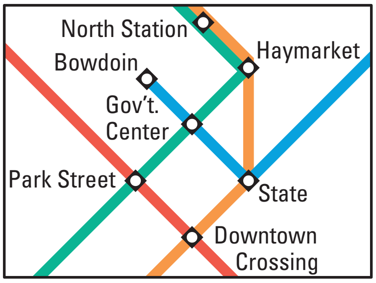
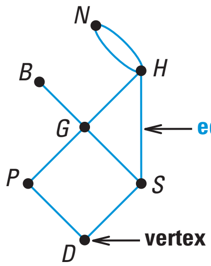
Appendix 1 Networks

GOAL ***Use properties of networks to solve real-world problems.***

Below is part of a Boston subway system map. The diagram to the right of the map represents the stops and subway lines connecting the stops. A diagram that uses connected points to show relationships is a **network**. A point on a network is a **vertex**, and a segment or arc connecting two vertices is an **edge**.

> **STUDENT HELP**
> **Study Tip**
> A network is sometimes called a *vertex-edge graph.*

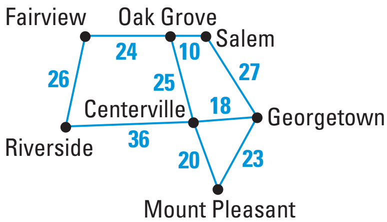




On a network, a **path** is an edge or sequence of edges that connect one vertex with another. In the network above, there are several paths from *B* to *D*. Some examples are *BGPD*, *BGSD*, and *BGHSD*.

EXAMPLE 1 *Finding the Shortest Path*

The network at the right shows highway distances (in miles) among cities.



a. Find three different paths from Riverside to Salem.

b. Find a path from Mount Pleasant to Salem that includes every city.

c. Find the shortest path from Fairview to Georgetown.

SOLUTION

a. There are many different paths from Riverside to Salem. Three of them are *RFOS*, *RCGS*, and *RCOS*.

b. There are several paths from Mount Pleasant to Salem that include every city. One possible path is *MGCRFOS*.

c. Consider the paths that are clearly shorter than others. The lengths of the shortest paths from Fairview to Georgetown are shown below.

FOSG	*FRCG*	*FOCG*
$24 + 10 + 27 = 61$	$26 + 36 + 18 = 80$	$24 + 25 + 18 = 67$

The shortest path is Fairview to Oak Grove to Salem to Georgetown.

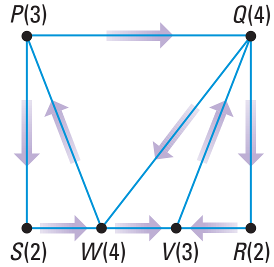
> **STUDENT HELP**
> **Study Tip**
> When listing paths, you can abbreviate the names of cities using the first letter of each city.

STUDENT HELP

Study Tip
If you can trace a network with your finger or a pencil without tracing the same edge more than once, the network is traceable.

The **degree of a vertex** is the number of edges that are connected to the vertex. In the diagram at the right, the degree of each vertex is in parentheses.

A network is **traceable** if there is a path that connects all the vertices and covers each edge of the network exactly once. In the diagram at the right, the network is traceable, and one path that demonstrates traceability is *PQRVQWPSWV*.



This network is traceable.

NETWORK TRACEABILITY

A network is traceable if and only if one of the following statements is true.

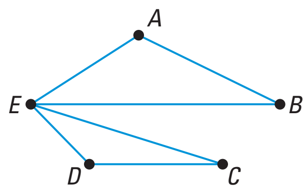
The degrees of all its vertices are even.
Exactly two of the degrees of its vertices are odd.

If the degrees of all the vertices of a traceable network are even, then you can begin tracing the network at any vertex. If two vertices of a traceable network are odd, then the tracing should begin or end at one of the odd vertices.

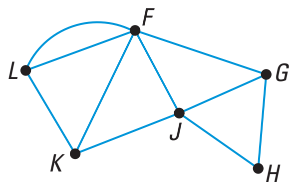
EXAMPLE 2 *Determining the Traceability of a Network*

Find the degree of each vertex of the network. Then use the network traceability rules to determine whether the network is traceable. If it is traceable, find a path that demonstrates traceability.

a.



b.



SOLUTION

a. The degree of *A* is 2, *B* is 2, *C* is 2, *D* is 2, and *E* is 4. The degrees of all the vertices of the network are even, so the network is traceable. One path that demonstrates traceability is *EABECDE*.

b. The degree of *F* is 5, *G* is 3, *H* is 2, *J* is 4, *K* is 3, and *L* is 3. Because the degrees of four of the vertices of the network are odd, the network is not traceable.

. . . .

A **circuit** is a path that starts and ends at the same vertex. In Example 2, part (a), the path *EABECDE* is a circuit because it is a path that starts and ends at vertex *E*.

Networks can also be used to connect several cities with roads. City planners want to keep the costs low by building the least possible number of roads. The planners can use *spanning trees* of the network of cities to decide how to build the roads.

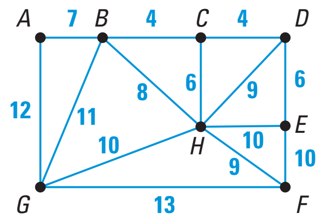
A **tree** is a diagram in which there is only one path connecting each pair of vertices. A **spanning tree** connects all the vertices of the network. If each edge has a value, then the **minimal spanning tree** of a network is the spanning tree that has the least value. You can use the following steps to find a minimal spanning tree.

FINDING A MINIMAL SPANNING TREE

1. Build the least expensive edge first.
2. Then build the edge that is next lowest in cost.
3. At each stage, build the edge that is next lowest in cost and *does not form a circuit.* Stop when all of the vertices in the network have been reached.

