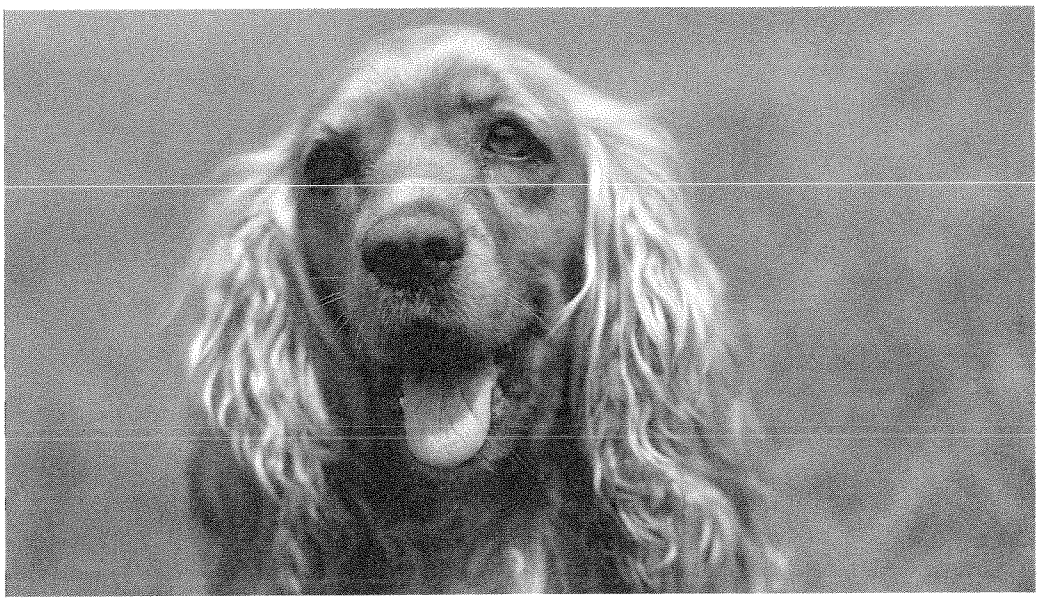
4

Decimals



Dogs range in height from just a few inches to nearly three feet. In Lesson 4.3, you will compare the heights of different breeds of dogs using decimals.

4

4.1 Cents Sense

Decimals as Special Fractions

Objectives

In this lesson, you will:

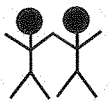
- Write decimals as special fractions.

Key Terms

- decimal

As societies formed and people began to barter, trade, and sell goods, the need for money arose. Early forms of money included shells, beads, and precious stones. As commerce expanded, people needed a more precise system. Initially, people used different weights of precious metals such as gold and silver, but this was cumbersome and not accurate. Eventually, systems using standard coins and paper currency developed. These systems were almost always based on decimals or multiples of 10.

Problem 1



A. In the United States, our monetary system is based on multiples of 10. Complete each statement.

_____ pennies = 1 dime

_____ dimes = 1 dollar

_____ pennies = 1 dollar

B. The U.S. monetary system also uses coins that are not multiples of 10. Complete each statement.

_____ pennies = 1 nickel

_____ nickels = 1 quarter

_____ dimes = 1 half-dollar

_____ quarters = 1 dollar

_____ half-dollars = 1 dollar

_____ nickels = 1 half-dollar

_____ quarters = 1 half-dollar

_____ nickels = 1 dime

In almost every case, each number you wrote was either 10 or a factor of 10. What are the exceptions to this?

C. Write the numbers of dollars, dimes, and pennies there are in each amount of money. Start by writing the greatest number of dollars in the amount, then the greatest number of dimes in the amount that remains, and then the number of pennies.

$4.25 = _____ dollars + _____ dimes + _____ pennies

$5.69 = _____ dollars + _____ dimes + _____ pennies

Two dollars and five cents = _____ dollars + _____ dimes + _____ pennies

$6.73 = _____ dollars + _____ dimes + _____ pennies

Four dollars and thirty-four cents = _____ dollars + _____ dimes + _____ pennies

Investigate Problem 1

1. What do you notice about the amount of money and the number of dollars, dimes, and pennies in Part (C) of Problem 1? Use a complete sentence in your answer.

Take Note

The word "cent" comes from the same root as the word "century" which means 100 years.

2. Complete each statement.

   1 penny = _____dime

   1 dime = _____ dollar

   1 penny = _____ dollar

   1 penny = _____ nickel

   1 nickel = _____ quarter

   1 dime = _____ half-dollar

   1 quarter = _____ dollar

   1 half-dollar = _____ dollar

   1 penny = _____ quarter

   1 quarter = _____ half-dollar

   1 nickel = _____ dime

3. What do you notice about all of the relationships in Question 2? Use a complete sentence in your answer.

4. Let's take a look at how we can write values of money using fractions. First, complete each statement.

   $2.35 = _____ dollars + _____ dimes + _____ pennies

   $3.67 = _____ dollars + _____ dimes + _____ pennies

   $5.89 = _____ dollars + _____ dimes + _____ pennies

   Rewrite each digit below as a part of a dollar. Use the relationships between dimes and dollars and between pennies and dollars that you found in Question 2.

   $2.35 = _____ dollars + (3 ____ of a dollar) + (5 ____ of a dollar)

   $3.67 = _____ dollars + (6 ____ of a dollar) + (7 ____ of a dollar)

   $5.89 = _____ dollars + (8 ____of a dollar) + (9 ____ of a dollar)

5. What do you notice about the fractions you used above? Use a complete sentence in your answer.

6. The fractions in Question 4 are special fractions with denominators that are multiples of what number?

Investigate Problem 1

7. Math Path: Decimals

A **decimal** is a number that is written using a system based on multiples of 10. This system is called the *base-ten place-value system*. You will learn more about this system in the next lesson. Each position of a digit in a decimal is 10 times the value of the position to its right. Let's take a look at two ways that we can write decimals. Complete each statement. The first statement is completed for you.

