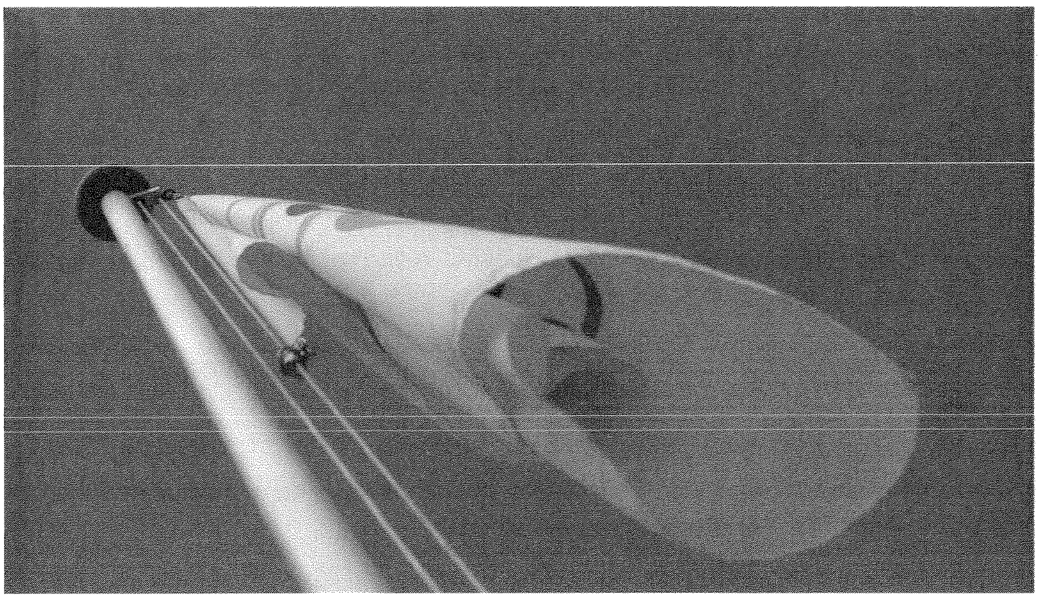
9

Geometric Figures and Their Properties



The highest flagpole in the world is in North Korea and stands 160 meters tall. In Lesson 9.5, you will use indirect measurement to find the height of a flagpole.

9

Geometry Introduction

Geometry is the branch of mathematics that studies the size and shape of things. The basic building blocks of geometry are points, lines, and planes. In this chapter, you will explore angles and polygons consisting of rays and line segments. You will also learn to make decisions about whether polygons are mathematically similar or exactly the same.

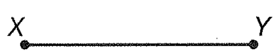
The following terms are common geometric terms that you should remember from earlier mathematics courses.

A *point* represents a position in space with no size.

Point *A* is represented by this dot.

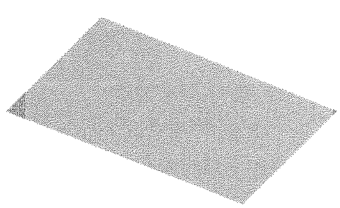
A *line* is a collection of points that extend infinitely in opposite directions. A line is named using two points on the line. For example, line *BC* is shown.

A *line segment* is a piece of a line with two endpoints. A line segment is named by its two endpoints. For example, line segment *XY* is shown.



A *ray* extends forever in one direction with one endpoint. A ray is named by its endpoint and another point on the ray. For example, ray *ST* is shown.

A *plane* is a flat surface with no thickness that extends forever in two dimensions.



9.1 Figuring All of the Angles

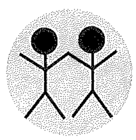
Angles and Angle Pairs

Objectives

In this lesson, you will:

- Determine measures of angles.
- Identify special angle pairs.

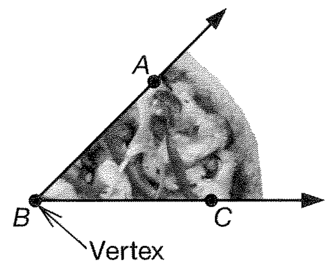
Key Terms



- angle
- vertex
- degrees
- right angle
- straight angle
- acute angle
- obtuse angle
- protractor
- complementary angle
- supplementary angle
- transversal
- congruent angles
- alternate interior angles
- alternate exterior angles
- corresponding angles
- vertical angles
- adjacent angles

Problem 1

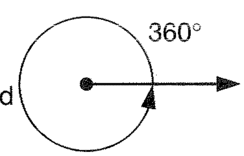
Your aunt is opening an Italian restaurant. You volunteer to help her on the weekends. One of your jobs is to slice the pizzas as they come out of the oven. You notice that the pieces have a smaller or larger angle at the center of the slice depending on the number of pieces into which the customer wants a pizza cut.



Imagine that the sides of a slice of pizza are two different rays with the same endpoint. These rays connect to form an **angle.** The endpoint is called the **vertex** of the angle.

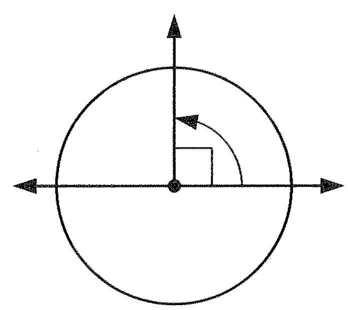
The angle in the figure at the right has several names: $\angle B$, $\angle ABC$, and $\angle CBA$. Notice that when you use three letters to name an angle, the vertex is always the middle letter.

The measure of an angle gives the size of the opening between its sides. The unit used to measure angles is **degrees** (°).

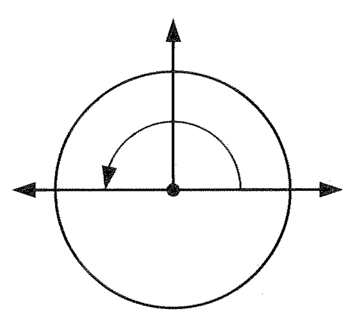


Imagine that you slice an extra-large round pizza into 360 equal pieces. The angle of each slice measures one degree (1°). There are 360° around a point (in this case, the center of the pizza).

The angle formed by a slice that is one quarter of the pizza measures 90°. This is a **right angle,** and is shown by placing a corner, ⌉ , at its vertex.

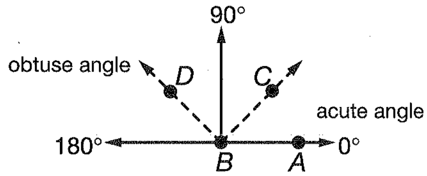

right angle

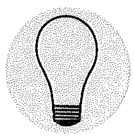
The angle formed by a slice that is one half of the pizza is a **straight angle.** Determine the measure of a straight angle. Use a complete sentence to explain how you determined the measure.


straight angle

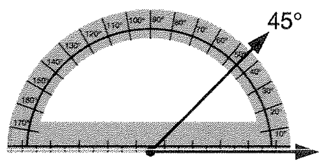
Investigate Problem 1

1. Angles that measure less than 90° are **acute angles.** Angles that measure between 90° and 180° are **obtuse angles.** The obtuse angle in the diagram below is ∠*ABD*. Name the acute angle.

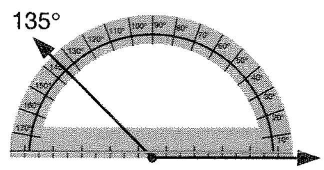




2. You can measure angles using a tool called a **protractor.** Align the center of the protractor with the vertex of the angle. Each line on the protractor has two measures—one for an acute angle and one for an obtuse angle.