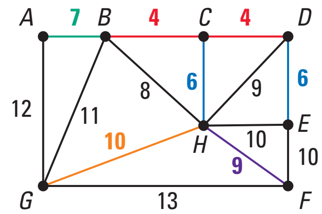
EXAMPLE 3 *Minimizing the Cost of a Network*

The diagram shows the costs (in thousands of dollars) of building walkways between several buildings on a college campus. How can all of the buildings be connected by walkways at the lowest cost? What is the cost?



SOLUTION

Find the minimal spanning tree for the network.



1. The least expensive routes are ***BC*** and ***CD***.
2. The routes next lowest in cost are ***CH*** and ***DE***.
3. The route next lowest in cost is ***AB***.
4. The route next lowest in cost is *BH*, but it is not used because it forms a circuit with *BC* and *CH*.
5. The routes next lowest in cost are *DH* and ***HF***. *DH* is not used because it forms a circuit with *CD* and *CH*.
6. The routes next lowest in cost are *HE, EF,* and ***GH***. *HE* is not used because it forms a circuit with *CH*, *CD*, and *DE*. *EF* is not used because it forms a circuit with *HF*, *CH*, *CD*, and *DE*.

▶ The buildings can be connected by walkways as shown in the diagram. The minimum cost of connecting all of the buildings by walkways is **4** + **4** + **6** + **6** + **7** + **9** + **10** = 46, or $46,000.

STUDENT HELP

Study Tip
In Step 6 of Example 3, all of the buildings have been connected, so you do not need to consider *GB*, *AG*, and *GF*.

In Examples 1, 2, and 3, networks are used to represent locations and the routes between them. Networks can also represent links between computers or even people. For example, networks can be used to solve scheduling problems, as shown in the following example.

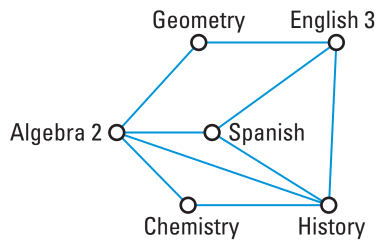
EXAMPLE 4 *Coloring Network Diagrams*

A curriculum director at a school is scheduling final exams. The director has a list of all the students who have more than one exam. Use a network to make an exam schedule so that no exams with students in common are given at the same time.

Geometry	History	Spanish	English 3	Chemistry	Algebra 2
Emma Annika	Max Davis Mattea	Toby Max	Annika Mattea Toby	Davis	Emma Max Davis

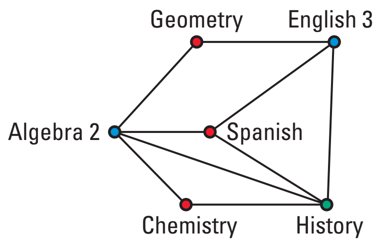
SOLUTION

Draw a vertex to represent each subject. Connect two vertices if they share at least one student.



Then color the vertices of the network so that two vertices that are connected by an edge are different colors. Use as few colors as possible.

1. Color any vertex. (**Geometry**)
2. Use the same color to color all the vertices that are *not* connected to each other or to the first vertex. (**Chemistry** and **Spanish**)
3. Use another color to color one of the remaining vertices. (**Algebra 2**) Use the same color to color all of the remaining vertices that are not connected to it or to each other. (**English 3**)
4. Repeat Step 3 until all vertices have been colored. (**History**)

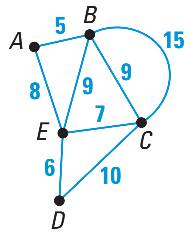


▶ The exams represented by vertices of the same color should be given at the same time because they do not have students in common. There should be three exam times.

Exam time 1	**Exam time 2**	**Exam time 3**
Geometry	Algebra 2	History
Chemistry	English 3	
Spanish		

EXERCISES

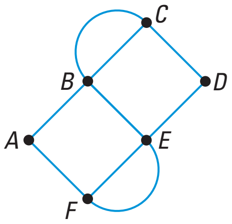
In Exercises 1–5, use the network at the right.



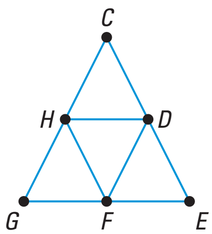
1. Find three different paths from *B* to *D*.
2. Find a path from *E* to *B* that includes every vertex. You can repeat edges and vertices.
3. Find a path from *A* to *D* that includes every edge. You can repeat edges and vertices.
4. Find the shortest path from *A* to *C*.
5. Find the shortest path from *A* to *D* that includes every vertex.

In Exercises 6–11, find the degree of each vertex of the network. Then use the network traceability rules to determine whether the network is traceable. If it is traceable, find a path that demonstrates traceability.

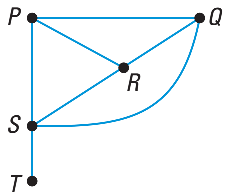
6.



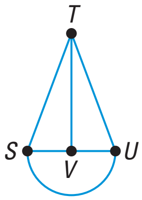
7.



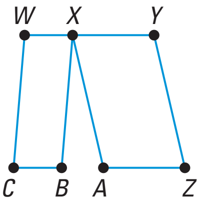
8.



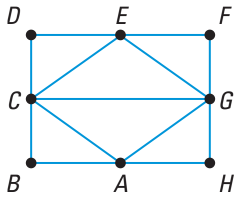
9.



10.



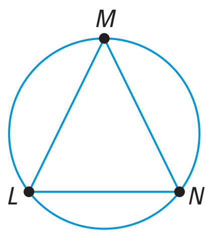
11.



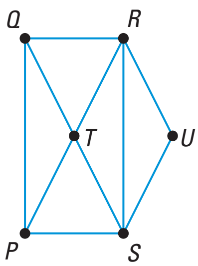
In Exercises 12–14, use the following information to tell whether you can find an Euler circuit for the network. Explain.