45.123 = 4 tens + 5 ones + 1 tenth + 2 hundredths + 3 thousandths

45.123 = ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

343.43 = ____ hundreds + ____ tens + ____ ones + ____ tenths + ____ hundredths

343.43 = ____ 100s + ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s

8. Use the results of Question 7 to explain why decimals are just special fractions. Use complete sentences in your answer.

Problem 2 Bolivar and Yen

People in Venezuela use a coin called a bolivar. People in Japan use a coin called a yen.

A. On a particular day, $.05 U.S. is equal to 107.365 bolivares. Complete each statement.

107.365 bolivares = ____ whole bolivar + (3 ____ of a bolivar) + (6 ____ of a bolivar) + (5 ____ of a bolivar)

332.095 bolivares = ____ hundreds + ____ tens + ____ ones + ____ tenths + ____ hundredths + ____ thousandths

332.095 bolivares = ____ 100s + ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

B. On a particular day, $5.00 U.S. is equal to 529.635 yen. Complete each statement.

529.635 yen = ____ whole yen + (6 ____ of a yen) + (3 ____ of a yen) + (5 ____ of a yen)

529.635 yen = ____ hundreds + ____ tens + ____ ones + ____ tenths + ____ hundredths + ____ thousandths

529.635 yen = ____ 100s + ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

4

4

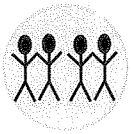
4.2 What's In a Place?

Place Value and Expanded Form

Objectives

In this lesson, you will:

- Represent decimals using a place-value chart.
- Write decimals in word form.
- Use expanded form to write decimals.



Key Terms

- place-value chart
- standard form
- expanded form

Problem 1 Softball stats

In softball, a batting average (AVG) is the number of hits divided by the number of times that a player bats. The home run ratio (HRR) is the number of times that a player bats divided by the number of home runs the player scores. The earned run average (ERA) is a measure of how many runs a pitcher allows.

The table shows the statistics for the top three players on your team.

Player	AVG	HRR	ERA
Sam	0.378	16.5	---
Keesha	---	---	1.89
Jamal	0.294	25.0	---

A. From Lesson 4.1, you know that decimals are special fractions whose digits have values based on their positions. Complete each statement.

0.378 = ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

16.5 = ____ ten + ____ ones + ____ tenths + ____ hundredths + ____ thousandths

1.89 = ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

0.294 = ____ tens + ____ ones + ____ tenths + ____ hundredths + ____ thousandths

25.0 = ____ 10s + ____ 1s + ____ $\frac{1}{10}$s + ____ $\frac{1}{100}$s + ____ $\frac{1}{1000}$s

B. In each decimal above, what determines the value of each digit? Use complete sentences to explain.

C. Which statistic uses decimals with digits only to the tenths place?

Which statistic uses decimals with digits only to the tenths and hundredths places?

Which statistic uses decimals with digits to the thousandths place?

Investigate Problem 1

1. There are actually two things that determine the value of a digit in a decimal. One of them is the numeral or digit. What is the other thing that determines the value of a digit? Write your answer using a complete sentence.

2. Math Path: Place Value

The chart below is a **place-value chart.** The decimal 453.269 is written in **standard form.** Complete the chart by writing the appropriate place value of each digit in the decimal in words in the first row. Then write the place value as a number or fraction in the second row.

Place-Value Chart							
Value in Words	hundreds			.			
Value as a Number or Fraction					$\frac{1}{10}$		
	4	5	3	.	2	6	9

3. For each number, identify the place value of the given digit.

   12,409.53

   What is the place value of 1? What is the place value of 4?

   What is the place value of 3? What is the place value of 0?

   34.5802

   What is the place value of 5? What is the place value of 0?

   What is the place value of 3? What is the place value of 2?

4. Each decimal is written in word form. Use your knowledge of place value to write the decimal as a number.

   Three hundred sixty-five and thirty-four hundredths =

   Eight thousand nine hundred seventy-one and twenty-one hundredths =

   Four hundred and six thousandths (Be careful here!) =

5. Write each decimal in word form.

   12.3 =

   360.23 =

   457.912 =

Take Note

Remember that when you write a number in word form, the "and" represents a decimal point. For instance, to write 425 in word form, you write four hundred twenty-five, *not* four hundred *and* twenty-five.

Problem 2

An official softball, according to the International Softball Federation, has a circumference (distance around) between 11.875 inches and 12.125 inches.

Sometimes it is necessary to write decimals in **expanded form.** In expanded form, a decimal is written as a sum of products. Each product has a power of 10 or a power of 0.1 as a factor.

We already wrote decimals in a similar form using fractions. First complete the following statement.

$11.875 = \_\_\_\_\ 10s + \_\_\_\_\ 1s + \_\_\_\_\ \frac{1}{10}s + \_\_\_\_\ \frac{1}{100}s + \_\_\_\_\ \frac{1}{1000}s$

Now write the decimal as a sum of products.

$11.875 = (\_\_ \times 10) + (\_\_ \times 1) + \left(\_\_ \times \frac{1}{10}\right) + \left(\_\_ \times \frac{1}{100}\right) + \left(\_\_ \times \frac{1}{1000}\right)$

Finally, write the decimal in expanded form.

