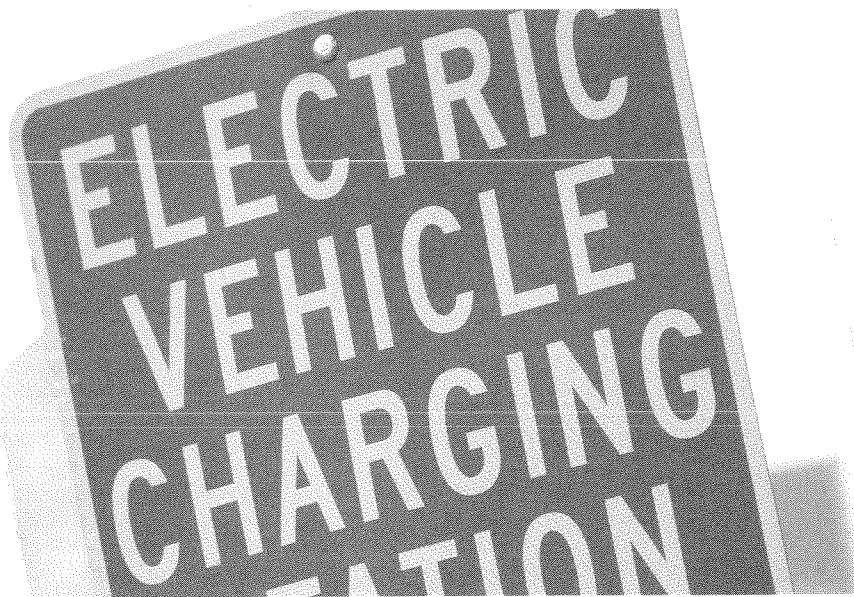# 8

Algebraic Problem Solving

Because they produce no emissions or pollution while parked or running, electric cars are zero-emissions vehicles, or ZEVs. In Lesson 8.7, you will use tables and graphs to determine the distance that an electric car can travel in a certain amount of time.

8

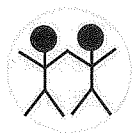# Life in a Small Town
Picture Algebra

Objectives

In this lesson, you will:
- Use picture algebra to represent and solve a problem.

Key Terms

- variable
- equation

Take Note

Recall that we can use a **variable** (a letter or symbol) to represent an unknown value.

In Lesson 1.1, we solved problems by drawing pictures or diagrams to help us understand the relationships between different quantities. In this lesson, you will use this process to solve problems.

Problem 1

Suppose that for social studies class, each student must choose a small town and write a report about the town's industry, resources, and economy. The town you choose has three main shops—Gift Gala, the Coffee Stop, and Flowers 'R Us. On a particular day, the shops together earned a total of $432. Gift Gala earned $52 more than Flowers 'R Us. The Coffee Stop earned $32 more than Flowers 'R Us. How much money did each shop earn that day?

A. In the space below, complete the picture to represent the problem. Label the known parts with their values. Do not worry about making the drawing to scale.

Gift Gala	?	
Coffee Stop	?	
Flowers 'R Us	?	

432
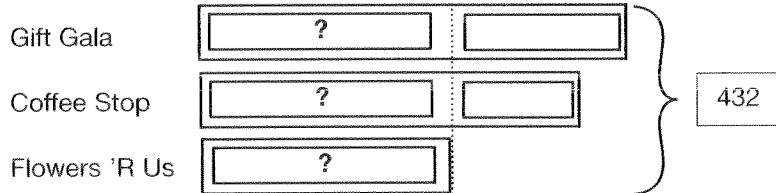
B. Use your picture to find out how much money each shop earned that day.

Gift Gala's earnings:

Coffee Stop's earnings:

Flowers 'R Us' earnings:

C. You can write an *equation* to represent the picture you drew above. An **equation** is a mathematical sentence that you make by placing an equals sign (=) between two expressions. One way to write an equation is to begin by writing the equation with words. One equation that you can write for this problem is:

$$\text{Gift Gala's earnings} + \text{Coffee Stop's earnings} + \text{Flowers 'R Us' earnings} = 432$$

You can label the unknown parts with a variable—use a "g" for Gift Gala's earnings, a "c" for the Coffee Stop's earnings, and an "f" for Flowers 'R Us' earnings. Write the equation above using variables instead of words.

Problem 2

There are two main sections in the town that you are studying, the Hill section and the River section. The town has 1456 people altogether in both sections. The number of people in the Hill section is 256 more than twice the number of people in the River section. How many people live in each section of town?

A. Draw a picture to represent the situation. (Hint: Begin by using a rectangle to represent the River section.) Label the unknown parts with the variable r, where r represents the number of people in the River section, and label the known parts with their values. Do not worry about making the drawing to scale.

B. Use the picture to find out how many people there are in each section of town.

   Number of people in River section:

   Number of people in Hill section:

Investigate Problem 2

1. Complete the statement to write a word equation for Problem 2.

   people in Hill Section
   2(people in River section) + ☐ + (people in River section) = 1456

2. Complete the statement below to write an equation using variables for Problem 2. In the equation, the variable r represents the number of people in the River section.

   ☐ r + ☐ + r = 1456

Problem 3

The town has two pizza parlors, Happy Days Pizza and Pizza Palace. On a given day, Happy Days Pizza delivers 35 fewer pizzas than Pizza Palace. Together they deliver a total of 125 pizzas. How many pizzas did each pizza parlor deliver that day?

A. Draw a picture to represent the situation. Label the unknown parts with a variable and the known parts with their values. Do not worry about making the drawing to scale.

B. Use the picture to find out how many pizzas each pizza parlor delivers.

Happy Days' deliveries:

Pizza Palace's deliveries:

C. Use complete sentences to explain how the picture you drew helped you solve the problem.

D. Use a complete sentence to explain how you know that your answers are correct.

Investigate Problem 3

1. Write a word equation to represent Problem 3.

2. Complete the statement to write an equation for Problem 3. In the equation, the variable p represents Pizza Palace's deliveries.

$$p + (p - \boxed{}) = \boxed{}$$

Form a group with another partner team to solve the more complicated problem below.