An *Euler circuit* covers every edge of the network exactly once and ends at its starting point.

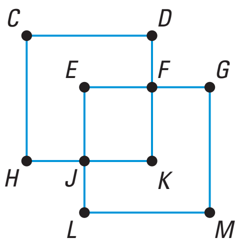
12.



13.

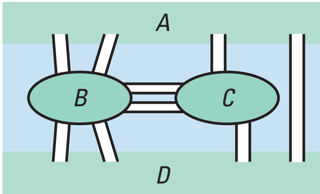


14.



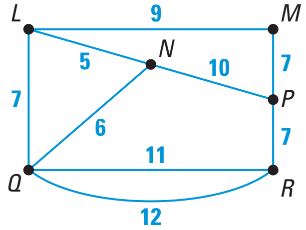
15. A *Hamilton circuit* includes every vertex of a network exactly once and ends at its starting point. Can you find a Hamilton circuit for any of the networks in Exercises 12–14? Explain.

16. Two islands, B and C, are joined to two cities, A and D, by several bridges. Is it possible to start at one place point, travel over each bridge exactly once, and return to your starting point? Explain your reasoning.

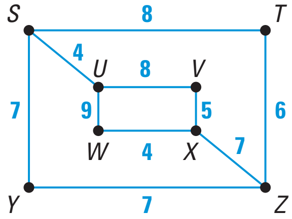


In Exercises 17 and 18, find the minimal spanning tree for each network.

17.



18.



19. An activities director needs to schedule meeting times for a school's activities. All of the students who participate in more than one activity are listed below. Use a network to make an activities schedule so that no activities with students in common meet at the same time.

Chess Club	Orchestra	Student Council	Yearbook Committee	Latin Club	French Club	Drama Club
Sharon Mike Carlos Erica	Brian Sharon	Jared Carlos Erica Angela	Angela Sharon	Mike Brian Jared	Danielle	Jared Danielle

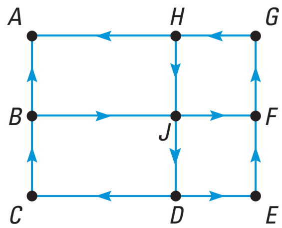
STUDENT HELP

Study Tip
A *directed graph,* or *digraph,* is a network that includes direction, like the one-way street map in Exercises 20 and 21.

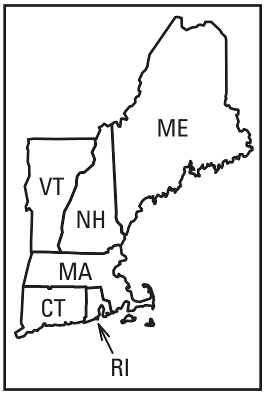
In Exercises 20 and 21, use the network of one-way streets in a town.

20. Find the path from E to J that includes the fewest number of one-way streets.

21. Is it possible to find a path from H to D on the one-way streets that includes every street? Explain.



22. A map of New England is shown at the right. Draw a vertex to represent each New England state. Connect the vertices for two states if they share a border. Use the method in Example 4 to find the least number of colors you need to use to color the map so that states that border each other are different colors.



Appendix 2 Matrices and Transformations

GOAL ***Use matrix operations to represent transformations of polygons in a coordinate plane.***

A **matrix** is a rectangular arrangement of numbers in **rows** and **columns**. The plural of matrix is *matrices*. Each number in a matrix is called an **entry** or **element**. The **dimensions** of a matrix are the number of rows and columns. The matrix below has three rows and four columns, so it is called a 3 × 4 (read "3 by 4") matrix.

column

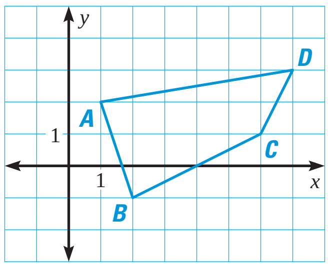
row → $\begin{bmatrix} 5 & 4 & 4 & 9 \\ -3 & 5 & 2 & 6 \\ 3 & -7 & 8 & 7 \end{bmatrix}$

The entry in the second row and third column is 2.

You can represent geometric figures using matrices. The coordinates of a point can be displayed in a 2 × 1 **point matrix** using the first row for the *x*-coordinate and the second row for the *y*-coordinate. A 2 × *n* **polygon matrix** can be used to represent the vertices of an *n*-sided polygon. Each column in the matrix represents a vertex. The columns follow the consecutive order of the vertices of the polygon.

Point $A(1, 2)$: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ← *x*-coordinate, ← *y*-coordinate

Quadrilateral *ABCD*: $\begin{matrix} A & B & C & D \end{matrix}$ $\begin{bmatrix} 1 & 2 & 6 & 7 \\ 2 & -1 & 1 & 3 \end{bmatrix}$



You can use matrix operations to represent transformations, such as dilations, translations, reflections, and rotations, in a coordinate plane.

DILATIONS You can represent the dilation of a polygon (with the center of the dilation at the origin) by multiplying each entry of the polygon matrix by the scale factor of the dilation. Multiplying a matrix by a real number is called **scalar multiplication**. An example of scalar multiplication is shown below.

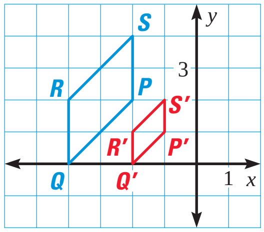
$$3 \cdot \begin{bmatrix} 0 & 5 & -2 \\ 4 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(0) & 3(5) & 3(-2) \\ 3(4) & 3(-1) & 3(2) \end{bmatrix} = \begin{bmatrix} 0 & 15 & -6 \\ 12 & -3 & 6 \end{bmatrix}$$

EXAMPLE 1 *Representing a Dilation*

The matrix $\begin{bmatrix} -2 & -4 & -4 & -2 \\ 2 & 0 & 2 & 4 \end{bmatrix}$ represents parallelogram *PQRS*. Find the image matrix that represents a dilation of *PQRS* centered at the origin with a scale factor of $\frac{1}{2}$. Then graph *PQRS* and its image.