$11.875 = (\_\_ \times 10) + (\_\_ \times 1) + (\_\_ \times 0.1) + (\_\_ \times 0.01) + (\_\_ \times 0.001)$

Investigate Problem 2

Take Note

Whenever you write a decimal in expanded form and one of the digits is 0, you do not need to include the expanded form for 0.

1. Write each decimal in expanded form.

   45.3 =

   709.65 =

   999.902 =

2. Each decimal is written as a sum of products. Write each decimal in expanded form, in standard form, and in word form.

   $(3 \times 10) + (6 \times 1) + \left(9 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right)$

   Expanded form:

   Standard form:

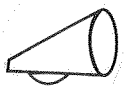
   Word form:

   $(6 \times 10) + \left(9 \times \frac{1}{10}\right) + \left(3 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right) + \left(2 \times \frac{1}{10{,}000}\right)$

   Expanded form:

   Standard form:

   Word form:



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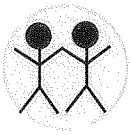
4.3 My Dog Is Bigger Than Your Dog

Decimals as Fractions: Comparing and Rounding Decimals

Objectives

In this lesson, you will:

- Write decimals as fractions.
- Compare and order decimals.
- Round decimals.



Key Terms

- round a decimal

Once we can read decimals and write them in expanded form, we also need to convert them to fractions or mixed numbers to compare them.

Problem 1

You and your friends belong to a kennel club. At the club meeting, you decide to measure your dogs. A dog's height is measured to the highest point on the dog's back between its shoulder blades. The table shows the heights of the dogs.

	Dog Breed	Height
Eddie	cocker spaniel	13.25 inches
Harold	beagle	12.625 inches
Lakeisha	Jack Russell terrier	12.50 inches
Yung	sheltie	13.375 inches

A. Work with your partner to write the decimal height of the cocker spaniel as a mixed number. First write the decimal in expanded form.

$$13.25 = (1 \times 10) + (3 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{100}\right) = 13 + \frac{2}{10} + \frac{5}{100}$$

B. Next, find the LCD of the fractions. Then rewrite the fractions and add.

$$13 + \frac{2}{10} + \frac{5}{100} = 13 + \frac{20}{100} + \frac{5}{100} = 13\frac{25}{100}$$

C. Write the height of the beagle in expanded form. Then write the fractions with like denominators and add.

$$12.625 = (\square \times 10) + (\square \times 1) + \left(\square \times \frac{1}{10}\right) + \left(\square \times \frac{1}{100}\right) + \left(\square \times \frac{1}{1000}\right)$$

$$= \square + \frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1000}$$

$$= \square + \frac{\square}{1000} + \frac{\square}{1000} + \frac{\square}{1000}$$

$$= \square + \frac{\square}{1000}$$

Take Note

To write a proper fraction with a denominator that is a power of 10 as a decimal, write the digits in the numerator and place the decimal point to the left of the digits. For example,

$\frac{425}{1000} = 0.425$.

Investigate Problem 1

1. Write each decimal as a mixed number.

$32.402 = (\square \times 10) + (\square \times 1) + \left(\square \times \frac{1}{10}\right) + \left(\square \times \frac{1}{100}\right) + \left(\square \times \frac{1}{1000}\right)$

$= \square + \frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1000}$

$= \square + \frac{\square}{1000} + \frac{\square}{1000}$

$= \square + \frac{\square}{1000}$

$29.08 =$

$=$

$=$

$12.0045 =$

$=$

$=$

$=$

$=$

$=$

$100.405 =$

$=$

$=$

$=$

$=$

Problem 2

Form a group with another partner team to discuss how you can compare the heights of the dogs to determine which is the tallest dog.

A. Harold says that you can always tell which of two decimals is greater by which number has the most digits. Is Harold correct? Use complete sentences to explain why or why not.

B. Eddie says that you can tell by looking at the last digit of each decimal. If the last digit is greater, then the decimal is greater. Is Eddie correct? Use complete sentences to explain why or why not.

C. Lakeisha says that neither Harold nor Eddie is correct. In fact, she says that you need to look at the place value of the right-most digit first. If these digits in each decimal have the same value, then you keep comparing the place value of each digit from right to left until the place value of one is greater than the other. She says that only then will you know which decimal is greater. Is Lakeisha correct? Use complete sentences to explain why or why not.

D. Finally, Yung says that you can write both of the decimals as mixed numbers or fractions and then decide which is greater. Is Yung correct? Use complete sentences to explain why or why not.

E. Do you have your own way to tell when one decimal is greater than another? Discuss this with your group. Then use your method to determine which dog is the tallest. Write your answer using complete sentences.

Investigate Problem 2

1. Sometimes you need to round whole numbers. Similarly, you sometimes need to round decimals. Round each whole number.

   567 rounded to the nearest ten =
   743 rounded to the nearest hundred =
   5432 rounded to the nearest thousand =

   In each case, how did you determine your answer? Use a complete sentence to explain.

2. Round each whole number.

   745 rounded to the nearest ten =
   850 rounded to the nearest hundred =

   In each case, how did you determine your answer? Were these whole numbers more difficult to round than those in Question 1? Use complete sentences to explain.

3. What rule did you use to round the whole numbers in Question 2?

4. **Math Path: Rounding Decimals**

   You can use a similar rule to round decimals.

   ### Rounding Decimals

   To **round a decimal** to a given place value, look at the digit to the right of the place where you want to round the decimal.

   If the digit is 4 or less, round down.

   If the digit is 5 or greater, round up.