Problem 4

On one of the farms outside of town, a water tank used for irrigation holds a total of 4568 gallons of water. The tank has three pipes through which the water drains to irrigate three different fields. Pipe B drains twice as much water as Pipe A. Pipe C drains 68 gallons more than Pipe B. Assume that the tank is drained completely before it is refilled. How many gallons does each pipe drain from the tank?

A. Draw a picture to represent the situation. Label the unknown parts with variables and the known parts with their values. Do not worry about making the drawing to scale.

B. Use the picture to find the number of gallons of water that will be drained by each pipe.

Gallons drained by Pipe A:

Gallons drained by Pipe B:

Gallons drained by Pipe C:

C. Use complete sentences to explain how the picture you drew helped you solve the problem.

D. Use complete sentences to explain the mathematical steps you used to solve the problem.

E. Write a word equation to represent Problem 4.

Problem 5

A company in town makes a special plastic piece that is used in automobiles. The company has five different machines that can produce the piece at different rates. For a particular job, the company needs to produce 2718 pieces.

Machine 2 produces twice as many pieces as Machine 1.

Machine 3 produces 15 more than three times as many pieces as Machine 1.

Machine 4, the oldest machine, produces 7 less pieces than Machine 1.

Machine 5 produces 10 more pieces than Machine 2.

How many pieces should each machine produce?

A. Draw a picture to represent the situation. Label the unknown parts with variables and the known parts with their values. Do not worry about making the drawing to scale.

B. Use the picture to find the number of pieces that each machine should produce.

Number of pieces produced by Machine 1:

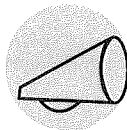Number of pieces produced by Machine 2:

Number of pieces produced by Machine 3:

Number of pieces produced by Machine 4:

Number of pieces produced by Machine 5:

C. Use complete sentences to explain the mathematical steps you used to solve the problem.
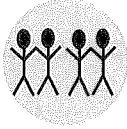
D. Write a word equation to represent Problem 5.

Investigate Problem 1

1. Math Path: Algebraic Expressions

An **algebraic expression** is a phrase that uses variables, numbers, and operations. Whenever we perform the same mathematical process over and over again, often we can write an algebraic expression to represent the situation. Write an algebraic expression to represent the total cost in the table. Let the variable p represent the price of the item.

Expression for total cost: _____

Use this expression to calculate the total cost of an item whose price is $15.67. Then use a complete sentence to describe how the expression helped you find the cost.

8

Problem 2

You want to rent DVDs from a local store. The store charges $3 for each DVD, but requires you pay a membership fee of $20.

Number of DVDs	Membership Fee	Total Cost
10		
25		
13		
32		

A. Complete the table to find the total cost of renting DVDs including the membership fee. In the table, what values changed?

B. In the table, what values did not change?

C. Did one value depend on another? Write complete sentences to explain.

D. Write an algebraic expression to represent the total cost of renting DVDs. Let the variable d represent the number of DVDs.

Expression for total cost: _____

Use this expression to calculate the total cost of renting 55 DVDs. Be sure to follow the proper order of operations.

What is the total cost of renting 42 DVDs?

Take Note

Order of Operations

1. Evaluate expressions inside grouping symbols like ( ) or [ ].

2. Multiply and divide from left to right.

3. Add and subtract from left to right.

Investigate Problem 2

1. You can replace the variables in an algebraic expression with any number. After you replace the variable, you can find the value of, or **evaluate,** the expression.

   Evaluate $t - 8$ when $t = -7, -9,$ and 12.

   Evaluate $4y$ when $y = -3, 0,$ and 5.

   Evaluate $3x - 6$ when $x = -1, 4,$ and 8.

   Evaluate $-2m + 8$ when $m = -5, -3,$ and 10.

   Evaluate $-2.5 + 7.2y$ when $y = -3, 0,$ and 7.

2. Sometimes it is more convenient to use a table to record the results from evaluating an expression. Complete each table.
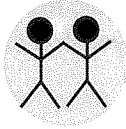
h	$3h - 5$
2	
−5	
9	
−4	

m	$16 - 5m$
−1	
0	
1	
2	

y	$-5y + 3.55$
1.3	
−2.4	
5.2	
−8.7	
−4.3	

z	$-\dfrac{2}{3}z + 2\dfrac{1}{3}$
6	
−6	
9	
−2	

3. Form a group with another partner team and compare your answers in Question 2. If you have any answers on which you do not agree, work together to find out why.

4. Explain how you evaluated the expressions to the other groups in your class.

5. Finding an algebraic expression that represents a problem is usually the most difficult part of solving these types of problems. Work with your partner to read the following situation and answer the questions.

You have $100 saved and you are spending it at the rate of $10 per week. How much money will you have left after the first week?

How much money will you have left after 4 weeks?

How much money will you have left after 10 weeks?

In this set of questions, what values changed? What values did not change?

Did one value depend on another? Use complete sentences to explain.

Write an algebraic expression to represent the situation above. In your expression, what does the variable represent?

6. Read the following situation and answer the questions.

A local store has 35 computer games in stock and receives 5 more each day. During this time period, no games are sold. How many computer games will the store have in 2 days?

How many computer games will the store have in 10 days?

How many computer games will the store have in 2 weeks?

In this set of questions, what values changed? What values did not change?

Did one value depend on another? Use complete sentences to explain.

Write an algebraic expression to represent this situation. In your expression, what does the variable represent?
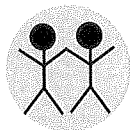
8.3

Selling Cars
Solving One-Step Equations
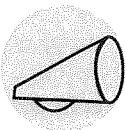
Objectives

In this lesson, you will:
- Solve one-step equations.

Key Terms

- solve
- one-step equation

Problem 1

An automobile dealer decided to hold a one-day sale in which the price on every car in stock was immediately reduced by $550. Complete the table below to find the sale price of each model of car.

Model	Regular Price	Sale Price
sedan	$25,890	
coupe	$27,700	
SUV	$35,980	

A. How did you find the sale price given the regular price? Use a complete sentence to explain.

B. Write an expression to represent the sale price given the regular price.

C. Write an equation that you can use to find the regular price of a van. (Hint: Place an equals sign between the expression that you wrote in part (B) and the van's sale price of $19,987.)

