

# 1.1

## Patterns and Inductive Reasoning

### What you should learn

**GOAL 1** Find and describe patterns.

**GOAL 2** Use inductive reasoning to make **real-life** conjectures, as in **Ex. 42**.

### Why you should learn it

▼ To make predictions based on observations, such as predicting full moons in **Example 6**.

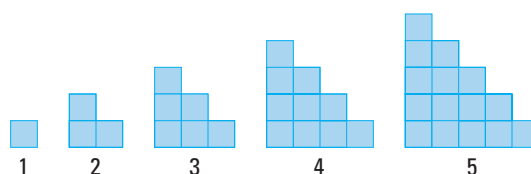


### GOAL 1 FINDING AND DESCRIBING PATTERNS

Geometry, like much of mathematics and science, developed when people began recognizing and describing patterns. In this course, you will study many amazing patterns that were discovered by people throughout history and all around the world. You will also learn to recognize and describe patterns of your own. Sometimes, patterns allow you to make accurate predictions.

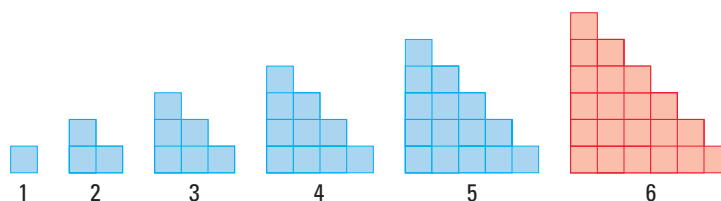
#### EXAMPLE 1 Describing a Visual Pattern

Sketch the next figure in the pattern.



#### SOLUTION

Each figure in the pattern looks like the previous figure with another row of squares added to the bottom. Each figure looks like a stairway.



► The sixth figure in the pattern has six squares in the bottom row.

#### EXAMPLE 2 Describing a Number Pattern

Describe a pattern in the sequence of numbers. Predict the next number.

- a. 1, 4, 16, 64, ...      b. -5, -2, 4, 13, ...

#### SOLUTION

- a. Each number is four times the previous number. The next number is 256.  
b. You add 3 to get the second number, then add 6 to get the third number, then add 9 to get the fourth number. To find the fifth number, add the next multiple of 3, which is 12.

► So, the next number is  $13 + 12$ , or 25.

## GOAL 2 USING INDUCTIVE REASONING

Much of the reasoning in geometry consists of three stages.

- 1 Look for a Pattern** Look at several examples. Use diagrams and tables to help discover a pattern.
- 2 Make a Conjecture** Use the examples to make a general *conjecture*.  
A **conjecture** is an unproven statement that is based on observations. Discuss the conjecture with others. Modify the conjecture, if necessary.
- 3 Verify the Conjecture** Use logical reasoning to verify that the conjecture is true in all cases. (You will do this in Chapter 2 and throughout this book.)

Looking for patterns and making conjectures is part of a process called **inductive reasoning**.



### EXAMPLE 3 Making a Conjecture

Complete the conjecture.

**Conjecture:** The sum of the first  $n$  odd positive integers is  $\underline{\quad ? \quad}$ .

#### SOLUTION

List some specific examples and look for a pattern.

**Examples:**

first odd positive integer:	$1 = 1^2$
sum of first <b>two</b> odd positive integers:	$1 + 3 = 4 = 2^2$
sum of first <b>three</b> odd positive integers:	$1 + 3 + 5 = 9 = 3^2$
sum of first <b>four</b> odd positive integers:	$1 + 3 + 5 + 7 = 16 = 4^2$

**Conjecture:** The sum of the first  $n$  odd positive integers is  $n^2$ .

.....

To prove that a conjecture is true, you need to prove it is true in *all* cases. To prove that a conjecture is false, you need to provide a single *counterexample*. A **counterexample** is an example that shows a conjecture is false.

### EXAMPLE 4 Finding a Counterexample

Show the conjecture is false by finding a counterexample.

**Conjecture:** For all real numbers  $x$ , the expression  $x^2$  is greater than or equal to  $x$ .

#### SOLUTION

The conjecture is false. Here is a counterexample:  $(0.5)^2 = 0.25$ , and 0.25 is *not* greater than or equal to 0.5. In fact, any number between 0 and 1 is a counterexample.

Not every conjecture is known to be true or false. Conjectures that are not known to be true or false are called *unproven* or *undecided*.

