

1.2

Points, Lines, and Planes

GOAL 1 USING UNDEFINED TERMS AND DEFINITIONS

What you should learn

GOAL 1 Understand and use the basic undefined terms and defined terms of geometry.

GOAL 2 Sketch the intersections of lines and planes.

Why you should learn it

▼ To name and draw the basic elements of geometry, including lines that intersect, as in the perspective drawing in Exs. 68–72.

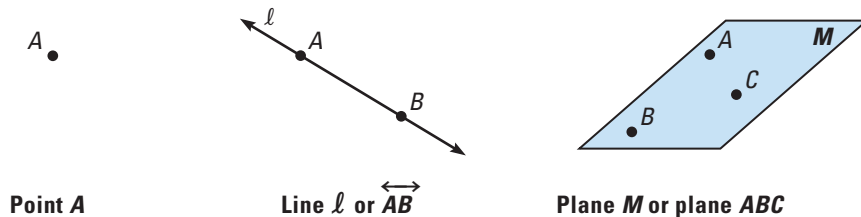


A **definition** uses known words to describe a new word. In geometry, some words, such as *point*, *line*, and *plane*, are **undefined terms**. Although these words are not formally defined, it is important to have general agreement about what each word means.

A **point** has no dimension. It is usually represented by a small dot.

A **line** extends in one dimension. It is usually represented by a straight line with two arrowheads to indicate that the line extends without end in two directions. In this book, lines are always straight lines.

A **plane** extends in two dimensions. It is usually represented by a shape that looks like a tabletop or wall. You must imagine that the plane extends without end, even though the drawing of a plane appears to have edges.



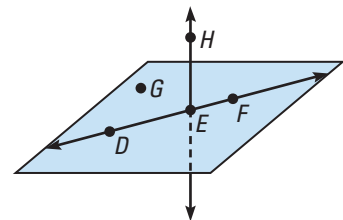
A few basic concepts in geometry must also be commonly understood without being defined. One such concept is the idea that a point *lies on* a line or a plane.

Collinear points are points that lie on the same line.

Coplanar points are points that lie on the same plane.

EXAMPLE 1 Naming Collinear and Coplanar Points

- Name three points that are collinear.
- Name four points that are coplanar.
- Name three points that are not collinear.



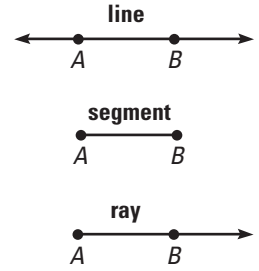
SOLUTION

- Points D , E , and F lie on the same line, so they are collinear.
- Points D , E , F , and G lie on the same plane, so they are coplanar. Also, D , E , F , and H are coplanar, although the plane containing them is not drawn.
- There are many correct answers. For instance, points H , E , and G do not lie on the same line.



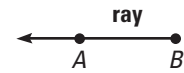
Another undefined concept in geometry is the idea that a point on a line is *between* two other points on the line. You can use this idea to define other important terms in geometry.

Consider the **line** AB (symbolized by \overleftrightarrow{AB}). The **line segment** or **segment** AB (symbolized by \overline{AB}) consists of the **endpoints** A and B , and all points on \overleftrightarrow{AB} that are between A and B .



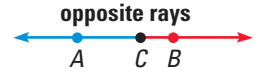
The **ray** AB (symbolized by \overrightarrow{AB}) consists of the **initial point** A and all points on \overleftrightarrow{AB} that lie on the same side of A as point B .

Note that \overleftrightarrow{AB} is the same as \overleftrightarrow{BA} , and \overline{AB} is the same as \overline{BA} . However, \overrightarrow{AB} and \overrightarrow{BA} are *not* the same.



They have different initial points and extend in different directions.

If C is between A and B , then \overrightarrow{CA} and \overrightarrow{CB} are **opposite rays**.

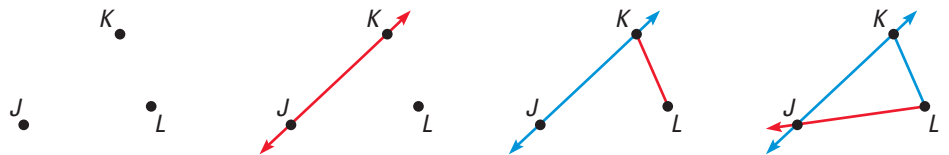


Like points, segments and rays are collinear if they lie on the same line. So, any two opposite rays are collinear. Segments, rays, and lines are coplanar if they lie on the same plane.