5. Round each decimal to the given place value in the table.

Number	Rounded to the nearest ten	Rounded to the nearest one	Rounded to the nearest tenth	Rounded to the nearest hundredth	Rounded to the nearest thousandth
23.1768					
45.3455					
125.3578					
435.9008					
236.0895					

4.4 Making Change and Changing Hours

Adding and Subtracting Decimals

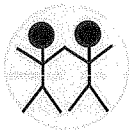
Objectives

In this lesson, you will:

- Add and subtract decimals.
- Represent decimals using base-ten pieces.

Key Terms

- base-ten pieces



Problem 1

Jenny works in her family's small convenience store. Her job is to take money and make change. She finds the total cost and then determines the correct change. Last week, a customer bought three items: one item that cost \$2.34, a second item that cost \$0.98, and a third item that cost three dollars and nine cents.

A. Help Jenny by finding the total cost of all three items. Show your work below.

B. The customer gave Jenny a ten dollar bill. How much change did Jenny give the customer? Show your work below.

C. Compare your work and your answers to Parts (A) and (B) with your partner.

D. Tom, Jenny's younger brother, added the three items as shown below.

$$\begin{array}{r} \$2.34 \\ \$.98 \\ +\ \$3.90 \\ \hline \$16.04 \end{array}$$

Does Tom's answer agree with your answer? Use complete sentences to explain why or why not.

Investigate Problem 1

1. With your partner, decide what Tom needs to learn in order to add and subtract money correctly. Write your answer using a complete sentence.

2. Form a group with another partner team. With your group, write the rule(s) that must be followed in order to correctly add or subtract money.

3. Let's take a look at the rules for adding and subtracting decimals. These rules are similar to the rules for adding and subtracting money. Suppose we want to add 4.05 and 12.341. What is one important step that we must do in order to get the correct answer? Use a complete sentence to answer the question.

4. Recall that decimals are special fractions. We can add the decimals by first writing them as mixed numbers. This will help us to understand how the rules for adding decimals were developed. Complete the statement to rewrite 12.341. The decimal 4.05 is already rewritten for you.

$$4.05 = (4 \times 1) + \left(0 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{100}\right) = 4 + \frac{0}{100} + \frac{5}{100} = 4\frac{5}{100}$$

$$12.341 = 12 + \frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1000} = 12 + \frac{\square}{1000} + \frac{\square}{1000} + \frac{\square}{1000} = 12\frac{341}{1000}$$

5. Now the addition problem is a mixed number addition problem. Complete the problem by writing the fractional parts with the LCD. Then find the sum of the mixed numbers. Finally, write the sum as a decimal.

$$4\frac{5}{100} = 4\frac{\square}{\square}$$

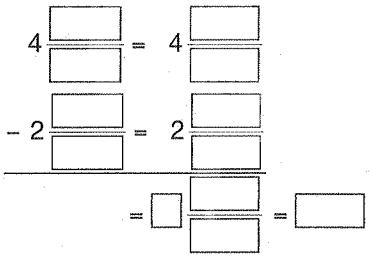
$$+12\frac{341}{1000} = 12\frac{\square}{\square}$$

$$= \square\frac{\square}{\square} = \square$$

Investigate Problem 1

6. Use what you learned in Question 5 to complete the decimal subtraction problem.

4.5 – 2.34 =



7. Work with your group to explain why we have the rule about decimal points for adding and subtracting decimals. Use the results of Questions 5 and 6. Explain your reasoning to each other. Then write your explanation using complete sentences.

8. Find each sum or difference. Be sure to properly apply the rule for decimals.

23.05 + 2.301 =

400.1 + 6.202 =

45.23 – 9.08 =

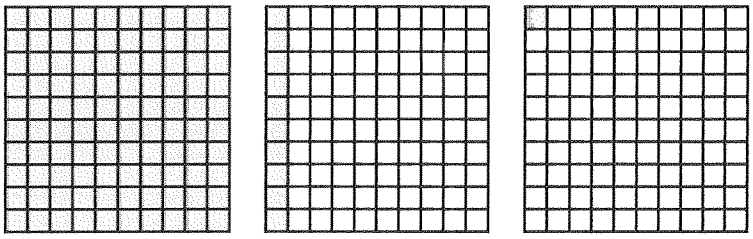
1.0003 – 0.9 =

203.901 – 2.0451 =

54.2 + 45.09 + 6.09 =

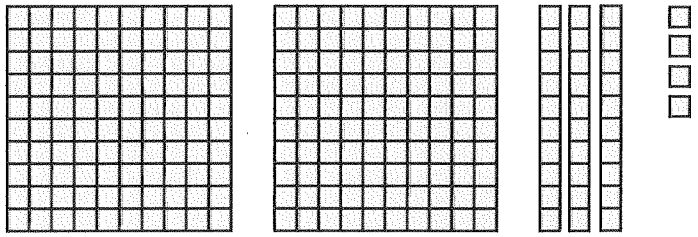
9. Math Path: Base-Ten Pieces

We can represent decimals using **base-ten pieces.**



1 one-piece 1 tenth-piece 1 hundredth-piece

What decimal do the base-ten pieces below represent?



Problem 2

Because of a special sale, Jenny increased the number of hours that she worked each day during a particular week. She kept track of the hours in a table.

Day	Number of Hours
Monday	1.75
Wednesday	1.5
Thursday	2.15

A. Draw the outlines of the base-ten pieces that you would use to represent each decimal in the table.

1.75

1.5

2.15

B. Work with your partner to determine how you can use the base-ten pieces to find the total number of hours that Jenny worked during the week. Draw the outlines of the base-ten pieces that you would use to represent the sum.

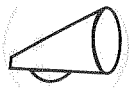
1.75 + 1.5 + 2.15 =

4

Investigate Problem 2

1. Draw the outlines of the base-ten pieces to represent each decimal and the sum.

2.09 + 0.93 =



4.5 Rules Make the World Go Round

Multiplying Decimals

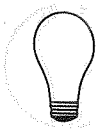
Objectives

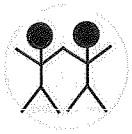
In this lesson, you will:

- Multiply decimals.