Model	Regular Price	Sale Price
van		$19,987
luxury sports car		$55,990

D. Use the equation to find the regular price of the van.

Investigate Problem 1
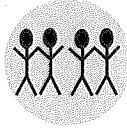
1. Write an equation that you can use to find the regular price of the luxury sports car in Problem 1. Use complete sentences to explain how you wrote the equation.

2. Use the equation that you wrote above to find the regular price of the luxury sports car. Use a complete sentence to explain how you found your answer.
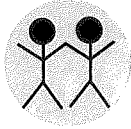
Problem 2

You read a report that says that only $\frac{7}{100}$ of all people who own car dealerships in the country are women.

A. There are about 20,000 people who own car dealerships in the country. How many of them are female?

B. In a group of 2000 people who own car dealerships attending a conference, about how many would you expect to be female?

C. How did you find the number of women car dealers, given the total number of car dealers? Use complete sentences to explain your answer.

D. Write an expression to represent the number of women car dealers, given the total number of car dealers.

E. Write an equation that you can use to find the total number of car dealers in a certain city, given that the number of women car dealers in the city is 14. (Hint: Place an equals sign between the expression that you wrote in Part (D) and the number of women car dealers in the city.)

F. Use the equation to find the total number of car dealers in the city.

Investigate Problem 2

1. Math Path: Solutions of Equations

In Problem 1 and Problem 2, you wrote equations to represent particular problem situations. To **solve** each equation, you found the values of the variable that make the equation true. In both problems, solving the equation required only one operation, so the equation is called a **one-step equation.**

What operation is used in each equation?

$$m + 6 = 13 \qquad 19 = x - 10 \qquad \frac{s}{5} = 4.2 \qquad 3y = 15$$

Solve each equation. That is, find the values of the variable that make the equation true. In each case, use a complete sentence to explain how you found the solution.

How does the operation that you used to solve each equation compare to the operation that is actually part of the equation? Use complete sentences to explain.

2.
Form a group with another partner team and compare your answers in Question 1. If you have any answers on which you do not agree, work together to find out why.

3.
Below is an example of a one-step equation and three different mathematical solutions.

$z + 6 = 14$	$z + 6 = 14$	$z + 6 = 14$
$z + 6 - 6 = 14 - 6$	$-6 = -6$	$z + 6 + (-6) = 14 + (-6)$
$z + 0 = 8$	$z + 0 = 8$	$z + 0 = 8$
$z = 8$	$z = 8$	$z = 8$

Explain each solution method and why it works.

4. Use each different solution method in Question 3 to solve each equation.

$x - 8 = 12$ $x - 8 = 12$ $x - 8 = 12$

$5c = 20.5$ $5c = 20.5$ $5c = 20.5$

$b + 75 = 12$ $b + 75 = 12$ $b + 75 = 12$

$\dfrac{v}{4} = 2.3$ $\dfrac{v}{4} = 2.3$ $\dfrac{v}{4} = 2.3$

5. Use any method shown in Question 3 to solve each equation.
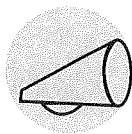
$2.45 + b = 5.45$ $6d = 2.46$

$\dfrac{t}{5} = 20.2$ $y \div \dfrac{3}{4} = \dfrac{7}{8}$

$230 = m - 78$ $2.5w = 27.35$

A Park Ranger's Work Is Never Done
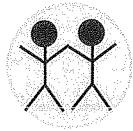## Solving Two-Step Equations

Objectives

In this lesson, you will:
- Solve two-step equations.

## Key Terms

- two-step equation
- inverse operations

Many situations can be modeled by equations that need more than one operation to solve them. The equations are called *multi-step equations*. In this lesson, we will work with **two-step equations,** equations that require two operations to solve.

Problem 1

At a local national park, the park rangers decide that they want to extend a wooden walkway through the forest to encourage people to stay on the path. The existing walkway is 150 feet long. The park rangers believe that they can build the additional walkway at a rate of about 5 feet per hour.

A. How many total feet of walkway will there be after the park rangers work 5 hours?

B. How many total feet of walkway will there be after the park rangers work 7 hours?

C. Define a variable for the amount of time that the rangers will work. Then use the variable to write an expression that represents the total number of feet of walkway built, given the amount of time that the rangers will work.

D. How many hours will the rangers need to work to have a total of 500 feet of walkway completed?

E. Use complete sentences to explain how you found the answer to Part (D).

F. What mathematical operations did you perform to find the answer to Part (D)?

G. Write an equation that you can use to find the amount of time needed for the rangers to have a total of 500 feet of walkway completed. (Hint: Place an equals sign between the expression that you wrote in Part (C) and the total length of the completed walkway.) Then find the value of the variable that will make this equation true.

Investigate Problem 1
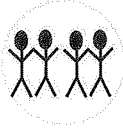
1. How many hours will the rangers need to work to have a total of 270 feet of walkway completed? Use complete sentences to explain how you found the answer.

   What mathematical operations did you perform to find the answer?

   Use complete sentences to explain why using these mathematical operations gives you the correct answer.

   Write an equation that you can use to find the amount of time it will take to have a total of 270 feet of walkway completed.

   Find the value of the variable that will make this equation true.

2. How many hours will the rangers need to work to have a total of 100 feet of walkway completed? Use a complete sentence to explain how you found the answer.

   What mathematical operations did you perform to find the answer?

   Write an equation that you can use to find the amount of time it will take to to have a total of 100 feet of walkway completed.

   Find the value of the variable that will make this equation true.

3. Form a group with another partner team and compare your answers in Questions 1 and 2. If you have any answers on which you do not agree, work together to find out why.

Problem 2

Part of a park ranger's job is to perform rescue missions for people and animals. Suppose that a bear cub has fallen into an abandoned mine shaft on the park grounds. The cub is 77 feet below the surface of the ground in the mine shaft. A ranger coaxed the cub to climb into a basket attached to a rope and is pulling up the cub at a rate of 7 feet per minute.

A. How many feet below the surface of the ground will the cub be in 6 minutes?

B. How many feet below the surface of the ground will the cub be in 11 minutes?

C. Define a variable for the amount of time spent pulling the cub up the shaft. Then use the variable to write an expression that represents the number of feet below the surface of the ground the cub is, given the number of minutes that the ranger has spent pulling up the cub.