EXAMPLE 2 Drawing Lines, Segments, and Rays

Draw three noncollinear points, J , K , and L . Then draw \overleftrightarrow{JK} , \overline{KL} , and \overrightarrow{LJ} .

SOLUTION



- 1 Draw J , K , and L .
- 2 Draw \overleftrightarrow{JK} .
- 3 Draw \overline{KL} .
- 4 Draw \overrightarrow{LJ} .

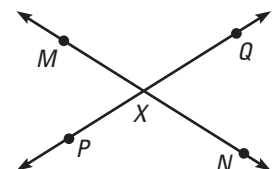
EXAMPLE 3 Drawing Opposite Rays

Draw two lines. Label points on the lines and name two pairs of opposite rays.

SOLUTION

Points M , N , and X are collinear and X is between M and N . So, \overrightarrow{XM} and \overrightarrow{XN} are opposite rays.

Points P , Q , and X are collinear and X is between P and Q . So, \overrightarrow{XP} and \overrightarrow{XQ} are opposite rays.



GOAL 2 SKETCHING INTERSECTIONS OF LINES AND PLANES

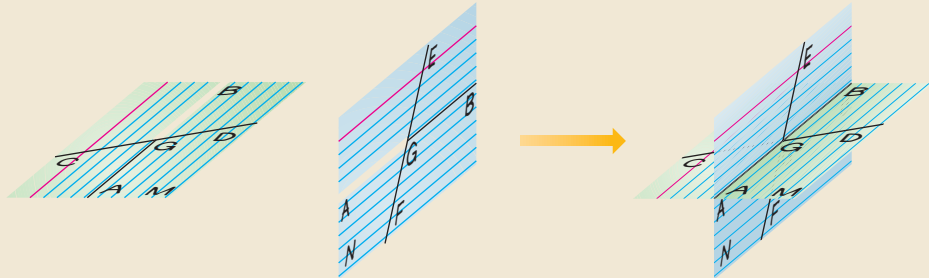
Two or more geometric figures **intersect** if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common.

ACTIVITY

Developing Concepts

Modeling Intersections

Use two index cards. Label them as shown and cut slots halfway along each card.



1. What is the intersection of \overline{AB} and \overline{CD} ? of \overline{AB} and \overline{EF} ?
2. Slide the cards together. What is the intersection of \overline{CD} and \overline{EF} ?
3. What is the intersection of planes M and N ?
4. Are \overleftrightarrow{CD} and \overleftrightarrow{EF} coplanar? Explain.

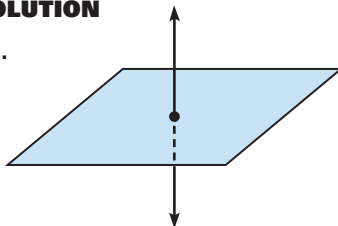
EXAMPLE 4 Sketching Intersections

Sketch the figure described.

- a. a line that intersects a plane in one point
- b. two planes that intersect in a line

SOLUTION

a.

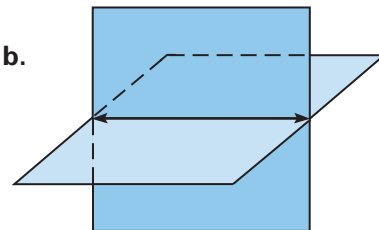


Draw a plane and a line.

Emphasize the point where they meet.

Dashes indicate where the line is hidden by the plane.

b.



Draw two planes.

Emphasize the line where they meet.

Dashes indicate where one plane is hidden by the other plane.

STUDENT HELP



HOMEWORK HELP

Visit our Web site
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 for extra examples.

NAMING COPLANAR POINTS Name a point that is coplanar with the given points.

28. $A, B,$ and C

29. $D, C,$ and F

30. $G, A,$ and D

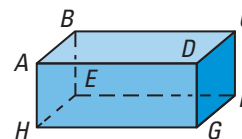
31. $E, F,$ and G

32. $A, B,$ and H

33. $B, C,$ and F

34. $A, B,$ and F

35. $B, C,$ and G



NAMING NONCOPLANAR POINTS Name all the points that are not coplanar with the given points.