SOLUTION

$$\underset{\text{scale factor}}{\frac{1}{2}} \cdot \underset{\text{polygon matrix}}{\overset{\quad P \quad\;\; Q \quad\;\; R \quad\;\; S}{\begin{bmatrix} -2 & -4 & -4 & -2 \\ 2 & 0 & 2 & 4 \end{bmatrix}}} = \underset{\text{image matrix}}{\overset{\quad P' \quad\; Q' \quad\; R' \quad\; S'}{\begin{bmatrix} -1 & -2 & -2 & -1 \\ 1 & 0 & 1 & 2 \end{bmatrix}}}$$



STUDENT HELP

Look Back
For help with the blue to red color scheme used in transformations, see p. 396.

. . . .

To add or subtract matrices, add or subtract the corresponding entries. You can add or subtract matrices only if they have the same dimensions.

EXAMPLE 2 *Adding and Subtracting Matrices*

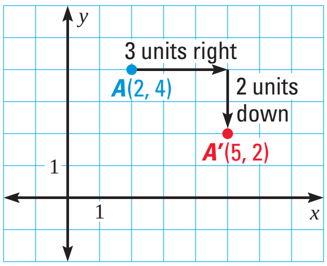
a. $\begin{bmatrix} 5 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 5+1 & -3+2 \\ 6+3 & -6+(-4) \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 9 & -10 \end{bmatrix}$

b. $\begin{bmatrix} 6 & -2 \\ -1 & 4 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 3 & -2 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} 6-1 & -2-4 \\ -1-3 & 4-(-2) \\ 5-0 & 8-(-7) \end{bmatrix} = \begin{bmatrix} 5 & -6 \\ -4 & 6 \\ 5 & 15 \end{bmatrix}$

. . . .

TRANSLATIONS You can use matrix addition to represent a translation in the coordinate plane. In the diagram below, point *A* is translated 3 units to the right and 2 units down. Therefore, the translation matrix is $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

To find the image matrix for a translation, add the translation matrix to the matrix that represents the figure that is translated.



$$\underset{\text{translation matrix}}{\begin{bmatrix} 3 \\ -2 \end{bmatrix}} + \underset{\text{point matrix}}{\overset{A}{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}} = \underset{\text{image matrix}}{\overset{A'}{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}}$$

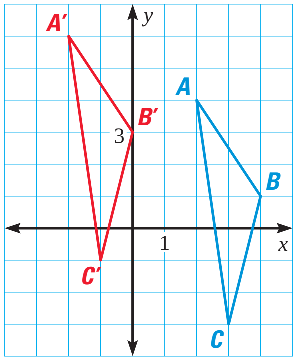
A translation matrix must have the same dimensions as the polygon matrix that represents the polygon being translated, as you will see in Example 3.

EXAMPLE 3 *Representing a Translation*

The matrix $\begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & -3 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation that shifts $\triangle ABC$ 4 units to the left and 2 units up. Then graph $\triangle ABC$ and its image.

SOLUTION

Because the polygon matrix has 3 columns, the translation matrix must also have 3 columns. The translation matrix that will shift a triangle 4 units to the left and 2 units up is $\begin{bmatrix} -4 & -4 & -4 \\ 2 & 2 & 2 \end{bmatrix}$. Add this to the polygon matrix to find the image matrix.



$$\underset{\text{translation matrix}}{\begin{bmatrix} -4 & -4 & -4 \\ 2 & 2 & 2 \end{bmatrix}} + \underset{\text{polygon matrix}}{\overset{\begin{matrix} A & B & C \end{matrix}}{\begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & -3 \end{bmatrix}}} = \underset{\text{image matrix}}{\overset{\begin{matrix} A' & B' & C' \end{matrix}}{\begin{bmatrix} -2 & 0 & -1 \\ 6 & 3 & -1 \end{bmatrix}}}$$

. . . .

You can use *matrix multiplication* to represent reflections and rotations. Matrix multiplication is very different from scalar multiplication, as you will see in Example 4.

EXAMPLE 4 *Multiplying Matrices*

Find AB if $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 & 0 \\ -1 & 6 & 7 \end{bmatrix}$.

SOLUTION

To write the entry in the first row and first column of the product AB, multiply each entry in the first *row* of A by its corresponding entry in the first *column* of B. Then add. Use a similar procedure to write the other entries of the product.

$$AB = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 & 0 \\ -1 & 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -2(2) + 3(-1) & -2(5) + 3(6) & -2(0) + 3(7) \\ 1(2) + 4(-1) & 1(5) + 4(6) & 1(0) + 4(7) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 8 & 21 \\ -2 & 29 & 28 \end{bmatrix}$$

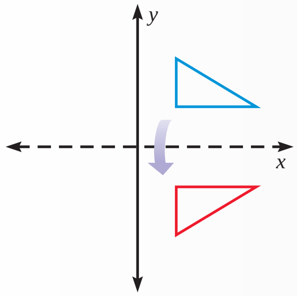
In Example 4, notice that the number of entries in a row of matrix A is equal to the number of entries in a column of matrix B. In general, the product of two matrices AB is defined only when the number of *columns* in A (the left matrix) is equal to the number of *rows* in B (the right matrix). If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

REFLECTIONS To represent a reflection in the x-axis or y-axis, you can multiply the polygon matrix by the appropriate reflection matrix. When you multiply, the reflection matrix should be on the left and the polygon matrix should be on the right.