Measuring an acute angle



Measuring an obtuse angle

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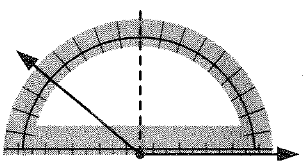
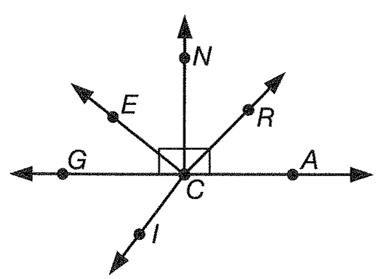
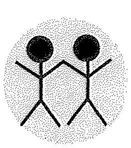


A right angle, which measures 90°, is a useful benchmark, or reference angle. If you compare your angle to a right angle and identify it as acute or obtuse *before* using a protractor to measure it, you will not be confused by which mark to use on the protractor. Would you estimate the measure of the angle at the left to be 40° or 140°? Use a complete sentence to explain your reasoning.

Use your protractor to measure the angles below. Don't forget to first identify each angle as *right*, *acute*, or *obtuse*. Record the angle measures in the table.

Angle	Type of Angle	Measure
∠*ACR*		
∠*ACN*		
∠*RCN*		
∠*ACE*		
∠*ECG*		
∠*ECI*		





3. Compare your measurements with your partner. If you have an angle measure that is more than 2° greater or less than your partner's measure of the same angle, measure the angle again together.

Investigate Problem 1

4. **Math Path: Complementary and Supplementary Angles**

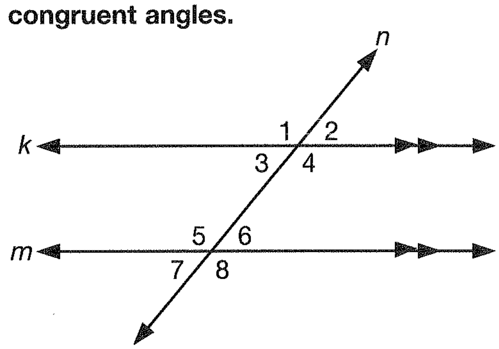
What is the sum of the measure of $\angle ACR$ and the measure of $\angle RCN$?

Two angles whose sum is 90° (a right angle) are **complementary angles.** Which pairs of angles in the figure in Question 2 are complementary angles?

Two angles whose sum is 180° (a straight angle) are **supplementary angles.** Which pairs of angles in the figure in Question 2 are supplementary angles?

5. **Math Path: Special Pairs of Angles**

In the figure below, parallel lines *k* and *m* are cut by line *n*, called a **transversal.** When a transversal intersects two parallel lines, certain pairs of angles are formed that have equal measures. When angles have equal measures, the angles are said to be **congruent angles.**



$\angle 3$ and $\angle 6$ are **alternate interior angles.**

$\angle 4$ and $\angle 5$ are alternate interior angles.

$\angle 1$ and $\angle 8$ are **alternate exterior angles.**

$\angle 2$ and $\angle 7$ are alternate exterior angles.

What do you notice about these angle pairs? Use a complete sentence in your answer.

The following angle pairs are **corresponding angles.**

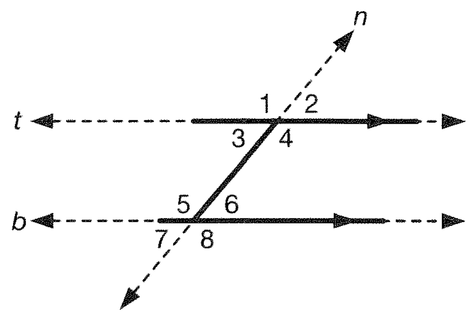
$\angle 1$ and $\angle 5$	$\angle 3$ and $\angle 7$
$\angle 2$ and $\angle 6$	$\angle 4$ and $\angle 8$

What do you notice about these angle pairs? Use a complete sentence in your answer.

9

Problem 2

Your aunt wants you to help her make new tables for the restaurant. You create a drawing of your design for the new tables. In the drawing, the top of the table, represented by line *t*, is parallel to the bench, represented by line *b*. The brace is represented by line *n*.



A. Use your protractor to measure all of the angles in the drawing and record them in the table. Remember to estimate before measuring.

Angle	Measure	Angle	Measure
∠1		∠5	
∠2		∠6	
∠3		∠7	
∠4		∠8	

Take Note

The symbol "*m*∠1" means "the measure of angle 1."

B. Complete the table at the right. What do you notice about these angle measures? Use complete sentences to describe your findings.

Sums of Angles	
m∠1 + *m*∠2	
m∠3 + *m*∠4	
m∠1 + *m*∠3	
m∠2 + *m*∠4	
m∠5 + *m*∠6	
m∠7 + *m*∠8	
m∠5 + *m*∠7	
m∠6 + *m*∠8	

C. Compare your observations with those of your partner. Are your observations the same or different?

D. Create a new set of angles at the right using two parallel lines and a transversal. Label the angles formed. Have your partner measure all of the angles.

9

Investigate Problem 2

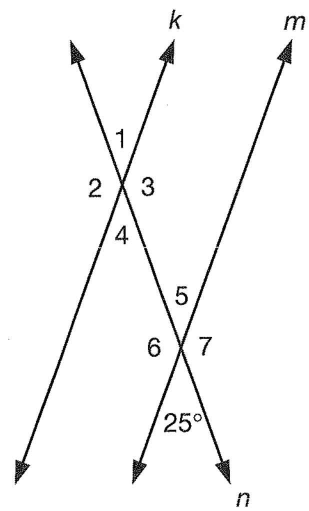
1. Math Path: Vertical and Adjacent Angles

A pair of opposite angles formed by two intersecting lines are **vertical angles.** Vertical angles have equal measures. Angles that share a side are **adjacent angles.**

Name all of the vertical angles in the drawing in Problem 2.

Name all of the adjacent angles in the drawing in Problem 2.

2. In the figure, line *k* is parallel to line *m*. Find the measures of all of the angles without using your protractor.



$m\angle 1 =$

$m\angle 2 =$

$m\angle 3 =$

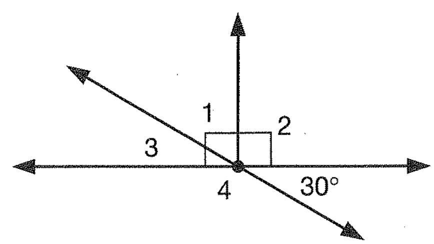
$m\angle 4 =$

$m\angle 5 =$

$m\angle 6 =$

$m\angle 7 =$

3. Find the measures of all of the angles in the figure without using your protractor.



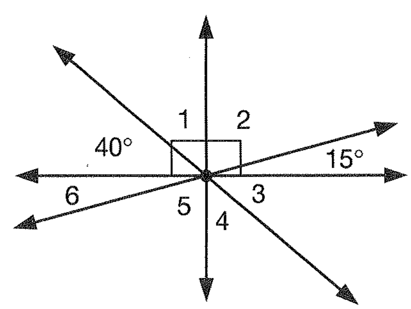
$m\angle 1 =$

$m\angle 2 =$

$m\angle 3 =$

$m\angle 4 =$

4. Find the measures of all of the angles in the figure without using your protractor.



$m\angle 1 =$

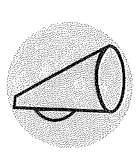
$m\angle 2 =$

$m\angle 3 =$

$m\angle 4 =$

$m\angle 5 =$

$m\angle 6 =$



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9.2 A Collection of Triangles
Classifying Triangles

Objectives

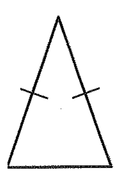
In this lesson, you will:
- Classify triangles by their side lengths.
- Classify triangles by their angle measures.