D. In how many minutes will the cub be 14 feet from the surface?

E. Use a complete sentence to explain how you found the answer to Part (D).

F. What mathematical operations did you perform to find the answer to Part (D)?

G. Use complete sentences to explain why using these mathematical operations gives you the correct answer.

H. Write an equation that you can use to find the number of minutes it takes for the cub to be 14 feet below the surface of the ground by setting the expression you wrote in Part (C) equal to 14. Then find the value of the variable that will make the equation true.

1. In how many minutes will the cub be 28 feet from the surface?

2. Use complete sentences to explain how you found the answer to Question 1.

3. What mathematical operations did you perform to find the answer to Question 1?

4. Use complete sentences to explain why using these mathematical operations gives you the correct answer.

5. Write an equation that you can use to find the number of minutes it takes for the cub to be 28 feet below the surface of the ground. Then find the value of the variable that will make the equation true.

6. **Math Path: Inverse Operations**

   In Problems 1 and 2, you needed to perform *inverse operations* to solve the equation. **Inverse operations** are two operations that undo each other. For example, adding 3 and subtracting 3 are inverse operations. Below are two different examples of ways to solve two-step equations. On each line, name the inverse operations. The first one is done for you.

   $$2m - 6 = 22$$
   $$2m - 6 + 6 = 22 + 6 \qquad \underline{\text{Subtract 6 and add 6.}}$$
   $$2m = 28$$
   $$\frac{2m}{2} = \frac{28}{2} \qquad \underline{\hspace{4cm}}$$
   $$m = 14$$

   $$2m - 6 = 22$$
   $$\underline{+6 = +6} \qquad \underline{\hspace{4cm}}$$
   $$2m = 28$$
   $$\frac{2m}{2} = \frac{28}{2} \qquad \underline{\hspace{4cm}}$$
   $$m = 14$$

7. Solve each two-step equation. Show your work.

$5v - 34 = 26$ $3x + 7 = 37$

$23 + 4x = 83$ $2.5c - 12 = 13$

$\dfrac{3}{4}x + 2 = 4\dfrac{2}{3}$ $-\dfrac{2}{3}b + \dfrac{2}{5} = 6\dfrac{4}{5}$
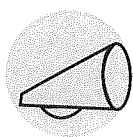
$-\dfrac{t}{5} - 9 = 21$ $2 = 2.27 - \dfrac{s}{4}$

$12m - 17 = 139$ $121.1 = -19.3 - 4d$

$-23z + 234 = 970$ $7685 = 345 - 5d$

8

8.5 Where's the Point?
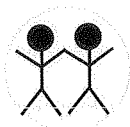### Plotting Points in the Coordinate Plane

Objectives

In this lesson, you will:
- Identify points in the coordinate plane.
- Graph points in the coordinate plane.

Key Terms

- Cartesian coordinate system
- x-axis
- y-axis
- origin
- coordinate plane
- ordered pair
- x-coordinate
- y-coordinate

In earlier mathematics classes, you may have created graphs such as pictographs and bar graphs to display information. These types of graphs do a good job of showing some information, but fall short of being able to show how one quantity relates to another quantity.

Problem 1

French mathematician René Descartes devised a method for assigning every point a unique name that describes its location.

A. Take a blank sheet of paper and draw two points somewhere on the paper. Do not allow your partner to see your paper. Turn to your partner and try to describe exactly where the points are on the paper so that your partner can draw two points in the exact same location on his or her paper. Then switch roles and have your partner try to do the same thing with you.

B. Were either of you successful? If so, use complete sentences to explain how you or your partner described the locations of the points.

C. Descartes's method was very straightforward and can be easily duplicated. Use a ruler to measure the distance from the left edge of the paper to the point. Then measure the distance from the bottom of the paper to the point. Now, use those measurements to have your partner duplicate your points on his or her paper. Then switch and have your partner measure and give you the distances to draw his or her points.

D. Were you more successful or less successful using the method in Part (C) than you were using the method in Part (A)? Use complete sentences to explain.

Investigate Problem 1
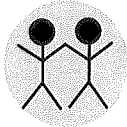
1. Descartes used a similar process to devise his coordinate system that you can duplicate. From the point below, draw a horizontal segment to the right of the point, across the entire width of the box. Then draw a vertical line segment up from the same point across the entire side of the box. Use the length of 1 unit shown below to mark every 1 unit on both lines, starting from the point.
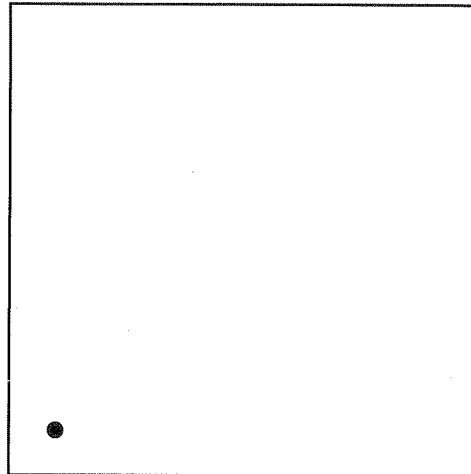
— 1 unit

Does your diagram look similar to your partner's diagram?

2. Place a point anywhere on your diagram. Then have your partner duplicate the point on his or her diagram. Now switch roles and try to duplicate your partner's point. Were you and your partner successful in duplicating the points? Why or why not? Use complete sentences to explain your answer.

3. Math Path: Cartesian Coordinate System

Your work in Questions 1 and 2 is an approximation of how Descartes designed what became known as the **Cartesian coordinate system.** The horizontal number line you drew is the **x-axis** and the vertical number line you drew is the **y-axis.** The point at which they cross is the **origin.** The intersection of these two number lines forms a **coordinate plane.** Label the x-axis, the y-axis, and the origin in the coordinate plane above.
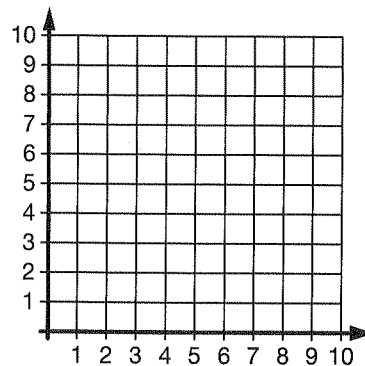
4. Each point on a graph can be represented by an ordered pair. An **ordered pair** consists of two numbers. The first number, called the **x-coordinate,** is the horizontal distance from the origin to the point. The second number, called the **y-coordinate,** is the vertical distance from the origin to the point. The ordered pair is written as (*x*-coordinate, *y*-coordinate). Write the ordered pair of the point that you and your partner drew in Question 2.