### EXAMPLE 5 Examining an Unproven Conjecture

In the early 1700s a Prussian mathematician named Goldbach noticed that many even numbers greater than 2 can be written as the sum of two primes.

**Specific Cases:**

$$\begin{array}{lll} 4 = 2 + 2 & 10 = 3 + 7 & 16 = 3 + 13 \\ 6 = 3 + 3 & 12 = 5 + 7 & 18 = 5 + 13 \\ 8 = 3 + 5 & 14 = 3 + 11 & 20 = 3 + 17 \end{array}$$

**Conjecture:** Every even number greater than 2 can be written as the sum of two primes.

This is called *Goldbach's Conjecture*. No one has ever proved that this conjecture is true or found a counterexample to show that it is false. As of the writing of this book, it is unknown whether this conjecture is true or false. It is known, however, that all even numbers up to  $4 \times 10^{14}$  confirm Goldbach's Conjecture.

### EXAMPLE 6 Using Inductive Reasoning in Real Life

#### FOCUS ON APPLICATIONS



#### CALENDARS

The earliest calendars were based on lunar or seasonal patterns. The ancient Egyptians were the first to introduce a calendar based on the solar year.

**MOON CYCLES** A full moon occurs when the moon is on the opposite side of Earth from the sun. During a full moon, the moon appears as a complete circle.



Use inductive reasoning and the information below to make a conjecture about how often a full moon occurs.

**Specific Cases:** In 2005, the first six full moons occur on January 25, February 24, March 25, April 24, May 23, and June 22.

#### SOLUTION

**Conjecture:** A full moon occurs every 29 or 30 days.

This conjecture is true. The moon revolves around Earth once approximately every 29.5 days.

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Inductive reasoning is important to the study of mathematics: you look for a pattern in specific cases and then you write a conjecture that you think describes the general case. Remember, though, that just because something is true for several specific cases does not *prove* that it is true in general.

# GUIDED PRACTICE

Vocabulary Check ✓

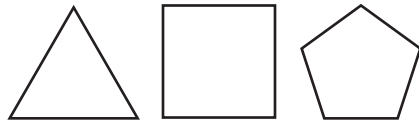
Concept Check ✓

Skill Check ✓

1. Explain what a *conjecture* is.
2. How can you prove that a conjecture is false?

Sketch the next figure in the pattern.

3.



4.



Describe a pattern in the sequence of numbers. Predict the next number.

5. 2, 6, 18, 54, ...

6. 0, 1, 4, 9, ...

7. 256, 64, 16, 4, ...

8. 3, 0, -3, 0, 3, 0, ...

9. 7.0, 7.5, 8.0, 8.5, ...

10. 13, 7, 1, -5, ...

11. Complete the conjecture based on the pattern you observe.

$$3 + 4 + 5 = 4 \cdot 3$$

$$6 + 7 + 8 = 7 \cdot 3$$

$$9 + 10 + 11 = 10 \cdot 3$$

$$4 + 5 + 6 = 5 \cdot 3$$

$$7 + 8 + 9 = 8 \cdot 3$$

$$10 + 11 + 12 = 11 \cdot 3$$

$$5 + 6 + 7 = 6 \cdot 3$$

$$8 + 9 + 10 = 9 \cdot 3$$

$$11 + 12 + 13 = 12 \cdot 3$$

Conjecture: The sum of any three consecutive integers is     ?

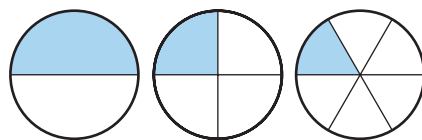
# PRACTICE AND APPLICATIONS

## STUDENT HELP

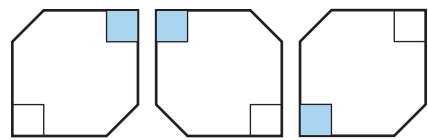
Extra Practice to help you master skills is on p. 803.

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern.

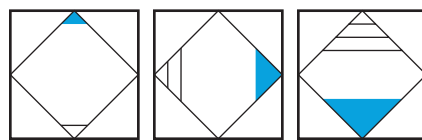
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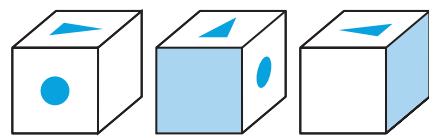
13.