Key Terms

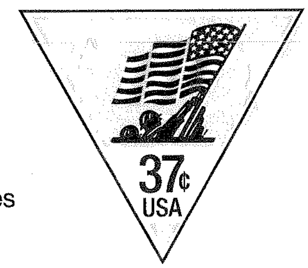
- triangle
- congruent sides
- equilateral triangle
- isosceles triangle
- scalene triangle
- acute triangle
- obtuse triangle
- right triangle

Take Note

When the lengths of the sides of a triangle are equal, they are marked with a dash.

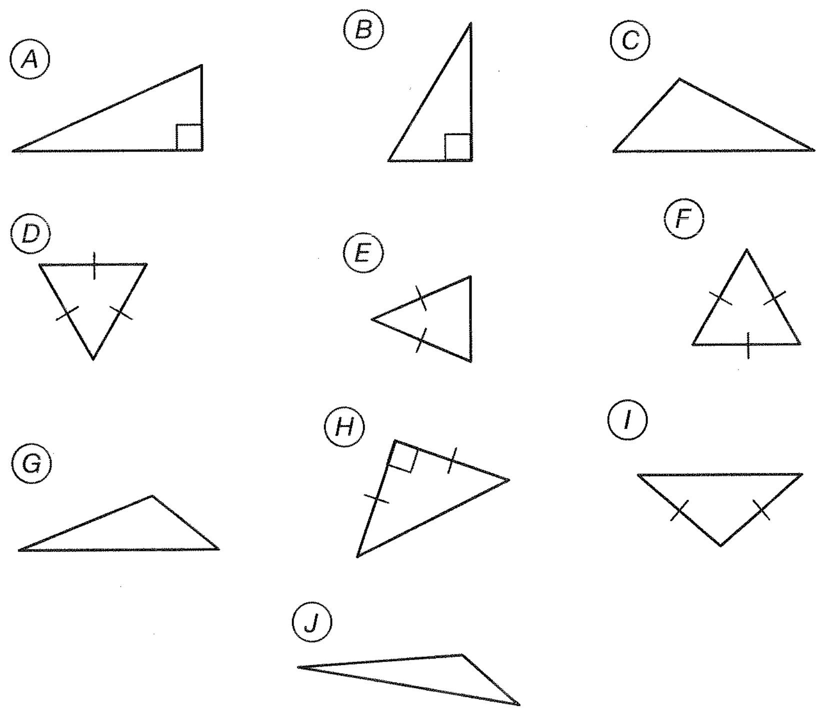


Problem 1



Most postage stamps are shaped as rectangles or squares. In some rare cases though, world governments have issued stamps shaped as triangles. A **triangle** is a figure that has three sides and three angles. In 1997, the United States issued its first triangular stamp.

Your cousin collects rare triangular stamps. The outlines of the stamps are shown below. Help your cousin by sorting the triangles that represent the stamps into groups so that each group is described by one characteristic. You can refer to the triangle by the letter.



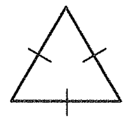
Group	Triangles	Characteristics
1		
2		
3		
4		

Investigate Problem 1

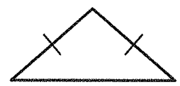
1. In the table in Problem 1, did some of the triangles fit into more than one category? If so, which ones?

2. **Math Path: Classifying Triangles**

   We can classify triangles by their side lengths. Just as congruent angles have the same measure, **congruent sides** have the same length.



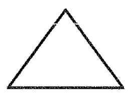
An **equilateral triangle** has three congruent sides.



An **isosceles triangle** has two congruent sides.

A **scalene triangle** has no congruent sides.

We can also classify triangles by their angles.

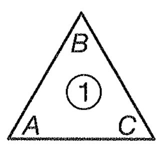


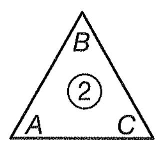
An **acute triangle** has three acute angles

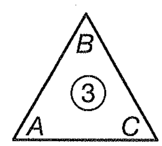
An **obtuse triangle** has one obtuse angle.

A **right triangle** has one right angle.

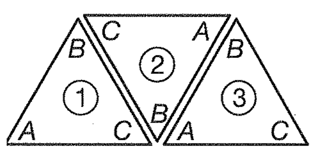
Cut out three exact copies of the same triangle. Label the angles *A*, *B*, and *C*.







Arrange the three triangles so that one of each of angle *A*, angle *B*, and angle *C* are next to each other.



What do you notice? Write your answer using a complete sentence.

Did you get the same result as others in your group? Use complete sentences to explain.

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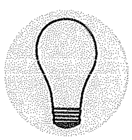
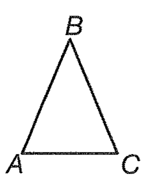
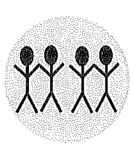


Take Note

Triangles can be named using the three vertices of the triangle in any order.



Triangle *ABC*, Triangle *BCA*, or Triangle *CAB*.

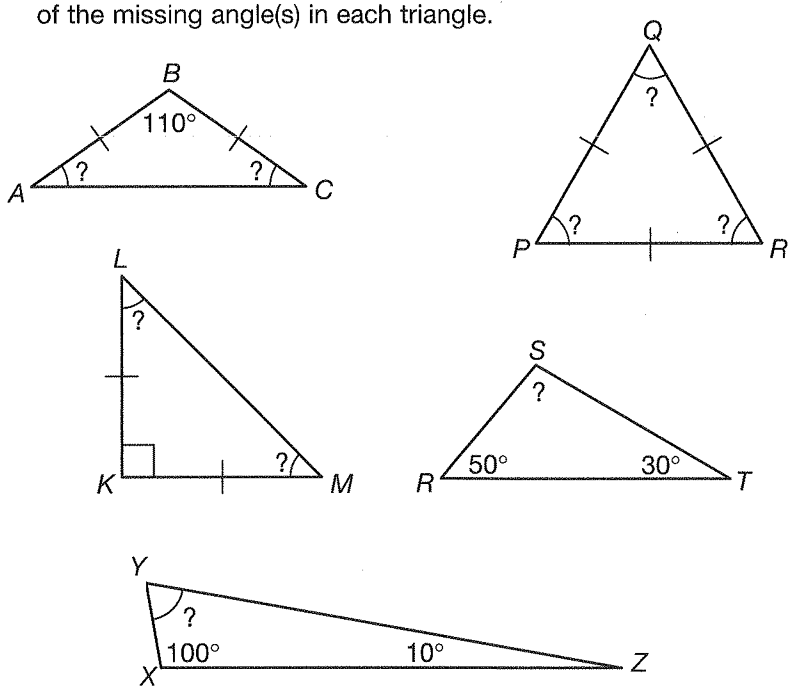


Investigate Problem 1

Take Note

In an isosceles triangle, the angles opposite the congruent sides have the same measure. In an equilateral triangle, all three angles have the same measure.

3. The straight line that is formed by angle *A*, angle *B*, and angle *C* is a visual representation of the fact that the sum of the measures of the three angles in a triangle is always 180°. Find the measure of the missing angle(s) in each triangle.



Problem 2

Decide whether it is possible to design a stamp that has each shape described below. If it is possible, draw an example. If it is not possible, explain why.

A right isosceles triangle

An acute isosceles triangle

An obtuse scalene triangle

An equilateral right triangle

An obtuse isosceles triangle

A scalene right triangle

An equilateral acute triangle

An obtuse equilateral triangle

Investigate Problem 2

1. With your partner, use a ruler to measure and mark 2 inches on a straw or piece of spaghetti. Cut the straw or piece of spaghetti into the 2-inch length. Repeat this process to make pieces whose lengths are whole inches from 3 inches up to 12 inches.

