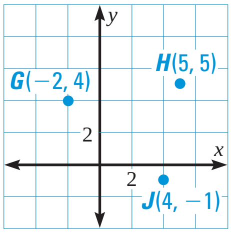
34.



35.

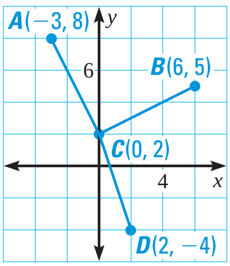


36.

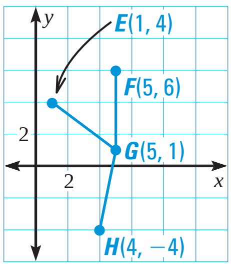


DISTANCE FORMULA **Find the lengths of the segments. Tell whether any of the segments have the same length.**

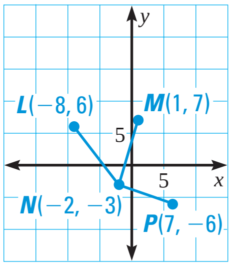
37.



38.



39.



CONGRUENCE **Use the Distance Formula to decide whether $\overline{PQ} \cong \overline{QR}$.**

40. $P(4, -4)$
$Q(1, -6)$
$R(-1, -3)$

41. $P(-1, -6)$
$Q(-8, 5)$
$R(3, -2)$

42. $P(5, 1)$
$Q(-5, -7)$
$R(-3, 6)$

43. $P(-2, 0)$
$Q(10, -14)$
$R(-4, -2)$

CAMBRIA INCLINE **In Exercises 44 and 45, use the information about the incline railway given below.**
In the days before automobiles were available, railways called “inclines” brought people up and down hills in many cities. In Johnstown, Pennsylvania, the Cambria Incline was reputedly the steepest in the world when it was completed in 1893. It rises about 514 feet vertically as it moves 734 feet horizontally.

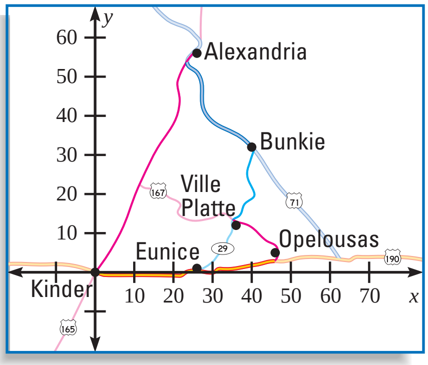
44. On graph paper, draw a coordinate plane and mark the axes using a scale that allows you to plot (0, 0) and (734, 514). Plot the points and connect them with a segment to represent the incline track.

45. Use the Distance Formula to estimate the length of the track.

Workers constructing the Cambria Incline

DRIVING DISTANCES In Exercises 46 and 47, use the map of cities in Louisiana shown below. Coordinates on the map are given in miles.

The coordinates of Alexandria, Kinder, Eunice, Opelousas, Ville Platte, and Bunkie are $A(26, 56)$, $K(0, 0)$, $E(26, 1)$, $O(46, 5)$, $V(36, 12)$, and $B(40, 32)$.



46. What is the shortest flying distance between Eunice and Alexandria?

47. Using only roads shown on the map, what is the approximate shortest driving distance between Eunice and Alexandria?

LONG-DISTANCE RATES In Exercises 48–52, find the distance between the two cities using the information given in the table, which is from a coordinate system used for calculating long-distance telephone rates.

Buffalo, NY	(5075, 2326)	**Omaha, NE**	(6687, 4595)
Chicago, IL	(5986, 3426)	**Providence, RI**	(4550, 1219)
Dallas, TX	(8436, 4034)	**San Diego, CA**	(9468, 7629)
Miami, FL	(8351, 527)	**Seattle, WA**	(6336, 8896)

48. Buffalo and Dallas

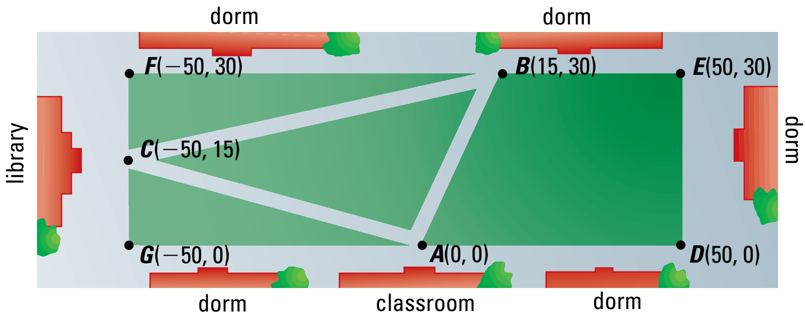
49. Chicago and Seattle

50. Miami and Omaha

51. Providence and San Diego

52. The long-distance coordinate system is measured in units of $\sqrt{0.1}$ mile. Convert the distances you found in Exs. 48–51 to miles.

CAMPUS PATHWAYS In Exercises 53 and 54, use the campus map below. Sidewalks around the edge of a campus quadrangle connect the buildings. Students sometimes take shortcuts by walking across the grass along the pathways shown. The coordinate system shown is measured in yards.