14.



15.



## STUDENT HELP

### HOMEWORK HELP

Example 1: Exs. 12–15, 24, 25

Example 2: Exs. 16–23, 26–28

Example 3: Exs. 29–33

Example 4: Exs. 34–39

Example 5: Exs. 40, 41

Example 6: Exs. 42, 43

DESCRIBING NUMBER PATTERNS Describe a pattern in the sequence of numbers. Predict the next number.

16. 1, 4, 7, 10, ...

17. 10, 5, 2.5, 1.25, ...

18. 1, 11, 121, 1331, ...

19. 5, 0, -5, -10, ...

20. 7, 9, 13, 19, 27, ...

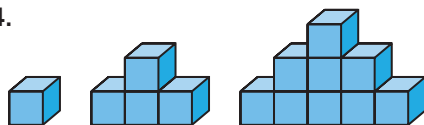
21. 1, 3, 6, 10, 15, ...

22. 256, 16, 4, 2, ...

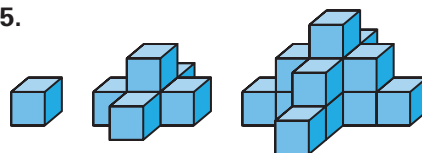
23. 1.1, 1.01, 1.001, 1.0001, ...

**VISUALIZING PATTERNS** The first three objects in a pattern are shown. How many blocks are in the next object?

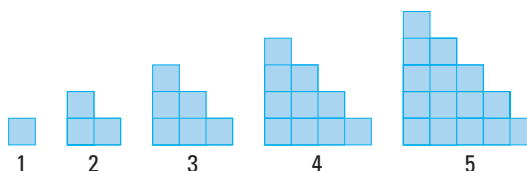
24.



25.



**MAKING PREDICTIONS** In Exercises 26–28, use the pattern from Example 1 shown below. Each square is 1 unit  $\times$  1 unit.



26. Find the distance around each figure. Organize your results in a table.

27. Use your table to describe a pattern in the distances.

28. Predict the distance around the twentieth figure in this pattern.

**STUDENT HELP**



**HOMEWORK HELP**

Visit our Web site  
www.mcdougallittell.com  
for help with Exs. 29–31.

**MAKING CONJECTURES** Complete the conjecture based on the pattern you observe in the specific cases.

29. **Conjecture:** The sum of any two odd numbers is \_\_\_\_?

$$1 + 1 = 2$$

$$7 + 11 = 18$$

$$1 + 3 = 4$$

$$13 + 19 = 32$$

$$3 + 5 = 8$$

$$201 + 305 = 506$$

30. **Conjecture:** The product of any two odd numbers is \_\_\_\_?

$$1 \times 1 = 1$$

$$7 \times 11 = 77$$

$$1 \times 3 = 3$$

$$13 \times 19 = 247$$

$$3 \times 5 = 15$$

$$201 \times 305 = 61,305$$

31. **Conjecture:** The product of a number  $(n - 1)$  and the number  $(n + 1)$  is always equal to \_\_\_\_?

$$3 \cdot 5 = 4^2 - 1$$

$$6 \cdot 8 = 7^2 - 1$$

$$4 \cdot 6 = 5^2 - 1$$

$$7 \cdot 9 = 8^2 - 1$$

$$5 \cdot 7 = 6^2 - 1$$

$$8 \cdot 10 = 9^2 - 1$$



**CALCULATOR** Use a calculator to explore the pattern. Write a conjecture based on what you observe.

32.  $101 \times 34 = \underline{\quad?}$

33.  $11 \times 11 = \underline{\quad?}$

$101 \times 25 = \underline{\quad?}$

$111 \times 111 = \underline{\quad?}$

$101 \times 97 = \underline{\quad?}$

$1111 \times 1111 = \underline{\quad?}$

$101 \times 49 = \underline{\quad?}$

$11,111 \times 11,111 = \underline{\quad?}$

**FINDING COUNTEREXAMPLES** Show the conjecture is false by finding a counterexample.


34. All prime numbers are odd.
35. The sum of two numbers is always greater than the larger number.
36. If the product of two numbers is even, then the two numbers must be even.
37. If the product of two numbers is positive, then the two numbers must both be positive.
38. The square root of a number  $x$  is always less than  $x$ .
39. If  $m$  is a nonzero integer, then  $\frac{m+1}{m}$  is always greater than 1.