REFLECTION MATRICES

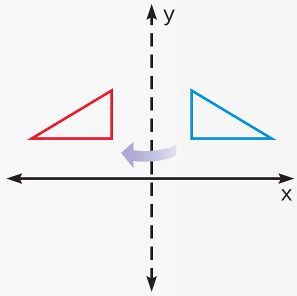
reflection in x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



reflection in y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



EXAMPLE 5 *Representing a Reflection*

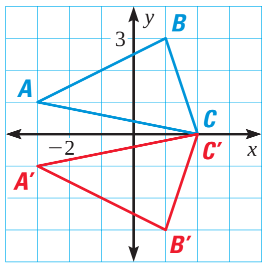
The vertices of $\triangle ABC$ are $A(-3, 1)$, $B(1, 3)$, and $C(2, 0)$. Find the image matrix that represents the reflection of $\triangle ABC$ in the x-axis. Then graph $\triangle ABC$ and its image.

SOLUTION

To reflect $\triangle ABC$ in the x-axis, multiply the polygon matrix by the appropriate reflection matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

$$\underset{\text{reflection matrix}}{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}} \underset{\text{polygon matrix}}{\overset{\quad A \quad B \quad C}{\begin{bmatrix} -3 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}}} = \begin{bmatrix} 1(-3) + 0(1) & 1(1) + 0(3) & 1(2) + 0(0) \\ 0(-3) + (-1)(1) & 0(1) + (-1)(3) & 0(2) + (-1)(0) \end{bmatrix}$$

$$= \underset{\text{image matrix}}{\overset{\quad A' \quad B' \quad C'}{\begin{bmatrix} -3 & 1 & 2 \\ -1 & -3 & 0 \end{bmatrix}}}$$

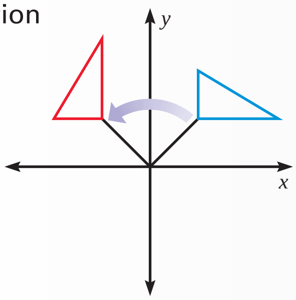


ROTATIONS As with reflections, there are matrices that you can multiply a polygon matrix by to produce counterclockwise rotations about the origin. When you multiply, the rotation matrix should be on the left and the polygon matrix should be on the right.

ROTATION MATRICES

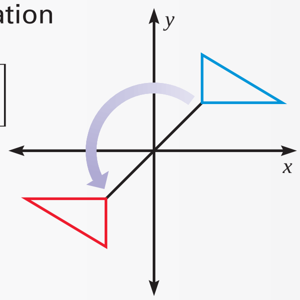
90° rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



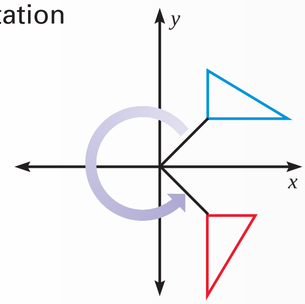
180° rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



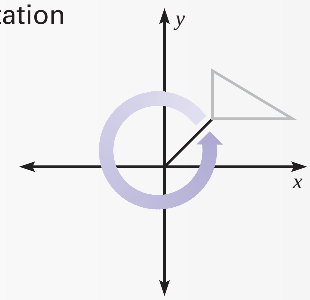
270° rotation

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



360° rotation

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



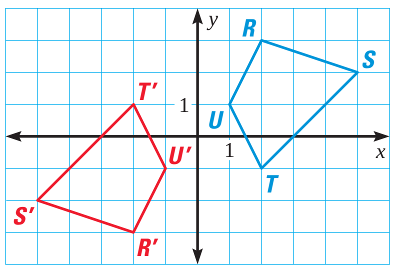
EXAMPLE 6 *Representing a Rotation*

The matrix $\begin{bmatrix} 2 & 5 & 2 & 1 \\ 3 & 2 & -1 & 1 \end{bmatrix}$ represents quadrilateral *RSTU*. Find the image matrix that represents a 180° rotation of *RSTU* about the origin. Then graph *RSTU* and its image.

SOLUTION

Multiply the appropriate rotation matrix and the polygon matrix.

$$\underset{\text{180° rotation}}{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}} \underset{\text{polygon matrix}}{\overset{\quad R \quad S \quad T \quad U}{\begin{bmatrix} 2 & 5 & 2 & 1 \\ 3 & 2 & -1 & 1 \end{bmatrix}}} = \underset{\text{image matrix}}{\overset{\quad R' \quad S' \quad T' \quad U'}{\begin{bmatrix} -2 & -5 & -2 & -1 \\ -3 & -2 & 1 & -1 \end{bmatrix}}}$$



EXERCISES

Write a point matrix that represents the ordered pair.

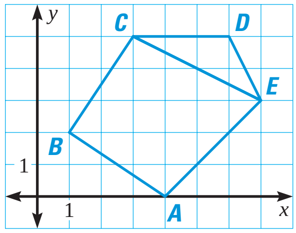
1. (3, 4) **2.** (–2, 6) **3.** (5, –8) **4.** (–1, –12)

In Exercises 5–7, use the diagram to write a polygon matrix that represents the polygon.

5. $\triangle CDE$

6. Quadrilateral *ABCE*

7. Pentagon *ABCDE*



Perform the matrix operation.

8. $5\begin{bmatrix} 3 & 6 & -2 \\ -1 & 0 & 2 \end{bmatrix}$

9. $[3 \quad 4] + [5 \quad 6]$

10. $\begin{bmatrix} -4 & -6 \\ 3 & -9 \end{bmatrix} - \begin{bmatrix} 7 & -1 \\ 2 & 5 \end{bmatrix}$

11. $\begin{bmatrix} 9 & -2 \\ 3 & -4 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ -5 & 3 \\ 7 & 1 \end{bmatrix}$

12. $[4 \quad 3]\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

13. $\begin{bmatrix} 6 & 5 & 0 \\ 2 & 4 & 8 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -5 & 4 \\ 7 & 2 \end{bmatrix}$

In Exercises 14–16, find the image matrix that represents a dilation of the polygon centered at the origin with the given scale factor. Then graph the polygon and its image.