Key Terms

- product





Problem 1

You are making a scale model of the planets for science class. If you make the Sun out of a ball that is 30 inches in diameter, you can make the planets to scale using the diameters in the table.

Planet	Diameter
Mercury	0.103
Venus	0.26
Earth	0.276
Mars	0.147
Jupiter	3.09
Saturn	2.6
Uranus	1.1
Neptune	1.07

A. If you changed your mind and wanted to use a ball with a 42-inch diameter for the Sun, you would need to multiply each diameter in the table by 1.4. To find the diameter for Saturn, you need to multiply 2.6×1.4.

You asked four of your friends to help. When you did, you got the four different answers below.

36.4 3.64 0.364 3.6

Which answer is correct? Use complete sentences to explain how you know.

B. Let's examine this problem by representing the decimals using improper fractions. By doing this, we may gain some insight into the rule for multiplying decimals. Work with your partner to complete the steps below.

When you multiply decimals, the number of decimal places in the **product** *is equal to the sum of the number of decimal places in the factors.*

Write the decimals as mixed numbers.
Then write the mixed numbers as improper fractions.
Perform the multiplication.
Write the answer as a mixed number.
Write the mixed number as a decimal.

$$2.6 = 2 + \frac{\square}{10} = \frac{\square}{10} + \frac{\square}{10} = \frac{\square}{10} \qquad 1.4 = 1 + \frac{\square}{10} = \frac{\square}{10} + \frac{\square}{10} = \frac{\square}{10}$$

$$\frac{\square}{10} \times \frac{\square}{10} = \frac{\square}{\square} = \square\frac{\square}{\square} = \square$$

Take Note

Remember that the result of multiplying two decimals is the *product* of the two decimals.

Investigate Problem 1

1. Discuss the result you got in Part (B) with your partner to see if your answer "explains" the rule for multiplying decimals.

2. To be sure that you understand the reasons behind the rule, multiply the decimals.

   $2.5 \times 1.001 =$

   $2.5 = 2\frac{\square}{10} = \frac{\square}{\square}$ $\qquad$ $1.001 = 1\frac{\square}{\square} = \frac{\square}{\square}$

   $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square} = \square$

3. Does your answer to Question 2 support your conclusion in Question 1? Use complete sentences to explain why or why not.

4. Find each product.

   $2.34 \times 2.5 =$ $\qquad$ $400.1 \times 1.01 =$ $\qquad$ $2.001 \times 3.4 =$

   $6.45 \times 3.1 =$ $\qquad$ $0.001 \times .03 =$ $\qquad$ $4.004 \times 0.002 =$

4

Problem 1 Revisited

A. Because you want to use a ball with a 42-inch diameter instead of a ball with a 30-inch diameter for the Sun, you need to multiply each diameter in the table by 1.4. The new diameter for Saturn is done for you. Complete the table.

Planet	Diameter (Sun is 30 in.)	New Diameter (Sun is 42 in.)
Mercury	0.103	
Venus	0.26	
Earth	0.276	
Mars	0.147	
Jupiter	3.09	
Saturn	2.6	3.64
Uranus	1.1	
Neptune	1.07	

B. Suppose that you wanted to use a ball with a 20-inch diameter instead of a 30-inch diameter for the Sun. Would the decimal that you multiply each diameter by be less than or greater than 1? Use complete sentences to explain.

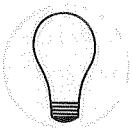
4.6 The Better Buy

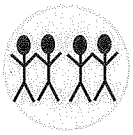
Dividing Decimals

Objectives

In this lesson, you will:

- Divide decimals by whole numbers.
- Divide decimals by decimals.





Key Terms

- dividend
- divisor
- quotient

Problem 1

You want to buy a special gift for your cousin, whose favorite hobby is knitting. You find a web site that sells handspun yarn. You can buy 2.4 ounces of llama yarn for $10.08 or 3.6 ounces of mohair yarn for $20.16. Which type of yarn is the most economical buy?

A. To determine the most economical buy, work with your partner to solve $10.08 \div 2.4$.

Complete the process below by first rewriting the decimals as mixed numbers. Then rewrite the mixed numbers as improper fractions and divide.

$10.08 = 10\frac{8}{100} = \frac{1008}{100}$ $\qquad$ $2.4 = 2\frac{\square}{\square} = \frac{\square}{\square}$

$\frac{1008}{100} \div \frac{\square}{\square} = \frac{1008}{100} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square} = \square\frac{\square}{\square}$

B. The result of Part (A) is a mixed number. Write the mixed number as a decimal.

$10.08 \div 2.4 = \square$

C. Use the method from Part (A) to divide 20.16 by 3.6.

$20.16 = 20\frac{\square}{\square} = \frac{\square}{\square}$ $\qquad$ $3.6 = 3\frac{6}{10} = \frac{36}{10}$

$\frac{\square}{\square} \div \frac{36}{10} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square} = \square$

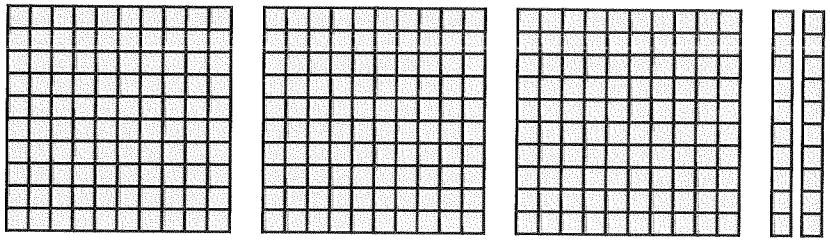
D. Based on your results from Parts (B) and (C), which type of yarn is the most economical buy? Use complete sentences to explain your answer.