   For each set in the table below, toss a pair of number cubes three times. Record the three numbers in the table under "Side Lengths." These numbers represent the lengths of the sides of a triangle. Use your straws or pieces of spaghetti to attempt to form a triangle with these side lengths. Record whether or not you can form a triangle.

	Side Lengths	Do the Side Lengths Form a Triangle? (Yes or No)
Set 1		
Set 2		
Set 3		
Set 4		
Set 5		
Set 6		
Set 7		

   What do you notice about the sets that form a triangle? What do you notice about the sets that do not form a triangle? Use complete sentences to record your observations.

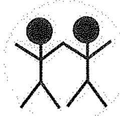
9.3 The Signs Are Everywhere

Quadrilaterals and Other Polygons

Objectives

In this lesson, you will:

- Classify quadrilaterals.
- Classify polygons.
- Find angle measures in polygons.

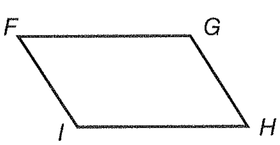


Key Terms

- quadrilateral
- trapezoid
- parallelogram
- rhombus
- rectangle
- square
- polygon
- diagonal
- regular polygon
- irregular polygon

Take Note

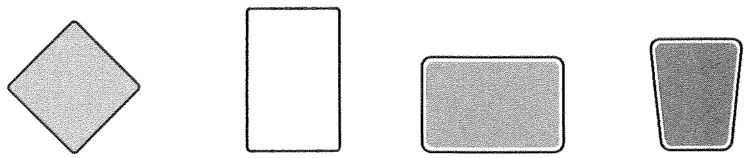
Quadrilaterals can be named using the four vertices by either listing them in order clockwise or counterclockwise.



Quadrilateral *FGHI* or Quadrilateral *FIHG*

Problem 1

You are teaching a bicycle safety course to a group of third graders. You want the class to learn the road signs that are put up for bicyclists. You tell them that there are standard shapes for road signs. Some of these shapes are *quadrilaterals.* A **quadrilateral** is a closed figure with four sides and four angles. Like triangles, quadrilaterals are classified by their sides and angles.



Warning signs **Regulation signs** **Guide signs** **Recreational area signs**

A. A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. Which types of road signs, if any, appear to be trapezoids?

B. A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel. Which types of road signs, if any, appear to be parallelograms?

C. A **rhombus** is a parallelogram with four sides of equal length. Which types of road signs, if any, appear to be rhombuses?

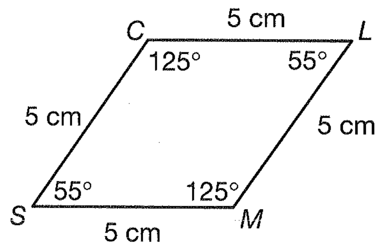
D. A **rectangle** is a parallelogram with 4 right angles. Which types of road signs, if any, appear to be rectangles?

E. A **square** is a parallelogram with 4 sides of equal length and 4 right angles. Which types of road signs, if any, appear to be squares?

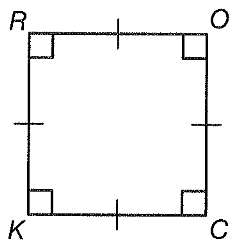
9

Investigate Problem 1

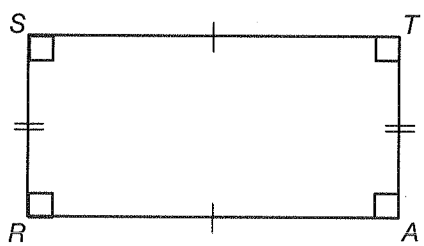
1. Write as many names as you can for quadrilateral *CLMS*.



Write five names for quadrilateral *ROCK*.



Write as many names as you can for quadrilateral *STAR*.



2. Work with your partner to answer each question. If your answer is "no," then draw a quadrilateral to justify your answer.

Are all rectangles parallelograms?

Are all parallelograms rectangles?

Are all squares rectangles?

Are all rectangles squares?

Are all rhombuses squares?

Are all squares rhombuses?

Are all quadrilaterals parallelograms?

Are all parallelograms quadrilaterals?

9

Problem 2

You want to teach your students to recognize other common road signs. Except for circular signs, almost all road signs are polygons. A **polygon** is a closed figure whose sides are line segments. Polygons can be classified by the number of their sides.

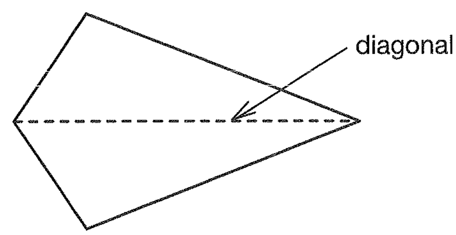
The prefix "penta-" means "five." Which road signs are pentagons?

The prefix "octa-" means "eight." Which road signs are octagons?

Investigate Problem 2

9

1. In Lesson 9.2, we learned that the sum of the measures of the angles of a triangle is 180°. What about the sum of the measures of the angles of other polygons? We can use triangles to help find this sum. A **diagonal** is a line segment in a polygon that connects two vertices that are not connected by a side. When you draw a diagonal in the quadrilateral below, you can see that the quadrilateral is made up of two triangles.



Because the sum of the measures of the angles of each triangle above is 180°, the sum of the measures of the angles of a quadrilateral is 2 × 180° or 360°. At the left, draw each polygon listed in the table. Then complete the table.

Polygon	Number of Sides	Number of Triangles	Sum of the Measures of the Angles of the Polygon
Quadrilateral	4	2	2 × 180° or 360°
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		

Investigate Problem 2

2. Math Path: Regular Polygons

In a **regular polygon,** all of the sides have the same length and all of the angles have the same measure. A polygon that is not regular is **irregular.** What are the measures of each angle of a regular pentagon? Use complete sentences to explain.

What are the measures of each angle of a regular hexagon? Use complete sentences to explain.

What are the measures of each angle of a regular octagon? Use complete sentences to explain.

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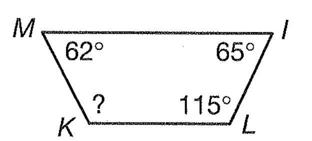
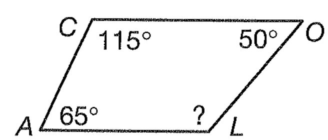
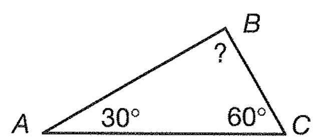
3. Find the measure of the missing angle of each polygon. For each polygon, explain your reasoning.



$m\angle K =$ ____________



$m\angle L =$ ____________



$m\angle B =$ ____________

4. In Lesson 9.2, we learned that the length of the third side of a triangle is always less than the sum of the lengths of the other two sides. Does this apply to quadrilaterals? Does the length of the fourth side of a quadrilateral always have to be less than the sum of the lengths of the other three sides? Work with your partner to write an explanation of your thinking.

9.4 How Does Your Garden Grow?

Similar Polygons

Objectives

In this lesson, you will:

- Determine whether polygons are similar.

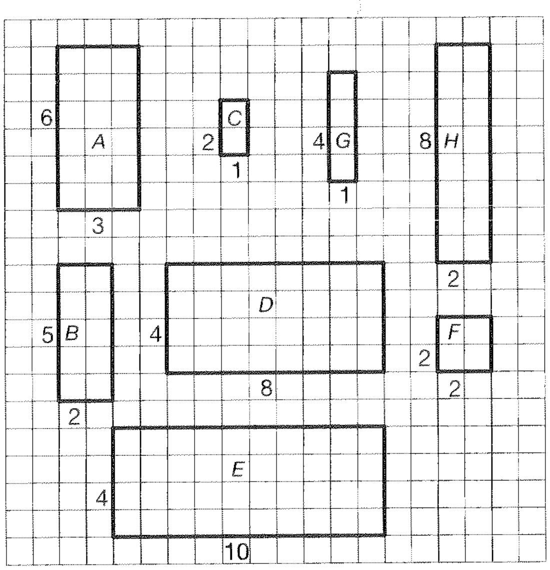
Key Terms

- similar polygons
- corresponding sides
- scale factor
- corresponding angles

If you have ever built a scale model of a car or an airplane or enlarged a picture on a photocopy machine, you have used similar polygons.