5. Usually a coordinate plane is drawn using a grid. The grid helps you to define the ordered pairs of the points. Use the coordinate plane to write the ordered pair that represents each point.

A (_____ , _____)

B (_____ , _____)

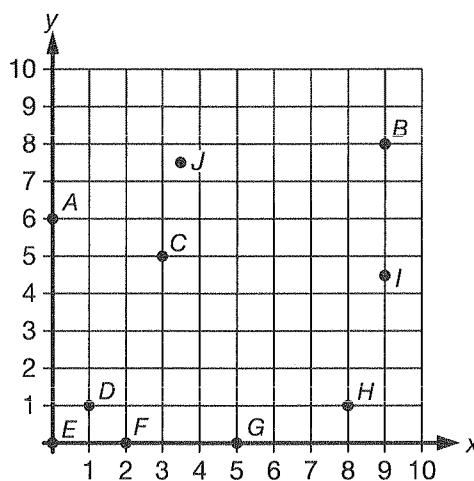C (_____ , _____)

D (_____ , _____)

E (_____ , _____)

F (_____ , _____)

G (_____ , _____)

H (_____ , _____)

I (_____ , _____)

J (_____ , _____)

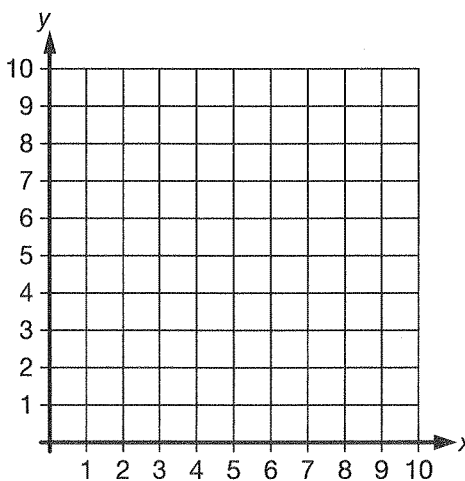**6.** Plot each point in the coordinate plane.

A(3, 4)

B(5, 7)

C(0, 3)

D(9, 0)

E(0, 0)

F(2.5, 3.1)

$G\left(3\frac{1}{3}, 4\frac{7}{8}\right)$

$H\left(8\frac{1}{4}, 7\frac{5}{6}\right)$

I(4.25, 7.5)

8

8.6 Get Growing!
Using Tables and Graphs

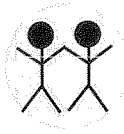## Objectives

In this lesson, you will:
- Make a table of values.
- Create a graph of ordered pairs.

Key Terms
- table
- graph

Now that we have a method of representing unique points, we can construct a "picture" or graph of a relationship between two quantities. We can express a relationship in several ways:

- Write an equation.
- Create a table of values.
- Construct a graph.

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Problem 1

Your friend Zoey is growing a plant for an experiment. She bought the plant when it was 3 centimeters high. She has measured the plant's height at the end of the day every day for the last month. She has found that the plant has grown about 2 centimeters per day.

A. How high was the plant at the end of the first day? Use a complete sentence in your answer.

B. How high was the plant at the end of the fifth day?

C. How high was the plant after the first week?

D. What quantities are changing? What quantities remain constant? Use complete sentences in your answer.

E. What quantity depends on the other quantity? Use a complete sentence in your answer.
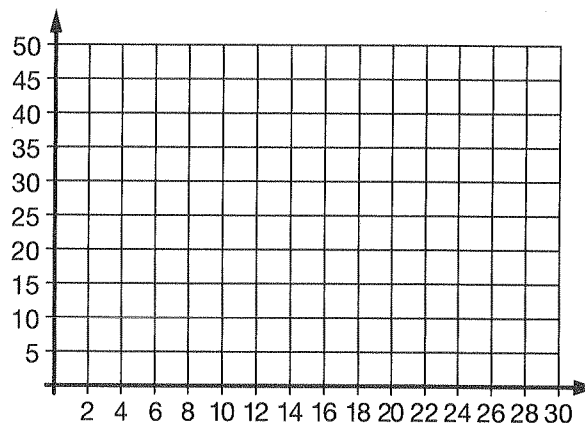
Investigate Problem 1
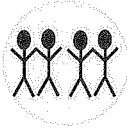
1. Define a variable for the amount of time in days that Zoey has been measuring the plant.

2. Write an expression that you can use to represent the plant's height in terms of the amount of time in days that Zoey has been measuring it.

3. Use the expression that you wrote in Question 2 and your answers to Parts (A), (B), and (C) in Problem 1 to complete the first six rows of the table.

Time (days)	Height (centimeters)
0	
1	
5	
10	
13	
	33
	45

4. To complete the last two rows of the **table**, write two equations by setting the expression that you wrote in Question 2 equal to each height. Then solve each equation to find the amount of time in days that it takes for the plant to reach each height.

5. Use the grid below to create a **graph** to represent the values in the table. Begin by labeling your axes. Use time for the horizontal axis and height for the vertical axis. The axes are already numbered.

6. Write each row in your table in Question 3 as an ordered pair. Note that the x-coordinate is the time and the y-coordinate is the height.

7. Plot each ordered pair on the prepared grid in Question 5.

8. What do you notice about points that you graphed in the coordinate plane above? Write your answer using a complete sentence.

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9. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

10. Use your graph to answer the following questions. Use complete sentences in your answers.

    When will the height of the plant be 19 centimeters?

    How tall will the plant be in 25 days?

11. How does the graph help you visualize the relationship between the amount of time and the plant height? Use complete sentences to explain.

12. Form a group with another partner team and compare your graphs and your answers to Question 10. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your work with the rest of the class.
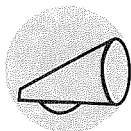
Problem 2
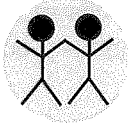
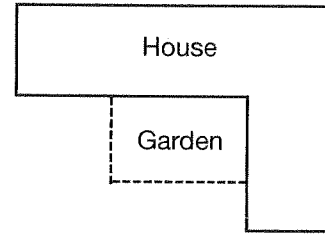Your friend Julio wants to plant a garden against his house and enclose the garden with 24 feet of fence that he bought. At the right is a diagram of where he wants to plant the garden. The garden will have a fence on two sides.