Investigate Problem 1

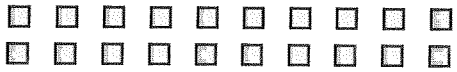
1. To find a method for dividing decimals that is not complicated, we need to look in a different direction. Complete the whole number division problems.

$13\overline{)273}$ $12\overline{)2802}$

2. In the expression $12\overline{)2802}$, you get a remainder. You can write the remainder as a fraction, and then write the fraction as a decimal by writing it with a denominator that is a power of 10: 10, 100, 1000, and so on. Let's look at how to write the fraction $\frac{3}{4}$ as a decimal. Remember that when you first learned to divide whole numbers, such as $8 \div 2$, you divided 8 into 2 equal groups. In the same way, you can think of the fraction $\frac{3}{4}$ as $3 \div 4$, or 3 divided into 4 equal groups. You know that $3 = \frac{30}{10}$. If you divide 30 tenths into 4 equal groups, how many tenths will be in each group? What is left over? Use the base-ten pieces to help you.



So, 3 is 4 groups of $\frac{7}{10}$ with $\frac{2}{10}$ as a remainder. You know that $\frac{2}{10} = \frac{20}{100}$. If you divide 20 hundredths into 4 equal groups, how many hundredths will be in each group? What is left over?



So, $\frac{2}{10}$ is 4 groups of $\frac{5}{100}$. Complete the statement to write $\frac{3}{4}$ as a decimal.

Four equal groups of 3 is $\frac{\square}{10} + \frac{\square}{100} = \frac{75}{100} = \square$.

Investigate Problem 1

3. There is a much less complicated process that uses what we know about whole number division. To find the **quotient** $4\overline{)3}$, first add a decimal point and a zero to the right of the **dividend** (see Step 1). When you do this, you are really rewriting the 3 as $\frac{30}{10}$. When you divide, you write $\frac{7}{10}$ as a decimal in the quotient (see Step 2). But after subtracting, you still have $\frac{2}{10}$ left over, which you can write as $\frac{20}{100}$ by adding an additional zero in the dividend (see Step 3). Divide again. Because there is no remainder, the problem is complete (see Step 4).

Step 1: $4\overline{)3.\square}$

Step 2:
$$\begin{array}{r} 0.\square \\ 4\overline{)3.0} \\ 2\,8 \\ \hline 2 \end{array}$$

Step 3:
$$\begin{array}{r} 0.7 \\ 4\overline{)3.00} \\ 2\,8 \\ \hline 2\square \end{array}$$

Step 4:
$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ 2\,8 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

4. Use Steps 1–4 to write $\frac{7}{8}$ as a decimal.

5. The method used in Question 3 works any time you want to divide a whole number or decimal by a whole number. Find each quotient.

$8\overline{)75}$ $23\overline{)983}$ $16\overline{)99.2}$

$45\overline{)10.35}$ $60\overline{)24.6}$ $25\overline{)7.4}$

Take Note

To summarize, when you divide a decimal by a whole number, divide as you would with whole numbers. The decimal point in the quotient should line up with the decimal point in the dividend.

Investigate Problem 1

When we want to divide by a decimal, we need to write the expression as an equivalent expression in which the **divisor** is a whole number. If we do this, we can use the method in Question 3. To find the quotient 8.66 ÷ 2.4, we first write the division as a fraction.

Because $8.66 = \frac{866}{100}$ and $2.4 = \frac{24}{10}$, we can write 8.66 ÷ 2.4 as $\frac{866}{100} \div \frac{24}{10} = \frac{\frac{866}{100}}{\frac{24}{10}}$. Remember that you can multiply any number by the multiplicative identity 1 and not change the value. So, multiply the fraction by $\frac{10}{10} = 1$. What is the numerator of the fraction as a decimal?

$$\frac{\frac{866}{10\,\cancel{100}}}{\frac{24}{\cancel{10}}} \times \frac{\cancel{10}}{\cancel{10}} = \frac{\frac{866}{10}}{24} = \frac{\square}{24}$$

This type of multiplication is what we need to do when we are dividing by a decimal. After multiplying to get a whole number, we can divide using the method in Question 3. Complete the division.

$24\overline{)86.6}$

6. Work with your partner to write a rule that summarizes what you do when dividing a decimal by a decimal. Then use the rule to find the quotients.

13.78 ÷ 5.3 =

$5.69\overline{)41.537}$

4.7 Bonjour!

Working with Metric Units

Objectives

In this lesson, you will:

- Use metric units to measure length, mass, and capacity.
- Choose the appropriate unit of measure.

Key Terms

- metric system
- meter
- gram
- liter

The customary system can be difficult and time-consuming to use because it depends on fractions. About 400 years ago, scientists proposed a measurement system based on decimals. France was the first nation to officially adopt this system, called the **metric system.** This system has become the basis for the International System of Units that is currently used in many countries.

Problem 1

For a project, everyone in your class chooses a pen pal. Your pen pal is from France. As you exchange letters, you notice that she uses centimeters and meters when she describes the length of something. She uses grams and kilograms when she writes about the mass of something. She uses liters and milliliters to discuss the capacity of something.

You wonder how these measurements compare to those used in the United States. She sends you the following information:

A meter is the length of a yardstick plus the length of a piece of chalk.

A centimeter is about the width of the tip of your pinky finger.

A gram is the mass of a large thumbtack. A nickel weighs 5 grams.