36. $N, K,$ and L

37. $S, P,$ and M

38. $P, Q,$ and N

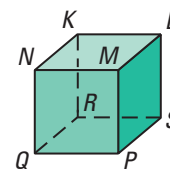
39. $R, S,$ and L

40. $P, Q,$ and R

41. $R, K,$ and N

42. $P, S,$ and K

43. $Q, K,$ and L



COMPLETING DEFINITIONS Complete the sentence.

44. \overline{AB} consists of the endpoints A and B and all the points on the line \overleftrightarrow{AB} that lie ?

45. \overrightarrow{CD} consists of the initial point C and all points on the line \overleftrightarrow{CD} that lie ?

46. Two rays or segments are collinear if they ?

47. \overrightarrow{CA} and \overrightarrow{CB} are opposite rays if ?

SKETCHING FIGURES Sketch the lines, segments, and rays.

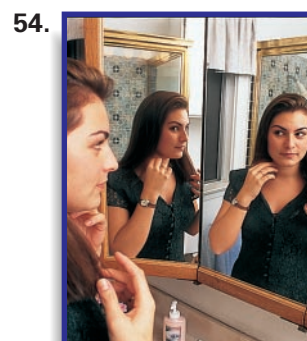
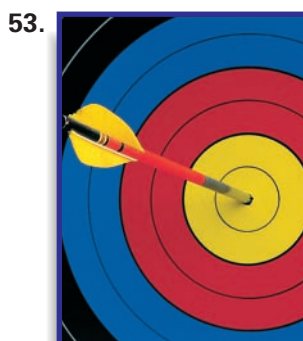
48. Draw four points $J, K, L,$ and $M,$ no three of which are collinear. Then sketch $\overrightarrow{JK}, \overrightarrow{KL}, \overrightarrow{LM},$ and $\overrightarrow{MJ}.$

49. Draw five points $P, Q, R, S,$ and $T,$ no three of which are collinear. Then sketch $\overrightarrow{PQ}, \overrightarrow{RS}, \overrightarrow{QR}, \overrightarrow{ST},$ and $\overrightarrow{TP}.$

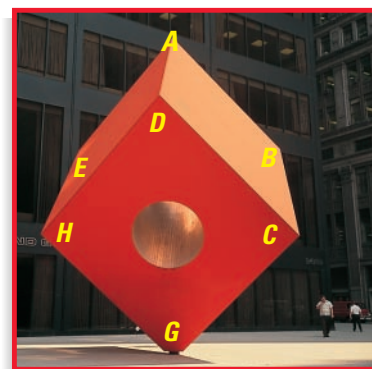
50. Draw two points, X and $Y.$ Then sketch $\overleftrightarrow{XY}.$ Add a point W between X and Y so that \overrightarrow{WX} and \overrightarrow{WY} are opposite rays.

51. Draw two points, A and $B.$ Then sketch $\overleftrightarrow{AB}.$ Add a point C on the ray so that B is between A and $C.$

 **EVERYDAY INTERSECTIONS** What kind of geometric intersection does the photograph suggest?



COMPLETING SENTENCES Fill in each blank with the appropriate response based on the points labeled in the photograph.



Red Cube, by sculptor Isamu Noguchi

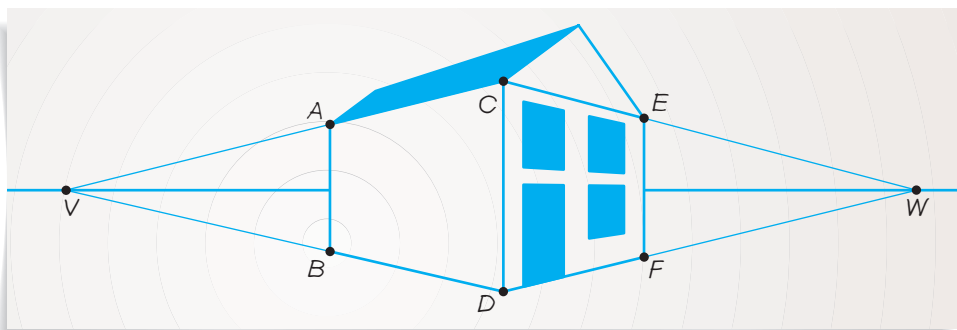
55. \overleftrightarrow{AB} and \overleftrightarrow{BC} intersect at ? .
56. \overleftrightarrow{AD} and \overleftrightarrow{AE} intersect at ? .
57. \overleftrightarrow{HG} and \overleftrightarrow{DH} intersect at ? .
58. Plane ABC and plane DCG intersect at ? .
59. Plane GHD and plane DHE intersect at ? .
60. Plane EAD and plane BCD intersect at ? .