14. $\triangle DEF$: $\begin{bmatrix} 1 & 3 & 6 \\ 3 & 7 & 2 \end{bmatrix}$; scale factor: 2

15. $\triangle XYZ$: $\begin{bmatrix} -3 & -3 & 6 \\ -3 & 6 & 3 \end{bmatrix}$; scale factor: $\frac{1}{3}$

16. *WXYZ*: $\begin{bmatrix} -4 & 4 & 8 & 4 \\ 4 & 8 & 0 & -4 \end{bmatrix}$; scale factor: $\frac{1}{4}$

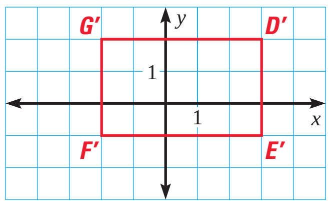
In Exercises 17–19, find the image matrix that represents the given translation of the polygon. Then graph the polygon and its image.

17. $\triangle FGH$: $\begin{bmatrix} -6 & -2 & 8 \\ 2 & -7 & 6 \end{bmatrix}$; shifted 3 units up

18. *KLMN*: $\begin{bmatrix} -2 & 2 & 3 & -1 \\ 1 & 1 & 3 & 3 \end{bmatrix}$; shifted 7 units to the left and 1 unit down

19. *ACEG*: $\begin{bmatrix} -2 & 4 & 2 & -1 \\ -3 & -2 & 4 & 1 \end{bmatrix}$; shifted 2 units to the right and 5 units up

20. Rectangle $D'E'F'G'$ was created by a translation that shifted *DEFG* 4 units to the left and 3 units down. Find the matrix that represents *DEFG*.



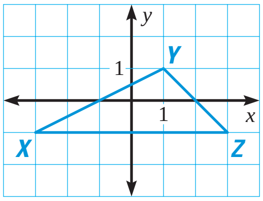
Find the image matrix that represents the given reflection of the polygon. Then graph the polygon and its image.

21. $\triangle BCD$: $\begin{bmatrix} -2 & -1 & 4 \\ 3 & -2 & 3 \end{bmatrix}$; reflected in the x-axis

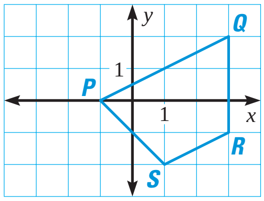
22. $NPRS$: $\begin{bmatrix} -6 & -5 & -2 & -2 \\ -2 & -4 & -5 & -3 \end{bmatrix}$; reflected in the y-axis

Write a matrix for the polygon. Then find the image matrix that represents the polygon after reflection in the given line.

23. x-axis



24. y-axis



Find the image matrix that represents the given counterclockwise rotation of the polygon. Then graph the polygon and its image.

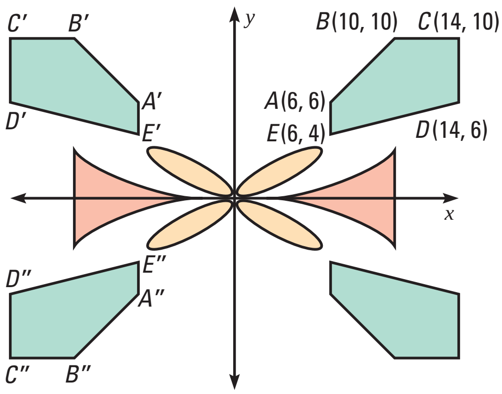
25. $\triangle FGH$: $\begin{bmatrix} 2 & 5 & 4 \\ 2 & 3 & 5 \end{bmatrix}$; 90°

26. $\triangle JKL$: $\begin{bmatrix} 4 & 3 & 5 \\ 1 & -3 & -3 \end{bmatrix}$; 180°

27. $CDEF$: $\begin{bmatrix} -3 & 1 & 1 & -3 \\ -2 & -5 & -7 & -4 \end{bmatrix}$; 270°

28. $MNPQ$: $\begin{bmatrix} 5 & 3 & 1 & 1 \\ 1 & -2 & -1 & 3 \end{bmatrix}$; 180°

In Exercises 29–33, use the diagram of a quilt pattern below.



29. Write a polygon matrix for pentagon $ABCDE$.

30. $A'B'C'D'E'$ is a reflection of $ABCDE$ in the y-axis. Use matrix multiplication to find the image matrix for $A'B'C'D'E'$.

31. $A''B''C''D''E''$ is a reflection of $A'B'C'D'E'$ in the x-axis. Use matrix multiplication to find the image matrix for $A''B''C''D''E''$.

32. What single transformation of $ABCDE$ results in $A''B''C''D''E''$?

33. Multiply the reflection matrices you used in Exercises 30 and 31. How does the product relate to your answer in Exercise 32?

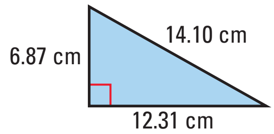
Appendix 3 Estimation and Measurement

GOAL ***Estimate the perimeter, area, surface area, and volume of figures; determine the precision and accuracy of measurements.***

When finding an exact answer is unnecessary, you can use estimation to approximate the perimeter, area, surface area, and volume of geometric figures.

EXAMPLE 1 *Estimating Perimeter and Area*

Estimate the perimeter and area of the triangle.