Problem 1

After reading a book about landscape design, you want to design two large flower beds that are the same shape but different sizes. You use rectangles to represent the flower beds. Polygons that have the same shape but not necessarily the same size are **similar polygons.** Complete the table to investigate the properties of similar polygons.



	Length, ℓ (units of longer side)	**Width, w (units of shorter side)**	**Ratio of Width to Length $\frac{w}{\ell}$**
A			
B			
C			
D			
E			
F			
G			
H			

9

Investigate Problem 1

1. Group the rectangles that have equal ratios of width to length. How many different groups do you have? What can you say about the rectangles in each group? Write your answer using a complete sentence.

2. **Math Path: Scale Factor**

   Corresponding sides of figures are in corresponding positions in different figures. A **scale factor** is the ratio of the lengths of the corresponding sides of two figures that are similar. You find the scale factor by taking the ratio of the lengths of the corresponding sides of the new figure to the original figure. A scale factor greater than 1 means that the original figure has been enlarged to form the new figure. A scale factor between 0 and 1 means that the original figure has been reduced to form the new figure. What are the scale factors of the similar rectangles?

   To enlarge rectangle *C* to rectangle *D*, the scale factor is _____.

   To reduce rectangle *E* to rectangle *B*, the scale factor is _____.

   To enlarge rectangle *G* to rectangle *H*, the scale factor is _____.

   To reduce rectangle *H* to rectangle *G*, the scale factor is _____.

   To enlarge rectangle *B* to rectangle *E*, the scale factor is _____.

3. Give an example of two squares and find the ratio of the sides of the squares. Do you think that all squares are similar to all other squares? Use complete sentences to explain why or why not.

4. The quadrilateral *MATH* is similar to the quadrilateral *POLG*. Complete each statement to find the ratios of the lengths of the corresponding sides.

   $\frac{MA}{PO} = \frac{\square}{\square}$ $\frac{AT}{OL} = \frac{\square}{\square}$ $\frac{TH}{LG} = \frac{\square}{\square}$ $\frac{HM}{GP} = \frac{\square}{\square}$

   Use complete sentences to explain what you discovered about the ratios of the lengths of the corresponding sides of similar polygons.

Take Note

When naming two figures that are similar, the vertices should be written in order so that they correspond. In Question 4, point *M* corresponds to point *P*, point *A* corresponds to point *O*, etc.

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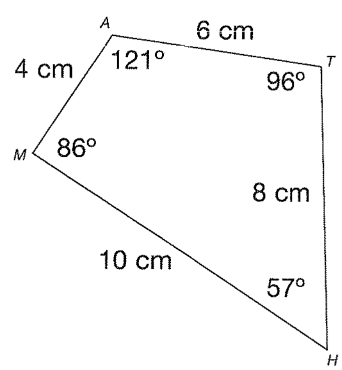
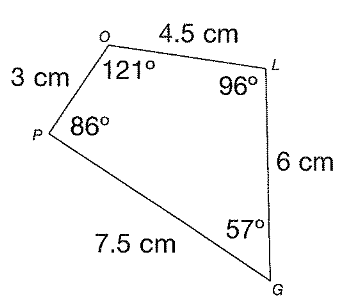




Investigate Problem 1

5. **Corresponding angles** of two figures are angles that are in corresponding positions in both figures. Write the measures of corresponding angles in quadrilateral *MATH* and quadrilateral *POLG*.

$m\angle M$: $m\angle A$: $m\angle T$: $m\angle H$:

$m\angle P$: $m\angle O$: $m\angle L$: $m\angle G$:

Use complete sentences to explain what you discovered about the measures of corresponding angles of similar polygons.

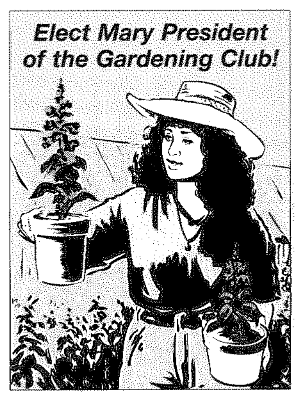
Problem 2

Your community has a gardening club, and you decide to run for president of the club. You want to make different sized posters with your smiling picture and hang them throughout the community.

9

A. Your best picture is 3 inches by 4 inches. You want each poster that you make to be similar to the original photo. What sizes of posters can you make? Complete the table.

Size	Scale Factor
3 in. × 4 in.	1



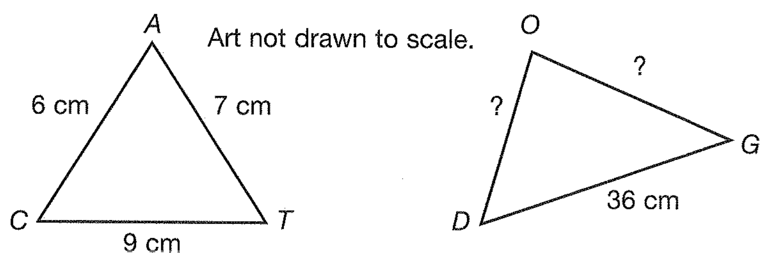
B. Choose one of the sizes from your table. For that size, show that the poster and the original photo are similar. (You can assume that all of the angles of the poster and the original photo are right angles.) Use complete sentences in your answer.

Investigate Problem 2

1. After you are elected, you plan to have a contest to design a poster that will encourage people in the community to plant trees. The rules of the contest will state that the design should be drawn on a small sheet of paper that is 3 inches by 9 inches. What sizes of poster paper should you buy to enlarge the poster so that it is similar to the drawing and has the same proportions? Show mathematically whether the sizes in the table will work.

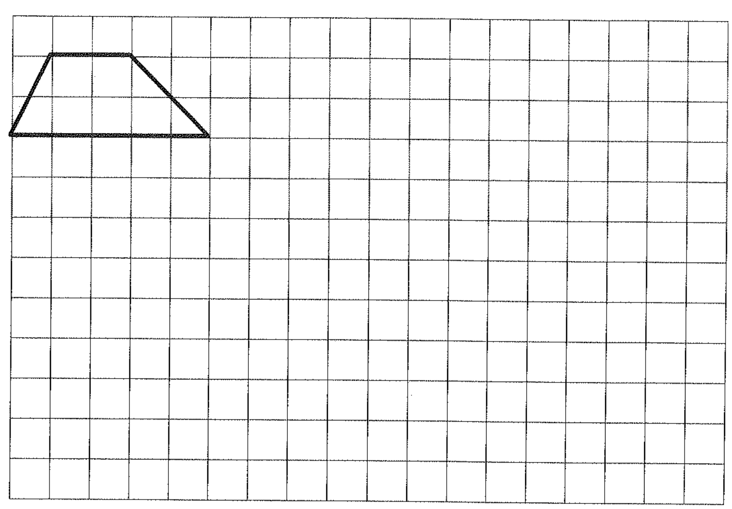
Poster Paper	
Length	**Width**
1 ft	3 ft
6 in.	12 in.
12 in.	36 in.
16 in.	48 in.
9 in.	36 in.