53. Find the distances from A to B, from B to C, and from C to A if you have to walk around the quadrangle along the sidewalks.

54. Find the distances from A to B, from B to C, and from C to A if you are able to walk across the grass along the pathways.

Test Preparation

55. **MULTIPLE CHOICE** Points K and L are on $\overline{AB}$. If $AK > BL$, then which statement must be true?

Ⓐ $AK < KB$ Ⓑ $AL < LB$ Ⓒ $AL > BK$

Ⓓ $KL < LB$ Ⓔ $AL + BK > AB$

56. **MULTIPLE CHOICE** Suppose point M lies on $\overline{CD}$, $CM = 2 \cdot MD$, and $CD = 18$. What is the length of MD?

Ⓐ 3 Ⓑ 6 Ⓒ 9 Ⓓ 12 Ⓔ 36

★ Challenge

THREE-DIMENSIONAL DISTANCE In Exercises 57–59, use the following information to find the distance between the pair of points.
In a three-dimensional coordinate system, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

EXTRA CHALLENGE
www.mcdougallittell.com

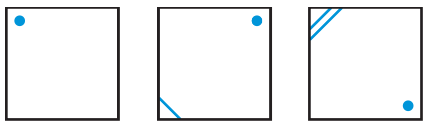
57. $P(0, 20, -32)$
$Q(2, -10, -20)$

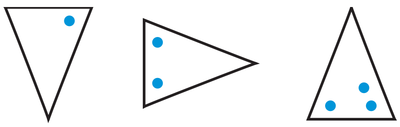
58. $A(-8, 15, -4)$
$B(10, 1, -6)$

59. $F(4, -42, 60)$
$G(-7, -11, 38)$

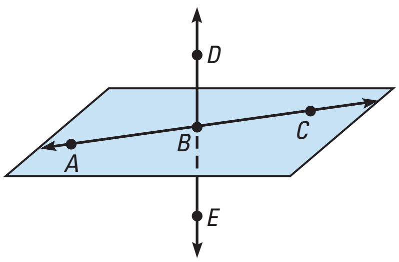
MIXED REVIEW

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. (Review 1.1)

60. 

61. 

EVALUATING STATEMENTS Determine if the statement is *true* or *false*. (Review 1.2)



62. E lies on $\overleftrightarrow{BD}$.

63. E lies on $\overrightarrow{BD}$.

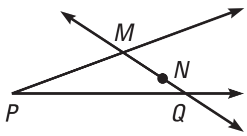
64. A, B, and D are collinear.

65. $\overrightarrow{BD}$ and $\overrightarrow{BE}$ are opposite rays.

66. B lies in plane ADC.

67. The intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{AC}$ is B.

NAMING RAYS Name the ray described. (Review 1.2 for 1.4)



68. Name a ray that contains M.

69. Name a ray that has N as an endpoint.

70. Name two rays that intersect at P.

71. Name a pair of opposite rays.

QUIZ 1

Self-Test for Lessons 1.1–1.3

Write the next number in the sequence. (Lesson 1.1)

1. 10, 9.5, 9, 8.5, . . .

2. 0, 2, -2, 4, -4, . . .

Sketch the figure described. (Lesson 1.2)

3. Two segments that do not intersect.

4. Two lines that do not intersect, and a third line that intersects each of them.

5. Two lines that intersect a plane at the same point.

6. Three planes that do not intersect.

7. **MINIATURE GOLF** At a miniature golf course, a water hazard blocks the direct shot from the tee at $T(0, 0)$ to the cup at $C(-1, 7)$. If you hit the ball so it bounces off an angled wall at $B(3, 4)$, it will go into the cup. The coordinate system is measured in feet. Draw a diagram of the situation. Find TB and BC. (Lesson 1.3)

MATH & History — Geometric Constructions

APPLICATION LINK www.mcdougallittell.com

THEN

MORE THAN 2000 YEARS AGO, the Greek mathematician Euclid published a 13 volume work called *The Elements*. In his systematic approach, figures are *constructed* using only a *compass* and a *straightedge* (a ruler without measuring marks).

An early printed edition of *The Elements*

NOW

TODAY, geometry software may be used to construct geometric figures. Programs allow you to perform constructions as if you have only a compass and straightedge. They also let you make measurements of lengths, angles, and areas.

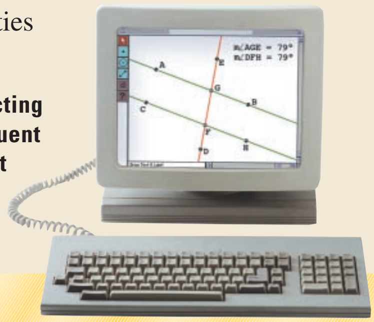
1. Draw two points and use a *straightedge* to construct the line that passes through them.

2. With the points as centers, use a *compass* to draw two circles of different sizes so that the circles intersect in two points. Mark the two points of intersection and construct the line through them.

3. Connect the four points you constructed. What are the properties of the shape formed?

c. 300 B.C. — Euclid develops The Elements.

1796 — Gauss proves constructing a shape with 17 congruent sides and 17 congruent angles is possible.



1990s — Geometry software duplicates the tools for construction on screen.