SOLUTION

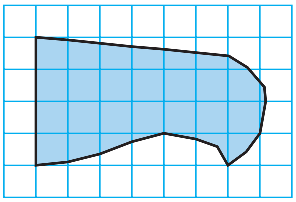
To estimate the perimeter and area, you can round the length of each side to the nearest centimeter and use mental math.

$$P \approx 7 + 12 + 14 = 33 \text{ cm} \qquad A = \frac{1}{2}bh \approx \frac{1}{2}(12)(7) = 42 \text{ cm}^2$$

▶ The perimeter of the triangle is about 33 centimeters, and the area is about 42 square centimeters.

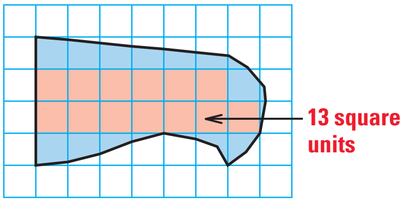
EXAMPLE 2 *Estimating the Area of an Irregular Figure*

Estimate the area, in square units, of the figure.

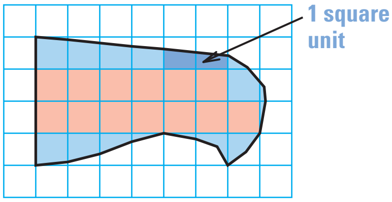


SOLUTION

1. Count the squares that are fully covered. There are 13 fully covered squares.



2. Group the partially covered squares so the combined area is about 1 square unit.

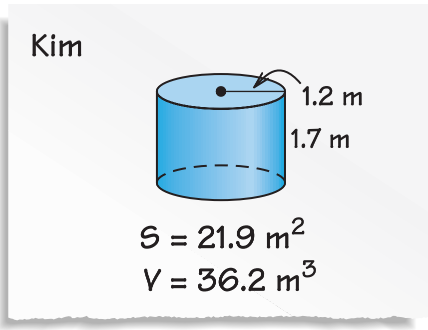


▶ The total area of all of the partially covered squares is about 8 square units, so the area of the figure is about 13 + 8 = 21 square units.

In Example 2, estimation was used when the exact dimensions of a geometric figure were not given. Estimation can also be used to help you check whether an answer is reasonable.

EXAMPLE 3 *Estimating to Check Reasonableness*

Kim was asked to find the surface area S and volume V of the right cylinder shown. Determine whether Kim's calculations shown at the right are reasonable.



SOLUTION

To check whether Kim's calculations are reasonable, estimate the surface area and volume by rounding the radius, height, and pi to the nearest whole number.

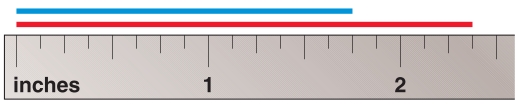
$S = 2\pi r^2 + 2\pi rh$	**Formula for surface area of cylinder**
$\approx 2(3)(1^2) + 2(3)(1)(2)$	**Substitute 3 for π, 1 for r, and 2 for h.**
$= 6 + 12$	**Multiply.**
$= 18 \text{ m}^2$	**Add.**
$V = \pi r^2 h$	**Formula for volume of cylinder**
$\approx 3(1^2)(2)$	**Substitute 3 for π, 1 for r, and 2 for h.**
$= 6 \text{ m}^3$	**Multiply.**

▶ Because 21.9 is close to 18, Kim's calculation of the surface area is reasonable. Because 36.2 is not close to 6, her calculation of the volume is not reasonable.

....

> **STUDENT HELP**
> **Study Tip**
> Of two measurements, the one with the smaller unit of measure is the *more precise* measurement.

MEASUREMENT All measurements are approximations. If you measure the segments below *to the nearest inch,* you find that the red segment is 2 inches, and the blue segment is also 2 inches. In this case, the measurement is to the nearest inch, so the **unit of measure** is 1 inch. The *precision* of a measurement is the unit of measure used.



Therefore, if you are told that an object measures 2 inches, you know that the exact length must be somewhere between $1\frac{1}{2}$ inches and $2\frac{1}{2}$ inches. The actual measurement could be $\frac{1}{2}$ inch greater than or less than 2 inches. Therefore, the **greatest possible error** of the measurement is $\frac{1}{2}$ inch. The greatest possible error of a measurement is always equal to half of the unit of measure.

EXAMPLE 4 *Finding Greatest Possible Error*

In an amusement park guide, the final drop of a log flume ride is listed as 52.3 feet. Find the unit of measure. Then find the greatest possible error.

SOLUTION

The measurement 52.3 feet is given to the nearest tenth of a foot.

Therefore, the unit of measure is $\frac{1}{10}$ foot.

The greatest possible error is $\frac{1}{2}$(unit of measure) $= \frac{1}{2}\left(\frac{1}{10}\right) = \frac{1}{20} = 0.05$ ft.

. . . .

Consider a bicycle tire with a diameter of 26 inches and a key ring with a diameter of 1 inch. In both cases, the greatest possible error is $\frac{1}{2}$ inch. The **relative error** of a measurement is ratio of the greatest possible error and the measured length.

$$\text{relative error of bicycle tire diameter} = \frac{0.5 \text{ in.}}{26 \text{ in.}} \approx 0.01923 \approx 1.9\%$$

$$\text{relative error of key ring diameter} = \frac{0.5 \text{ in.}}{1 \text{ in.}} = 0.5 = 50\%$$

As you can see, the $\frac{1}{2}$ inch error has a much greater effect on the diameter of the smaller object, the key ring. Of two measurements, the one with the smaller relative error is the *more accurate* measurement.

EXAMPLE 5 *Finding Relative Error*

An air hockey table is 3.7 feet wide. An ice rink is 85 feet wide. Find the relative error of each measurement. Then tell which measurement is more accurate.