**GOLDBACH'S CONJECTURE** In Exercises 40 and 41, use the list of the first prime numbers given below.

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...}

40. Show that Goldbach's Conjecture (see page 5) is true for the even numbers from 20 to 40 by writing each even number as a sum of two primes.
41. Show that the following conjecture is not true by finding a counterexample.

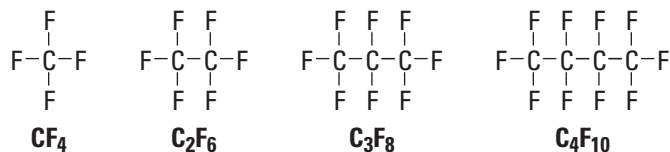
**Conjecture:** All *odd* numbers can be expressed as the sum of two primes.

42.  **BACTERIA GROWTH** Suppose you are studying bacteria in biology class. The table shows the number of bacteria after  $n$  doubling periods.

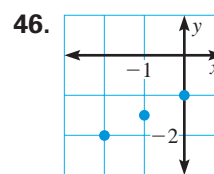
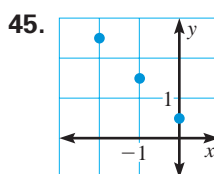
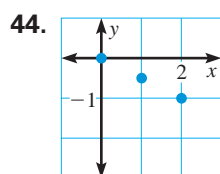
$n$ (periods)	0	1	2	3	4	5
Billions of bacteria	3	6	12	24	48	96

Your teacher asks you to predict the number of bacteria after 8 doubling periods. What would your prediction be?

43. **SCIENCE CONNECTION** Diagrams and formulas for four molecular compounds are shown. Draw a diagram and write the formula for the next two compounds in the pattern.



44.  **USING ALGEBRA** Find a pattern in the coordinates of the points. Then use the pattern to find the  $y$ -coordinate of the point (3, ?).



**FOCUS ON CAREERS**



**REAL LIFE LABORATORY TECHNOLOGIST**

Laboratory technologists study microscopic cells, such as bacteria. The time it takes for a population of bacteria to double (the *doubling period*) may be as short as 20 min.

**CAREER LINK**  
www.mcdougallittell.com

## Test Preparation

47. **MULTIPLE CHOICE** Which number is next in the sequence?

45, 90, 135, 180, ...

- (A) 205      (B) 210      (C) 215      (D) 220      (E) 225

48. **MULTIPLE CHOICE** What is the next figure in the pattern?

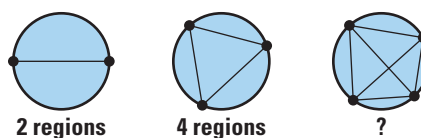


- (A)      (B)      (C)      (D)      (E)

## ★ Challenge

**DIVIDING A CIRCLE** In Exercises 49–51, use the information about regions in a circle formed by connecting points on the circle.

If you draw points on a circle and then connect every pair of points, the circle is divided into a number of regions, as shown.



49. Copy and complete the table for the case of 4 and 5 points.

Number of points on circle	2	3	4	5	6
Maximum number of regions	2	4	?	?	?

50. Make a conjecture about the relationship between the number of points on the circle and number of regions in the circle.

51. Test your conjecture for the case of 6 points. What do you notice?

### EXTRA CHALLENGE

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## MIXED REVIEW

**PLOTTING POINTS** Plot in a coordinate plane. (Skills Review, p. 792, for 1.2)

52. (5, 2)      53. (3, -8)      54. (-4, -6)      55. (1, -10)

56. (-2, 7)      57. (-3, 8)      58. (4, -1)      59. (-2, -6)

**EVALUATING EXPRESSIONS** Evaluate the expression. (Skills Review, p. 786)

60.  $3^2$       61.  $5^2$       62.  $(-4)^2$       63.  $-7^2$

64.  $3^2 + 4^2$       65.  $5^2 + 12^2$       66.  $(-2)^2 + 2^2$       67.  $(-10)^2 + (-5)^2$

**FINDING A PATTERN** Write the next number in the sequence. (Review 1.1)

68. 1, 5, 25, 125, ...

69. 4.4, 40.4, 400.4, 4000.4, ...

70. 3, 7, 11, 15, ...

71. -1, +1, -2, +2, -3, ...