2. Triangle *CAT* is similar to triangle *DOG*. Find the missing lengths.



DO = ____________ *GO* = ____________

3. Draw two quadrilaterals that are similar to the quadrilateral below so that one uses a scale factor of 2 and another uses a scale factor of 3.



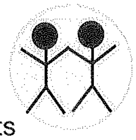
9.5 Shadows and Mirrors

Indirect Measurement

Objectives

In this lesson, you will:

- Use similar triangles to find measurements indirectly.



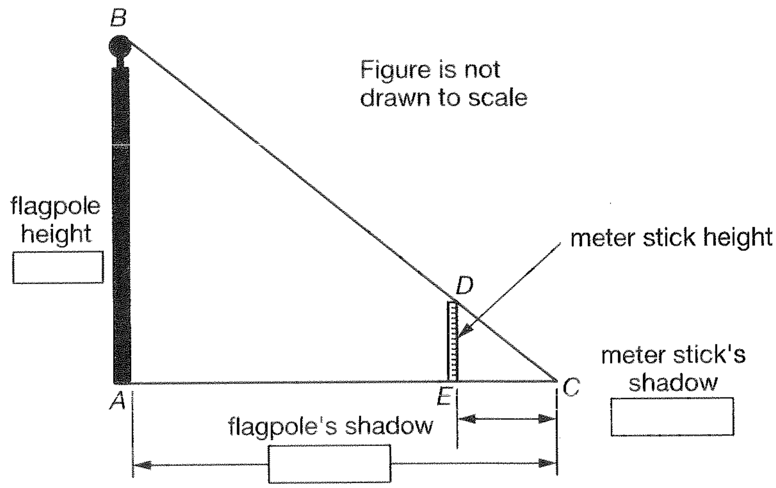
Key Term

- similar triangles
- indirect measurement

Problem 1

You can use **similar triangles** to measure the height of a flagpole, telephone pole, or any object that is not easily measured directly. This type of measurement is called **indirect measurement.**

A. On a sunny day, hold a meter stick so that it makes a right angle with the ground. Measure the length of the meter stick's shadow. Next, measure the length of the flagpole or telephone pole's shadow. Use your measurements to label the diagram.



B. In the diagram, name the triangles that are similar. Discuss with your partner how you know that the triangles are similar.

C. In Lesson 9.4, we saw that the ratios of the lengths of the corresponding sides of similar polygons are equal. Use this fact to complete the proportion below. Then solve the proportion to find the height of the flagpole.

$$\frac{\text{Height of flagpole}}{\text{Height of meter stick}} = \frac{\text{Length of flagpole's shadow}}{\text{Length of meter stick's shadow}}$$

$$\frac{?}{\text{1 meter}} = \frac{\square \text{ meters}}{\square \text{ meter(s)}}$$

9

Investigate Problem 1

1. Suppose that your meter stick casts a 0.5-meter shadow and the flagpole casts a 6-meter shadow. Draw and label the similar triangles involved. Then write and solve a proportion to find the approximate height of the flagpole.

2. A second method for indirectly measuring the height of a flagpole is to use a mirror. Have your partner measure you from the ground to your eye level. Then measure a convenient distance from the base of the flagpole to a level spot. Place the mirror flat on the ground at that level spot. Now, back up from the mirror until you can see the top of the flagpole in the mirror. Mark the spot at which you are standing. Then measure the distance from the mirror to that spot. Use your measurements to label the diagram.

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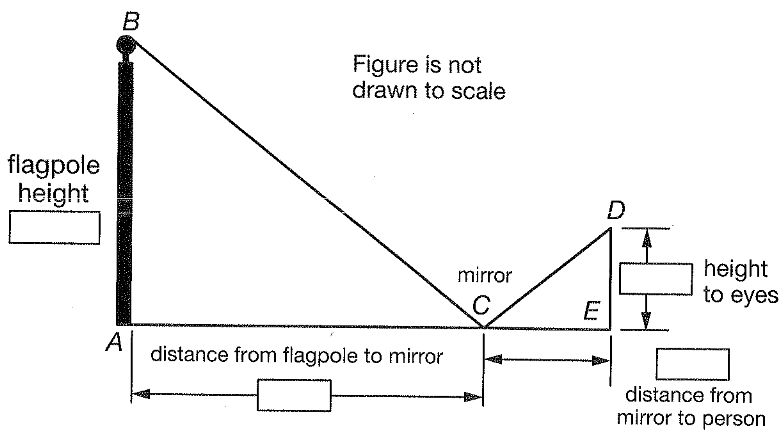


Use the diagram to write and solve a proportion to find the height of the flagpole.

$$\frac{\text{Height of flagpole}}{\text{Height to eyes}} = \frac{\square}{\square}$$

$$\frac{?}{\square \text{ meters}} = \frac{\square \text{ meters}}{\square \text{ meters}}$$

The height of the flagpole is ______ meters.

Investigate Problem 1

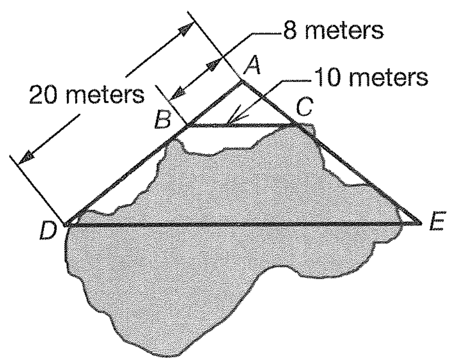
3. Suppose that you used the method in Question 2 and found the measurements below. Use similar triangles to find the height of the flagpole.

height to eyes = 150 centimeters

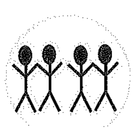
distance from mirror to person = 100 centimeters

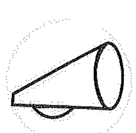
distance from flagpole to mirror = 600 centimeters

4. Miguel and Bill want to measure the length of a lake in Carnegie Park. On a diagram, they mark this length as segment *DE*. To indirectly measure the length, first they locate a point *A* from which they can see the edge of the lake on both sides at point *D* and point *E*. From point *A*, they use chalk to mark straight lines to both edges of the lake. Then they stake points *B* and *C* so that the line that they mark from point *B* to point *C* is parallel to segment *DE*. Then they measure the distances as shown in the diagram. How long is the lake?



Use complete sentences to describe how you found the length of the lake. Then share your solution with another partner team.





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9.6 A Geometry Game

Congruent Polygons

Objectives

In this lesson, you will:

- Use properties of congruent and similar polygons.
- Determine whether polygons are congruent.

Key Terms

- congruent polygons
- congruent

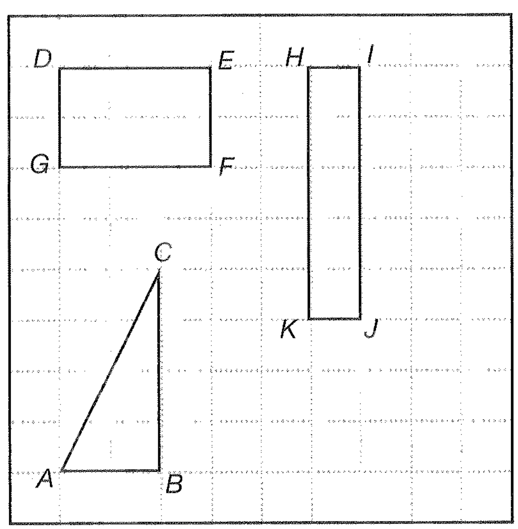
In Lesson 9.4, we saw that polygons that have the same shape but not necessarily the same size are similar polygons. **Congruent polygons** are polygons that have the same shape and the same size.