A kilogram is about the mass of a large book.

A liter is about the capacity of a large bottle of water.

A milliliter is about the capacity of an eyedropper.

A. Your pen pal writes that she has a "chien" that has a mass of 9.5 kilograms. Is she talking about a car or a dog? Use complete sentences to explain your answer.

B. Your pen pal writes that her "écritoire" at school is 0.65 meters wide. Is she talking about a desk or a book? Use complete sentences to explain your answer.

Investigate Problem 1

1. Math Path: The Metric System

One advantage of the metric system is the way in which units are named. The naming convention is built on six main prefixes.

kilo-	which means 1000	deci-	which means $\frac{1}{10}$
hecto-	which means 100	centi-	which means $\frac{1}{100}$
deka-	which means 10	milli-	which means $\frac{1}{1000}$

By putting each prefix together with a root word for a base unit of length, mass, or capacity, you have all of the units of the metric system.

The **meter** is the metric base unit of length.

The **gram** is the metric base unit of mass.

The **liter** is the metric base unit of capacity.

For example, a *kilometer* is 1000 meters, and a *milliliter* is $\frac{1}{1000}$ of a liter. Work with your partner to write each unit of measure and its meaning in the table below.

kilo-	hecto-	deka-	unit	deci-	centi-	milli-
kilometer 1000 meters			meter 1 meter			
			liter 1 liter			
			gram 1 gram			

4

Take Note

Abbreviations for commonly-used metric units are listed:
kilometer: km
centimeter: cm
meter: m
liter: l
milliliter: mL
kilogram: kg
gram: g

2. Because this system is based on powers of 10, making conversions is straightforward. Complete each conversion.

$$5.4 \text{ kilograms} \times \frac{\square \text{ grams}}{1 \text{ kilogram}} = \square \text{ grams}$$

$$235 \text{ milliliters} \times \frac{1 \text{ liter}}{\square \text{ milliliters}} = \square \text{ liter}$$

$$37.8 \text{ dekameters} \times \frac{1 \text{ kilometer}}{\square \text{ dekameters}} = \square \text{ kilometer}$$

Investigate Problem 1

3. Complete each row of the table by converting the given measures to all of the other units in the row.

kilo-	hecto-	deka-	unit	deci-	centi-	milli-
1 kilometer	10 hectometers		1000 meters			
			$\frac{1}{10}$ liter	1 deciliter		
					$\frac{1}{10}$ centimeter	1 millimeter

4. Because conversions in the metric system are based on powers of 10, we can convert by multiplying or dividing by powers of 10. We also know that when we multiply by 10, the answer can be found by adding a zero to the right of a whole number or by moving the decimal one place to the right.

What are the rules for multiplying a decimal and a whole number by 100? Use complete sentences in your answer.

What are the rules for multiplying a decimal and a whole number by 1000? Use complete sentences in your answer.

What are the rules for multiplying a decimal and a whole number by any other multiple of 10? Use complete sentences in your answer.

What are the rules for dividing a decimal and a whole number by 10? Use complete sentences in your answer.

What are the rules for dividing a decimal and a whole number by 100? Use complete sentences in your answer.

What are the rules for dividing a decimal and a whole number by 1000? Use complete sentences in your answer.

What are the rules for dividing a decimal and a whole number by any other power of 10? Use complete sentences in your answer.

Investigate Problem 1

5. The rules from Question 4 allow you to easily convert measurements. For example, to convert kilometers to meters, you *multiply* the number of kilometers by 1000. To convert centimeters to meters, you *divide* the number of centimeters by 10.

Use the rules from Question 4 to convert each measurement.

10 hectometers = _____ decimeters

2 kilograms = _____ grams

25 millimeters = _____ meter

34.5 centimeters = _____ dekameter

23 decigrams _____ dekagram

2.34 meters = _____ centimeters

6. Conversions between the U.S. customary system and the metric system are not as easy as converting within the metric system. Here are some ways to think about how the two systems are related.

A meter is a little longer than a yard.

A liter is a little more than a quart.

A kilogram is a little heavier than 2 pounds.

For each of the objects in the table, select the most appropriate metric unit and customary unit to use to measure them.

Item	Metric Unit	Customary Unit
height of a tree		
mass of a pin		
volume of water in a tub		
length of your classroom		
width of this page		
mass of a person		
volume of liquid in a cup		

7. Use what you have learned about metric units to write a paragraph to your pen pal. In the paragraph, describe the length of an object, the mass of a second object, and the capacity of a third object using metric units.

Paragraphs will vary but should include a customary measure of each: length, mass, and capacity.

Looking Back at Chapter 4

Key Terms

decimal ○ p. 113
place-value chart ○ p. 116
standard form ○ p. 116
expanded form ○ p. 117
round a decimal ○ p. 122
base-ten pieces ○ p. 125
product ○ p. 127
quotient ○ p. 131
dividend ○ p. 131
divisor ○ p. 132
metric system ○ p. 133
meter ○ p. 134
gram ○ p. 134
liter ○ p. 134

Summary

Writing Decimals as Special Fractions (p. 113)

In a decimal, each position of a digit is 10 times the value of the position to its right.

Examples

26.356 = 2 tens + 6 ones + 3 tenths + 5 hundredths + 6 thousandths

26.356 = 2 10s + 6 1s + 3 $\frac{1}{10}$s + 5 $\frac{1}{100}$s + 6 $\frac{1}{1000}$s

532.48 = 5 hundreds + 3 tens + 2 ones + 4 tenths + 8 hundredths

532.48 = 5 100s + 3 10s + 2 1s + 4 $\frac{1}{10}$s + 8 $\frac{1}{100}$s

Writing Decimals in Word Form as Numbers (p. 116)

To write a decimal in word form as a number, use your knowledge of place value. Remember that the word "and" represents the decimal point.