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A. Suppose that the width of the garden is 4 feet. In order to have enough fencing to enclose the garden and to use all of the fencing, what length should Julio make the garden? Use a complete sentence in your answer.

B. What is the area of the garden with the dimensions in part (A)? Write your answer using a complete sentence.

C. Suppose that the width of the garden is 6 feet. In order to have enough fencing to enclose the garden and to use all of the fencing, what length should Julio make the garden? What is the area of the garden with these dimensions?

D. Suppose that the width of the garden is 12 feet. In order to have enough fencing to enclose the garden and to use all of the fencing, what length should Julio make the garden? What is the area of the garden with these dimensions?

E. If the area of the garden is 140 square feet, what are the width and length of the garden? Does this question have only one answer? Use complete sentences to explain your answer.

Investigate Problem 2

1. In Problem 2, what quantities are changing? What quantities remain constant? Use complete sentences in your answer.

2. What quantity depends on the other quantity? Use a complete sentence in your answer.

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3. Define a variable for the width of the garden.

4. Write an expression that you can use to represent the area of the garden.

5. Use the expression that you wrote in Question 4 and your answers to Parts (C), (D), and (E) in Problem 2 to complete the table.

Width (feet)	Area (square feet)
0	
4	
6	
12	
	140
	140
18	
20	
24	

6. Write each row in your table in Question 5 as an ordered pair. The x-coordinate is the width and the y-coordinate is the area.

7. Use the grid below to create a graph to represent the values
in the table. Begin by labeling your axes. Use width for the
horizontal axis and area for the vertical axis. The axes are
already numbered.
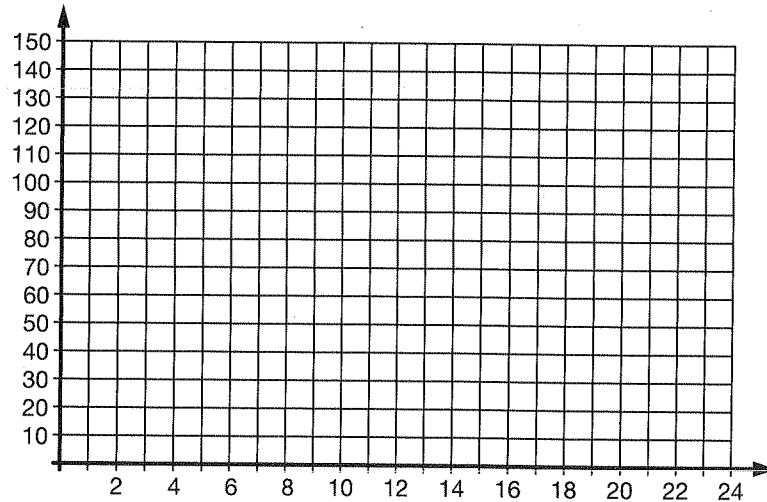
8. Plot each ordered pair on the grid.

9. What do you notice about points that you graphed in
the coordinate plane above? Write your answer using a
complete sentence.

10. Connect the points with a smooth curve. The result should be an
upside down U-shaped curve.

11. Use your graph to answer the following questions.

What are the dimensions of a garden that has an area of
119 square feet?

What is the area of a garden that has a width of 15 feet?
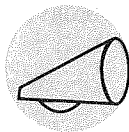
8.7 Saving Energy
Solving Problems Using Multiple Representations

Objectives

In this lesson, you will:
- Use equations, tables, and graphs to solve problems.

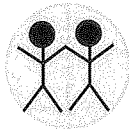We now have several powerful tools to help us solve complex problems including picture algebra, expressions, equations, tables, and graphs. These tools are **multiple representations,** or different ways of visualizing a problem. Each of these representations is useful in different ways.

Key Terms

- multiple representations

Problem 1

Your uncle in California drives an electric car. The average speed of his car is 15 miles per hour.

A. How far will your uncle's car travel in 5 hours? Use a complete sentence in your answer.

B. How far will your uncle's car travel in 10 hours? Use a complete sentence in your answer.

C. How far will your uncle's car travel in 30 minutes? Use a complete sentence in your answer.

D. How far will your uncle's car travel in six and one-half hours? Use a complete sentence in your answer.

E. Your uncle drove his car for 135 miles at the average speed. How many hours did he drive? Use a complete sentence in your answer.

Investigate Problem 1

1. What are the two quantities that are changing in Problem 1?

2. What quantity depends on the other quantity? Use a complete sentence in your answer.

3. Work on completing the table below by first filling in the names of the quantities in Problem 1 and their units of measure. Then fill in the table with your answers from Problem 1. The other table entries will be filled in later.

	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 1, Part (A)	5	
Problem 1, Part (B)	10	
Problem 1, Part (C)		
Problem 1, Part (D)		
Problem 1, Part (E)		135
Question 6		112.5
Question 7		$123\frac{3}{4}$
Expression		

4. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row of the table under Column 1.

5. Write an expression that you can use to represent the quantity in Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row of the table under Column 2.

6. You want to determine how long it will take the car to travel 112.5 miles. Use the expression you wrote in Question 5 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

7. You want to determine how long it will take the car to travel $123\frac{3}{4}$ miles. Use the expression that you wrote in Question 5 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

Investigate Problem 1

8. Use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use time for the horizontal axis and distance for the vertical axis. The axes are already numbered.
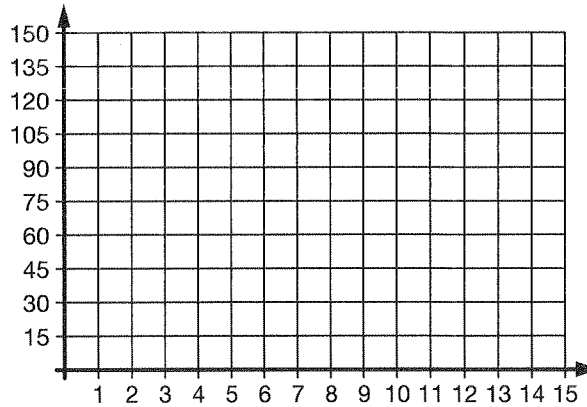
9. Write each row in your table in Question 3 as an ordered pair. Note that the x-coordinate is the amount of time that the car traveled and the y-coordinate is the distance the car has traveled.