SKETCHING FIGURES Sketch the figure described.

61. Three points that are coplanar but not collinear.
62. Two lines that lie in a plane but do not intersect.
63. Three lines that intersect in a point and all lie in the same plane.
64. Three lines that intersect in a point but do not all lie in the same plane.
65. Two lines that intersect and another line that does not intersect either one.
66. Two planes that do not intersect.
67. Three planes that intersect in a line.

 **TWO-POINT PERSPECTIVE** In Exercises 68–72, use the information and diagram below.

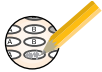
In *perspective drawing*, lines that do not intersect in real life are represented in a drawing by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*.



The diagram shows a drawing of a house with two vanishing points. You can use the vanishing points to draw the hidden parts of the house.

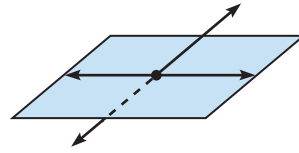
68. Name two lines that intersect at vanishing point V .
69. Name two lines that intersect at vanishing point W .
70. Trace the diagram. Draw \overleftrightarrow{EV} and \overleftrightarrow{AW} . Label their intersection as G .
71. Draw \overleftrightarrow{FV} and \overleftrightarrow{BW} . Label their intersection as H .
72. Draw the hidden edges of the house: \overline{AG} , \overline{EG} , \overline{BH} , \overline{FH} , and \overline{GH} .

Test Preparation



73. **MULTIPLE CHOICE** Which statement(s) are true about the two lines shown in the drawing to the right?

- I. The lines intersect in one point.
- II. The lines do not intersect.
- III. The lines are coplanar.



- (A) I only (B) I and II only (C) I and III only
- (D) II and III only (E) I, II, and III

74. **MULTIPLE CHOICE** What is the intersection of \overleftrightarrow{PQ} and \overleftrightarrow{QP} ?

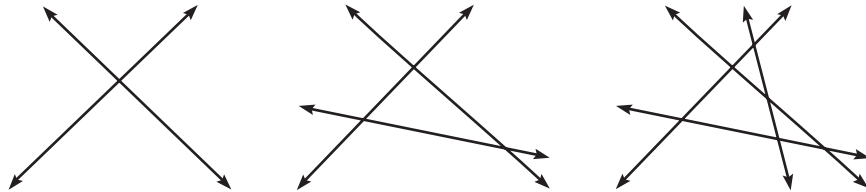
- (A) \overleftrightarrow{PQ} (B) \overline{PQ} (C) P and Q (D) P only (E) Q only

75. **MULTIPLE CHOICE** Points K , L , M , and N are not coplanar. What is the intersection of plane KLM and plane KNL ?

- (A) K and L (B) M and N (C) \overline{KL} (D) \overleftrightarrow{KL}
- (E) The planes do not intersect.

★ Challenge

76. **INTERSECTING LINES** In each diagram below, every line intersects all the other lines, but only two lines pass through each intersection point.



Can you draw 5 lines that intersect in this way? 6 lines? Is there a pattern to the number of intersection points?

EXTRA CHALLENGE

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MIXED REVIEW

DESCRIBING PATTERNS Describe a pattern in the sequence of numbers. Predict the next number. (Review 1.1)

77. 1, 6, 36, 216, ...

78. 2, -2, 2, -2, 2, ...

79. 8.1, 88.11, 888.111, 8888.1111, ...

80. 0, 3, 9, 18, 30, ...

OPERATIONS WITH INTEGERS Simplify the expression. (Skills Review, p.785)

81. $0 - 2$

82. $3 - 9$

83. $9 - (-4)$

84. $-5 - (-2)$

85. $5 - 0$

86. $4 - 7$

87. $3 - (-8)$

88. $-7 - (-5)$

RADICAL EXPRESSIONS Simplify the expression. Round your answer to two decimal places. (Skills Review, p. 799, for 1.3)

89. $\sqrt{21 + 100}$

90. $\sqrt{40 + 60}$

91. $\sqrt{25 + 144}$

92. $\sqrt{9 + 16}$

93. $\sqrt{5^2 + 7^2}$

94. $\sqrt{3^2 + (-2)^2}$

95. $\sqrt{(-3)^2 + 3^2}$

96. $\sqrt{(-5)^2 + 10^2}$