Problem 1

Because congruent polygons have the same size and shape, their corresponding sides are congruent. That is, the lengths of their corresponding sides are equal. Likewise, the corresponding angles of congruent polygons are **congruent.** That is, the measures of corresponding angles are equal. Congruent polygons have a scale factor of 1.

Design a geometry game board using triangles and rectangles. Include the following on your game board on the next page.

- A triangle congruent to triangle *ABC*
- A triangle similar to triangle *ABC* with a scale factor of 0.5
- A triangle similar to triangle *ABC* with a scale factor of 1.5
- A rectangle congruent to rectangle *DEFG*
- A rectangle similar to rectangle *DEFG* with a scale factor of 0.5
- A rectangle similar to rectangle *DEFG* with a scale factor of 2
- A rectangle congruent to rectangle *HIJK*
- A rectangle similar to rectangle *HIJK* with a scale factor of 0.5
- A rectangle similar to rectangle *HIJK* with a scale factor of 2.5



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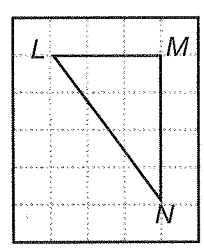
Investigate Problem 1

1. Form a group with another partner team. Each partner team should use the game board that they designed. Take turns asking the questions to each team below. If a team answers the question correctly, they can color in the polygon that pertains to that question. The team with the most polygons colored in at the end of the allowed time wins the game.

- The measure of $\angle B$ is about 26.6°. What is the measure of the angle that corresponds to angle B in the triangle that is congruent to triangle ABC?
- The length of IJ is 5 units. What is the length of the corresponding side in the rectangle with a scale factor of 2.5?
- The measure of $\angle A$ is about 63.4°. What is the measure of the angle that corresponds to angle A in the triangle that is similar to triangle ABC with a scale factor of 1.5?
- The length of DG is 2 units. What is the length of the corresponding side in the rectangle with a scale factor of 0.5?
- The ratio of the length of the shorter side of a rectangle to the longer side is 0.2. Is this rectangle similar to rectangle $DEFG$ or rectangle $HIJK$?
- The length of side BC is 4 units. What is the length of the corresponding side in the triangle with a scale factor of 0.5?
- The measure of $\angle F$ is 90°. What is the measure of the angle that corresponds to angle F in the rectangle that is similar to rectangle $DEFG$ with a scale factor of 2?
- The perimeter of a rectangle is the distance around the rectangle. What is the perimeter of the rectangle that is similar to rectangle $HIJK$ with a scale factor of 0.5?
- What is the perimeter of the rectangle that is congruent to rectangle $DEFG$?

2. If time permits, add a right triangle, triangle LMN, to your game board with side lengths of 3 units, 4 units, and 5 units, like the one at the right. Then add a similar, but not congruent, triangle and a congruent triangle to the board. Work with your partner to design additional questions that you can ask the other partner team about the triangles. Then continue to play the game using the questions that you created.



9

Looking Back at Chapter 9

Key Terms

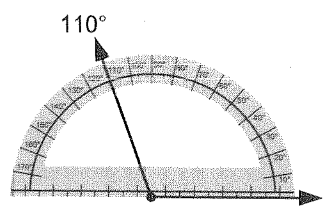
angle p. 273
vertex p. 273
degrees p. 273
right angle p. 273
straight angle p. 273
acute angle p. 274
obtuse angle p. 274
protractor p. 274
complementary angles p. 275
supplementary angles p. 275
transversal p. 275
congruent angles p. 275
alternate interior angles p. 275
alternate exterior angles p. 275
corresponding angles p. 275
vertical angles p. 277
adjacent angles p. 277
triangle p. 279
equilateral triangle p. 280
isosceles triangle p. 280
scalene triangle p. 280
acute triangle p. 280
obtuse triangle p. 280
right triangle p. 280
quadrilateral p. 283
trapezoid p. 283
parallelogram p. 283
rhombus p. 283
rectangle p. 283
square p. 283
polygon p. 285
diagonal p. 285
regular polygon p. 286
irregular polygon p. 286
similar polygons p. 287
corresponding sides p. 288
scale factor p. 288
corresponding angles p. 289
similar triangles p. 291
indirect measurement p. 291
congruent polygons p. 295
congruent p. 295

Summary

Finding Measures of Angles (p. 274)

To determine the measure of an angle, use a protractor. Align the center of the protractor with the vertex of the angle. Each line on the protractor has two measures—one for an acute angle and one for an obtuse angle.

Example

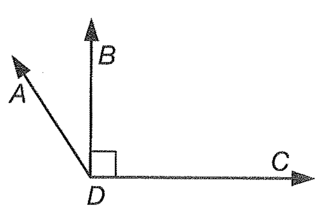


The measure of the angle is 110°.

Identifying Acute, Right, and Obtuse Angles (p. 274)

Angles that measure less than 90° are acute angles. Angles that measure between 90° and 180° are obtuse angles. Angles that measure exactly 90° are right angles.

Examples



∠*ADB* is an acute angle.

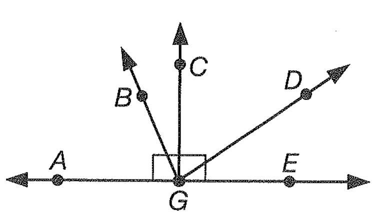
∠*BDC* is a right angle.

∠*ADC* is an obtuse angle.

Identifying Complementary and Supplementary Angles (p. 275)

Two angles whose sum is 90° are complementary angles. Two angles whose sum is 180° are supplementary angles.

Examples



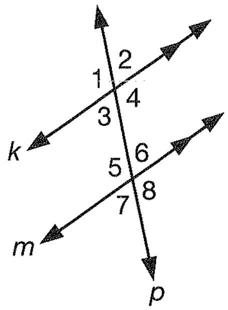
∠*AGB* and ∠*BGC* are complementary.

∠*AGC* and ∠*CGE* are supplementary.

Identifying Special Pairs of Angles (p. 275, 277)

The pairs of angles formed by parallel lines that are cut by a transversal include alternate interior angles, alternate exterior angles, and corresponding angles. Vertical angles are a pair of opposite angles formed by two intersecting lines. Each of these special pairs of angles are congruent. Angles that share a side are adjacent angles and are not necessarily congruent.

Examples



The pairs of alternate interior angles are ∠4 and ∠5 and ∠3 and ∠6.

The pairs of alternate exterior angles are ∠1 and ∠8 and ∠2 and ∠7.

The pairs of corresponding angles are ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, and ∠4 and ∠8.

The pairs of vertical angles are ∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, and ∠6 and ∠7.

The pairs of adjacent angles are ∠1 and ∠2, ∠2 and ∠4, ∠4 and ∠3, ∠3 and ∠1, ∠5 and ∠6, ∠6 and ∠8, ∠8 and ∠7, and ∠7 and ∠5.

Finding Measures of Angles Without Using a Protractor (p. 276–277)

To find the measures of all the angles without using a protractor, use what you know about pairs of angles.

Examples

∠1 and the angle that measures 110° are vertical angles. So, $m\angle 1 = 110°$.

∠1 and ∠2 are supplementary angles. So, $m\angle 2 = 180° - 110° = 70°$.

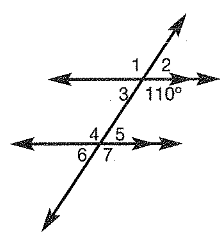
∠2 and ∠3 are vertical angles. So, $m\angle 3 = 70°$.

∠4 and ∠1 are corresponding angles. So, $m\angle 4 = 110°$.

∠5 and ∠3 are alternate interior angles. So, $m\angle 5 = 70°$.

∠5 and ∠6 are vertical angles. So, $m\angle 6 = 70°$.