10. Plot each ordered pair on the prepared grid in Question 8.

11. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

12. Use your equation to answer the following questions.

   Your uncle drove his car for 450 miles at the average speed. How many hours did he drive?

   How far will your uncle's car travel in 29 hours?

13. Join your group with another and compare your graphs and your answers to Question 12. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your work with the rest of the class.

Problem 2

When you use energy-efficient light bulbs, you can save up to $4.32 on your monthly electric bill for each bulb that you use (assuming that a bulb burns for 12 hours per day and electricity costs $.30 per kilowatt hour). Suppose that your family's monthly electric bill is $80.

A. What will the electric bill be if you change 2 bulbs to energy-efficient bulbs? Use a complete sentence in your answer.

B. What will the electric bill be if you change 4 bulbs to energy-efficient bulbs? Use a complete sentence in your answer.

C. What will the electric bill be if you change 5 bulbs to energy-efficient bulbs? Use a complete sentence in your answer.

**D.** If the electric bill is $54.08, how many bulbs were changed to energy-efficient bulbs? Use a complete sentence in your answer.

E. If the electric bill is $45.44, how many bulbs were changed to energy-efficient bulbs? Use a complete sentence in your answer.

Investigate Problem 2

1. What are the two quantities that are changing in Problem 2? Use a complete sentence in your answer.
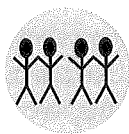
2. What quantity depends on the other quantity? Use a complete sentence in your answer.

3. Work on completing the table below by first filling in the names of the quantities in Problem 2 and their units of measure. Then fill in the table with your answers from Problem 2. The other table entries will be filled in later.

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	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 2, Part (A)	2	
Problem 2, Part (B)	4	
Problem 2, Part (C)	5	
Problem 2, Part (D)		54.08
Problem 2, Part (E)		45.44
Question 6		36.80
Question 7		28.16
Expression		

4. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row of the table under Column 1.

5. Write an expression that you can use to represent the quantity in the Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row of the table under Column 2.

6. You want to determine how many bulbs were changed to energy-efficient bulbs if the monthly electric bill is $36.80. Use the expression that you wrote in Question 5 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

7. You want to determine how many bulbs were changed if the monthly electric bill is $28.16. Use the expression that you wrote in Question 5 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

8. Use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use the number of bulbs for the horizontal axis and the amount of the electrical bill for the vertical axis. The axes are already numbered.
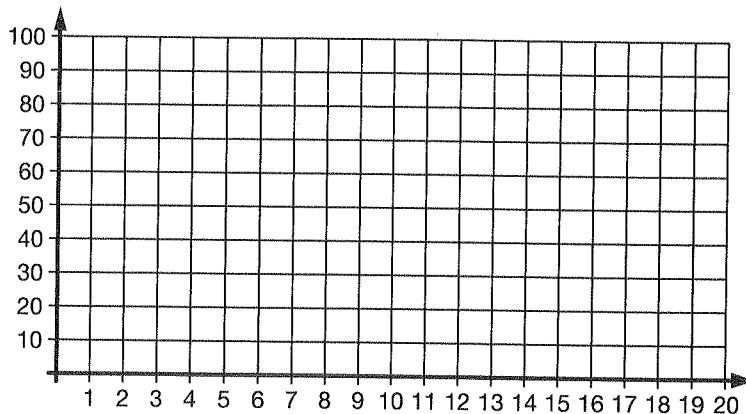
9. Write each row in your table in Question 3 as an ordered pair. Note that the x-coordinate is the number of bulbs and the y-coordinate is the amount of the electric bill.

10. Plot each ordered pair on the prepared grid in Question 8.

11. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

12. Use your graph to answer the following questions. Write your answer using a complete sentence.

    What will the electric bill be if you change 15 bulbs to energy-efficient bulbs?

    If the electric bill is $58.40, how many bulbs were changed?

13. Join your group with another and compare your graphs and your answers to Question 12. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your work with the rest of the class.

Looking Back at Chapter 8

Key Terms

variable · p. 231
equation · p. 231
algebraic expression · p. 238
evaluate · p. 239
solve · p. 243
one-step equation · p. 243
two-step equation · p. 245

inverse operations · p. 248
Cartesian coordinate
   system · p. 252
x-axis · p. 252
y-axis · p. 252
origin · p. 252
coordinate plane · p. 252

ordered pair · p. 253
x-coordinate · p. 253
y-coordinate · p. 253
table · p. 256
graph · p. 256
multiple representation · p. 261

Summary

Using Picture Algebra (p. 231)

You can use a picture or diagram to help you solve word problems. You can also use the picture to write an equation that represents the situation.

Example

On a given day, Pretty Petals and Coral's Florals sell 46 roses altogether. Pretty Petals sells 12 more roses than Coral's Florals. Draw a picture to represent the situation.
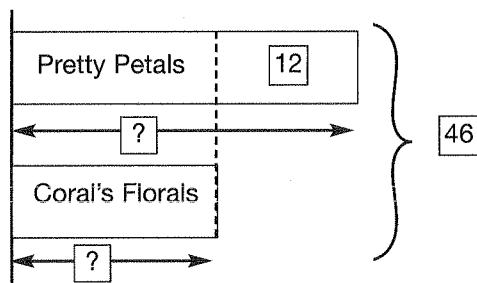
From the picture, you can see that Coral's Florals sold 17 roses and Pretty Petals sold, 17 + 12, or 29 roses.

Because the two shops sold 46 roses altogether, you can write the following word equation.

Number of roses sold + Number of roses sold = 46 roses
by Pretty Petals   by Coral's Florals

Let p represent the number of roses sold by Pretty Petals and c represent the number of roses sold by Coral's Florals. A variable equation for the situation is

$p + c = 46.$

Writing Algebraic Expressions (p. 238)

To write an algebraic expression for a situation, use variables, numbers, and operations to represent the situation.