SOLUTION

Air hockey table

unit of measure = 0.1 ft

$$\text{greatest possible error} = \frac{1}{2}(0.1) = 0.05 \text{ ft}$$

$$\text{relative error} = \frac{0.05 \text{ ft}}{3.7 \text{ ft}} \approx 0.0135 \approx 1.4\%$$

Ice rink

unit of measure = 1 ft

$$\text{greatest possible error} = \frac{1}{2}(1) = 0.5 \text{ ft}$$

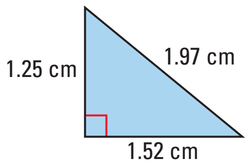
$$\text{relative error} = \frac{0.5 \text{ ft}}{85 \text{ ft}} \approx 0.00588 \approx 0.6\%$$

▶ Because the width of the ice rink has the smaller relative error, it is the more accurate measurement.

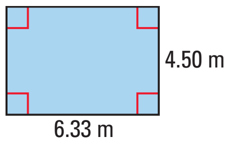
EXERCISES

Estimate the perimeter and area of the figure by rounding each measurement to the nearest whole number.

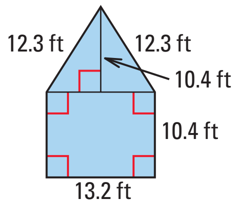
1.



2.



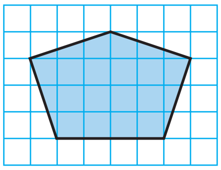
3.



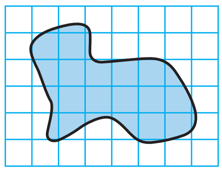
4. You are part of a community clean-up crew. Your task is to paint a wall that measures $24\frac{1}{2}$ feet by $13\frac{1}{4}$ feet. One can of paint covers 400 square feet. Estimate the number of cans of paint you will need to paint the entire wall. Explain your reasoning.

Estimate the area, in square units, of the figure.

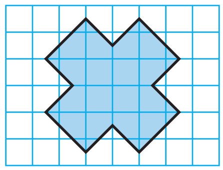
5.



6.

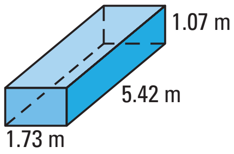


7.

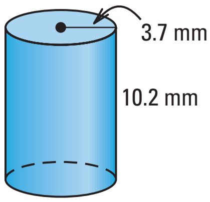


Estimate the volume and surface area of the solid by rounding each measurement to the nearest whole number. Use 3 for π.

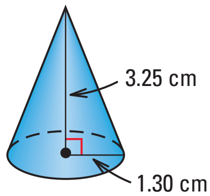
8.



9.



10.



In Exercises 11 and 12, determine whether the student's calculations for surface area and volume are reasonable. Explain your reasoning.

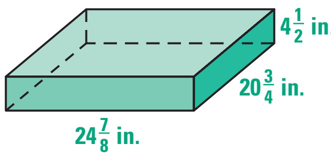
11. Right rectangular prism:
$l = 14.3$ m, $w = 9.9$ m, $h = 4.1$ m

$S = 481.58 \text{ m}^2 \quad V = 175.537 \text{ m}^3$

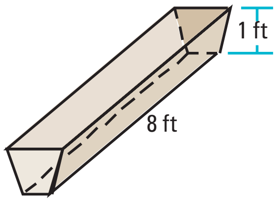
12. Right cylinder:
$r = 9.75$ m, $h = 6.21$ m

$S = 597.0 \text{ m}^2 \quad V = 1853.7 \text{ m}^3$

13. You and a friend are making seat cushions with the dimensions shown. Your friend estimates that you will need about 2300 cubic inches of filler and about 700 square inches of fabric for each cushion. Are your friend's estimates reasonable? Explain.



14. You want to fill the flower box at the right with soil. One bag of soil contains 1 cubic foot of soil. Estimate the number of bags of soil you need to fill the flower box knowing only its height and width. Explain your reasoning.



Find the unit of measure. Then find the greatest possible error.

15. 54 in.

16. 3.15 cm

17. 3 yd

18. 0.32 m

19. $2\frac{3}{8}$ in.

20. $\frac{2}{16}$ ft

21. 17.21 mm

22. 1.033 km

23. 11.0 mm

24. Consider two measurements that both a have greatest possible error of $\frac{1}{16}$ inch. Explain how the error could be problematic in one circumstance, but not in another.

25. You want to estimate the amount of paper you need to make book covers for your textbooks. Of 1 foot, 1 inch, and $\frac{1}{16}$ inch, which unit of measure should you use to measure the dimensions of your textbooks? Explain your reasoning.

Find the relative error of each measurement.

26. 18 in.

27. 6 ft

28. 104 mi

29. 17.5 cm

30. 4.25 m

31. 6.5 yd

32. $\frac{1}{16}$ in.

33. 5.0 mm

34. 1.1 km

In Exercises 35–38, find the greatest possible error of the measurement. Then find the relative error.

35. The diameter of the Earth is *12,756 kilometers.*

36. In one football game, your high school quarterback threw for *212 yards.*

37. The thickness of a quarter is *1.75 millimeters.*

38. The distance between Los Angeles and San Diego is *116 miles.*

39. Explain the difference between the precision of a measurement and the accuracy of a measurement. Provide an example to support your answer.

Tell which measurement is more precise. Then tell which measurement is more accurate.

40. 12 in.; 15 in.

41. 1.75 ft; 18.2 ft

42. 16.0 cm; 100 m

43. 53 yd; 12.0 ft

44. 532 cm; 43.224 km

45. $2\frac{1}{8}$ mi; $3\frac{1}{16}$ km

46. $7\frac{1}{2}$ in.; $3\frac{1}{4}$ ft

47. 18.0 yd; 125 in.

48. 715 ft; 112.3 mi