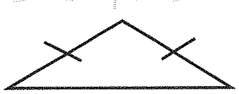
∠4 and ∠7 are vertical angles. So, $m\angle 7 = 110°$.



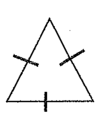
Classifying Triangles by Side Lengths (p. 280)

An equilateral triangle has three congruent sides. An isosceles triangle has exactly two congruent sides. A scalene triangle has no congruent sides.

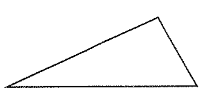
Examples



The triangle has exactly two congruent sides. So, the triangle is isosceles.



The triangle has three congruent sides. So, the triangle is equilateral.



The triangle has no congruent sides. So, the triangle is scalene.

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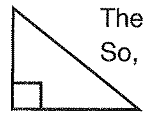
Classifying Triangles by Angles (p. 280)

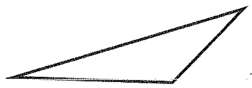
An acute triangle has three acute angles. An obtuse triangle has one obtuse angle. A right triangle has one right angle.

Examples

The triangle has 3 acute angles.
So, the triangle is acute.



The triangle has 1 right angle.
So, the triangle is right.

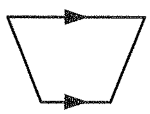


The triangle has 1 obtuse angle.
So, the triangle is obtuse.

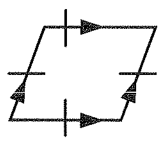
Classifying Quadrilaterals (p. 283)

To classify each quadrilateral below, identify it by its angles and sides.

Examples

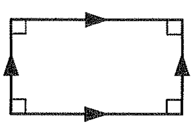


The quadrilateral has exactly one pair of parallel sides.
So, the quadrilateral is a trapezoid.



The quadrilateral is a parallelogram with four sides of equal length. So, the quadrilateral is a rhombus.

9

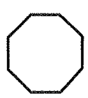


The quadrilateral is a parallelogram with four right angles. So, the quadrilateral is a rectangle.

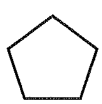
Classifying Polygons (p. 285)

To name a polygon, count the number of its sides.

Examples



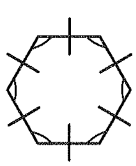
The polygon has 8 sides.
So, the polygon is an octagon.



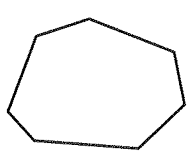
The polygon has 5 sides.
So, the polygon is an pentagon.

Identifying Regular and Irregular Polygons (p. 286)

In a regular polygon, all of the sides have the same length and all of the angles have the same measure. A polygon that is not regular is irregular.



The polygon's sides all have the same length and the angles all have the same measure. So, the polygon is regular.

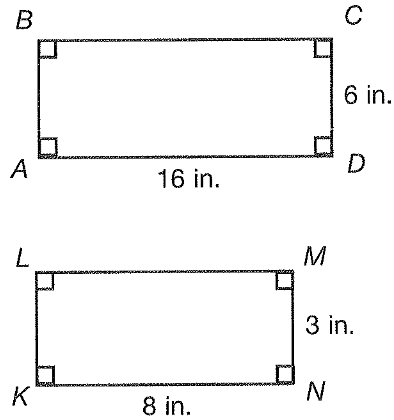


The polygon's sides do not all have the same length and all the angles do not have the same measure. So, the polygon is irregular.

Identifying Similar and Congruent Polygons (p. 287)

Two polygons are similar if they have the same shape, but not necessarily the same size. Two polygons are congruent if they have the same size and shape. To determine whether polygons with the same shape are similar, find the ratios of the lengths of the corresponding sides. If the polygons are similar, all of the ratios will be the same.

Example



$$\frac{AD}{KN} = \frac{BC}{LM} = \frac{16 \text{ in.}}{8 \text{ in.}} = 2$$

$$\frac{AB}{KL} = \frac{DC}{NM} = \frac{6 \text{ in.}}{3 \text{ in.}} = 2$$

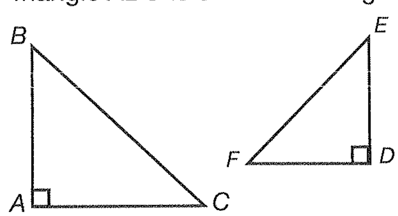
Because the ratios are equal, the rectangles are similar.

Identifying Corresponding Angles and Corresponding Sides (p. 288–289)

The corresponding sides of two congruent or similar figures are the sides that are in corresponding positions. The corresponding angles of two congruent or similar figures are the angles that are in corresponding positions.

Example

Triangle *ABC* is similar to triangle *DEF*.



The corresponding sides of the triangles are sides *AB* and *DE*, sides *BC* and *EF*, and sides *CA* and *FD*.

The corresponding angles of the triangles are $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$.

Using Indirect Measurement (p. 291)

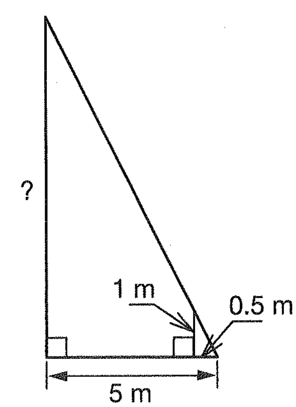
To measure a length indirectly, create two similar triangles and then write and solve a proportion that uses the ratios of the corresponding lengths of the similar triangles.

Example

You want to measure the height of a tree in your backyard.

To measure it indirectly, you make a right angle with a meter stick and the ground and line up the end of the meter stick's shadow with the end of the tree's shadow.

The shadow of the meter stick is 0.5 meter and the shadow of the tree is 5 meters. Now use a proportion to solve for the height of the tree.



$$\frac{\text{Height of tree}}{\text{Height of meter stick}} = \frac{\text{Length of tree's shadow}}{\text{Length of meter stick's shadow}}$$

$$\frac{x}{1 \text{ meter}} = \frac{5 \text{ meters}}{0.5 \text{ meter}}$$

$$0.5x = 5$$

$$x = 10$$

The height of the tree is 10 meters.

Looking Ahead to Chapter 10

Focus

In Chapter 10, you will find perimeters of rectangles, find circumferences of circles, and find the areas of rectangles, circles, parallelograms, triangles, trapezoids, and composite figures. You will find squares and square roots of numbers, and use the Pythagorean theorem to solve problems.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 10.

Use mental math to find the sum.

1. 27 + 94 **2.** 13 + 45 **3.** 9 + 5 + 21

Find the product.

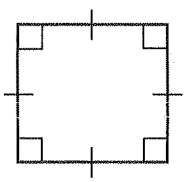
4. 3.4 × 5.7 **5.** 9.1 × 2.6 **6.** 1.52 × 7.8

Evaluate the power.

7. 9^2 **8.** 20^2 **9.** 45^2

Write as many names as you can for the quadrilateral.

10.



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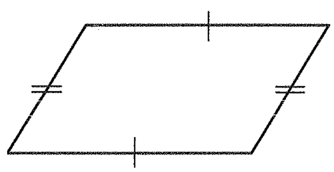

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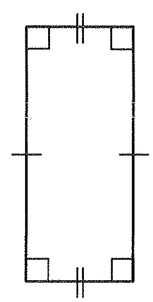
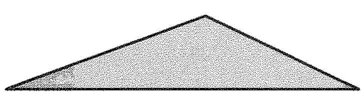


10

Your aunt is using the triangle to make a quilt. Describe the triangle as acute, right, or obtuse.

13.



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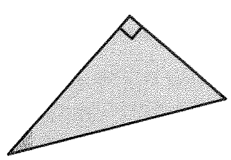

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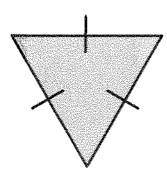

Key Terms