Example

A small pizza costs $6. There is a delivery fee of $1. Let the variable p represent the number of pizzas. Use the variable to write the expression for the total cost.

Expression for total cost: $6p + 1$

Evaluating Algebraic Expressions (p. 239)

To evaluate an algebraic expression, replace the variable with any number and evaluate the expression.

Example   To evaluate $3x - 7$ when $x = 3$, replace x with 3 and evaluate.

$$3 \cdot 3 - 7 = 9 - 7 = 2$$

To evaluate $3x - 7$ when $x = -2$, replace x with -2 and evaluate.

$$3 \cdot (-2) - 7 = -6 - 7 = -13$$

Solving One-Step Equations (p. 243)

To solve an equation, find the value of the variable that makes the equation true. A one-step equation requires one operation to find this value.

Example   To solve the equation $4x = 20$, use one operation to find the value that makes the equation true.

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

Solving Two-Step Equations (p. 248)

To solve a two-step equation, find the value of the variable that makes the equation true. Use inverse operations to find this value.

Example   To solve the equation $5y + 3 = 13$, use two operations to find the value that makes the equation true.
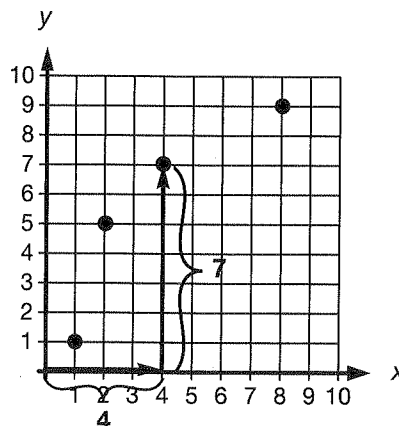
$$5y + 3 - 3 = 13 - 3$$

$$5y = 10$$

$$\frac{5y}{5} = \frac{10}{5}$$

$$y = 2$$

Plotting Points in the Coordinate Plane (p. 253)

To plot a point (x, y) in the coordinate plane, start at the origin. First move x units horizontally and then move y units vertically. Draw a point.

Examples   The points (1, 1), (2, 5), (4, 7), and (8, 9) are plotted in the coordinate plane.

The points (1, 1), (2, 5), (4, 7), (8, 9) plotted.

Making a Table of Values (p. 256)

To make a table of values for a situation, first write an expression that represents the situation. In your table, record the quantity names from your expression, the units of measure, and the expression. Then evaluate the expression for different values. Enter these values and the answers in your table.

Example

You have $15. Your parents will give you $5 a week for allowance. You want to write an expression that will represent the amount of money that you will have after a certain number of weeks. Let t represent the number of weeks you have been earning an allowance. An expression that represents this situation is $5t + 15$. In your table, let the first column represent the number of weeks and the second column represent the total amount of money you have saved.

| Quantity Name	Time	Total Amount of Money
Unit of Measure	Weeks	Dollars
	1	20
	2	25
	3	30
	4	35
	5	40
Expression	t	$5t + 15$

Constructing a Graph (pp. 256–257)

To construct a graph from a table of values, first label the axes on a grid by using the table values as a guide. Next, write each row of numbers in your table as an ordered pair. Then plot each ordered pair on the grid. Finally, connect the points with a smooth line.

Example

x	y	Ordered Pair
1	5	(1, 5)
2	8	(2, 8)
3	11	(3, 11)
4	14	(4, 14)
5	17	(5, 17)
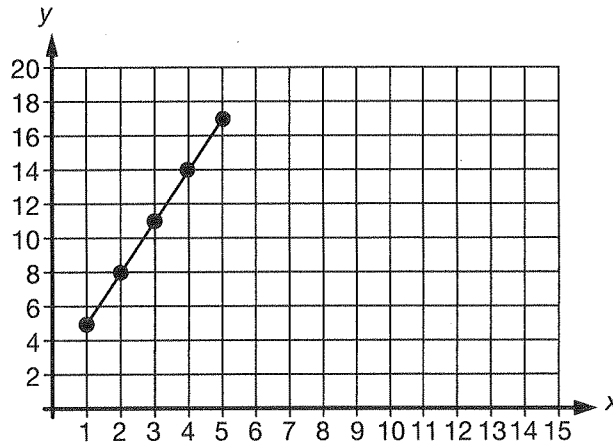
Looking Ahead to Chapter 9

Focus   In Chapter 9, you will work with angles and angle pairs, triangles, quadrilaterals, and other polygons. You will learn to find the lengths of sides and measures of angles. You will also learn how to find a measurement indirectly.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 9.

Use a ruler to find the length of each line segment below in centimeters.

1. _____

2. _____

Find the sum.

3. 66 + 24   4. 86 + 94   5. 53 + 37

9   **Read the problem scenario below.**

There are 20 marbles in a bag. Nine are red, 4 are orange, 3 are yellow, and the rest are blue.

6. Write a ratio that represents the number of red marbles to the total number of marbles.

7. Write a ratio that represents the number of orange marbles to the total number of marbles.

8. Write a ratio that represents the number of yellow marbles to the total number of marbles.

9. Write a ratio that represents the number of marbles that are not blue to the total number of marbles.

Key Terms

angle   p. 273
vertex   p. 273
degrees   p. 273
right angle   p. 273
straight angle   p. 273
acute angle   p. 274
obtuse angle   p. 274
protractor   p. 274
complementary angles   p. 275
supplementary angles   p. 275
transversal   p. 275
congruent angles   p. 275
alternate interior angles   p. 275
alternate exterior angles   p. 275

corresponding angles   p. 275
vertical angles   p. 277
adjacent angles   p. 277
triangle   p. 279
equilateral triangle   p. 280
isosceles triangle   p. 280
scalene triangle   p. 280
acute triangle   p. 280
obtuse triangle   p. 280
right triangle   p. 280
quadrilateral   p. 283
trapezoid   p. 283
parallelogram   p. 283
rhombus   p. 283

rectangle   p. 283
square   p. 283
polygon   p. 285
diagonal   p. 285
regular polygon   p. 286
irregular polygon   p. 286
similar polygons   p. 287
corresponding sides   p. 288
scale factor   p. 288
corresponding angles   p. 289
similar triangles   p. 291
indirect measurement   p. 291
congruent polygons   p. 295
congruent   p. 295

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