4

Examples

Seventy-three and sixteen hundredths = 73.16

Four hundred fifty-six and three hundred twenty-one thousandths = 456.321

Five hundred and nine hundredths = 500.09

Writing Decimals in Word Form (p. 116)

When writing a decimal in word form, remember to write only the decimal point as the word "and."

Examples

18.145 = Eighteen and one hundred forty-five thousandths

207.98 = Two hundred seven and ninety-eight hundredths

1001.007 = One thousand one and seven thousandths

Writing Decimals in Expanded Form (p. 117)

To write a decimal in expanded form, write the decimal as a sum of products using fractions and then write the decimal in expanded form.

Examples

$$57.12 = \left(5 \times 10\right) + \left(7 \times 1\right) + \left(1 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right)$$

$$57.12 = \underline{(5 \times 10) + (7 \times 1) + (1 \times 0.1) + (2 \times 0.01)}$$

$$31.054 = \left(3 \times 10\right) + \left(1 \times 1\right) + \left(5 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right)$$

$$31.054 = \underline{(3 \times 10) + (1 \times 1) + (5 \times 0.01) + (4 \times 0.001)}$$

Writing Decimals in Standard Form (p. 117)

To write a decimal in standard form, find the sum of the products.

Examples

$(7 \times 10) + (3 \times 1) + (8 \times 0.1) + (5 \times 0.01) + (6 \times 0.001)$

Standard form: 73.856

$(8 \times 100) + (1 \times 1) + (2 \times 0.1) + (4 \times 0.01) + (9 \times 0.001)$

Standard form: 801.249

Writing Decimals as Mixed Numbers (p. 119)

To write a decimal as a mixed number, first write the decimal as a sum of products using fractions. Then write the whole number part. Finally, write the fractions with like denominators and add.

Example

$$927.415 = (\underline{9} \times 100) + (\underline{2} \times 10) + (\underline{7} \times 1) + \left(\underline{4} \times \frac{1}{10}\right) + \left(\underline{1} \times \frac{1}{100}\right) + \left(\underline{5} \times \frac{1}{1000}\right)$$

$$= \underline{927} + \frac{4}{10} + \frac{1}{100} + \frac{5}{1000}$$

$$= \underline{927} + \frac{400}{1000} + \frac{10}{1000} + \frac{5}{1000}$$

$$= \underline{927}\ \frac{415}{1000}$$

Comparing and Ordering Decimals (p. 121)

To compare decimals, compare the digits in corresponding place values from left to right. If the digits in a place value are the same, keep comparing the digits in corresponding place values from left to right until the digit in the place value of one decimal is greater than the digit in the same place value of the other decimal.

Example

714.563 714.539 714.529 714.599

The digits in the hundreds, tens, ones, and tenths places are identical. The digits in the hundredths place are different. So, the numbers from least to greatest are 714.529, 714.539, 714.563, and 714.599.

Rounding Decimals (p. 122)

To round a decimal, look at the digit to the right of the place where you want to round the decimal. If the digit is 4 or less, round down. If the digit is 5 or greater, round up.

Examples

Number	Rounded to the Nearest Ten	Rounded to the Nearest One	Rounded to the Nearest Tenth	Rounded to the Nearest Hundredth	Rounded to the Nearest Thousandth
64.2367	60	64	64.2	64.24	64.237
358.7491	360	359	358.7	358.75	358.749
981.0172	980	981	981.0	981.02	981.017

Adding and Subtracting Decimals (p. 123)

To add or subtract decimals, arrange the numbers vertically so that the decimal points line up. Then add or subtract as you would with whole numbers.

Examples

$$\begin{array}{r} 3.056 \\ +\ 2.14 \\ \hline 5.196 \end{array} \qquad \begin{array}{r} 15.7 \\ -\ 8.32 \\ \hline 7.38 \end{array} \qquad \begin{array}{r} 2.052 \\ 6.743 \\ +\ 1.859 \\ \hline 10.654 \end{array} \qquad \begin{array}{r} 10.619 \\ 3.047 \\ +\ 7.681 \\ \hline 21.347 \end{array}$$

Multiplying Decimals (p. 127)

When multiplying decimals, the total number of decimal places in the product is equal to the sum of the numbers of decimal places in the factors.

Examples $3.004 \times 4.9 = 14.7196$

$$\begin{array}{r} 3.004 \\ \times\ \ 4.9 \\ \hline 2\ 7036 \\ 12\ 0160 \\ \hline 14.7196 \end{array}$$

The factors have a total of 4 decimal places, so the product has 4 decimal places.

4

Dividing Decimals by Whole Numbers (p. 131)

To divide a decimal by a whole number, use long division. As necessary, add a decimal point and zeros after the dividend.

Examples

$$\begin{array}{r} 0.16 \\ 49\overline{)7.84} \\ 49 \\ \hline 294 \\ 294 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0.35 \\ 32\overline{)11.20} \\ 96 \\ \hline 160 \\ 160 \\ \hline 0 \end{array}$$

Dividing Decimals by Decimals (p. 132)

To divide a decimal by a decimal, first multiply the divisor and dividend by an appropriate multiple of 10, then use long division.

Examples

$4.56\overline{)14.82} \rightarrow 456\overline{)1482.00}$ Multiply the divisor and dividend by 100.

$$\begin{array}{r} 3.25 \\ 456\overline{)1482.00} \\ \underline{1368} \\ 114\,0 \\ \underline{912} \\ 22\,80 \\ \underline{22\,80} \\ 0 \end{array}$$

Converting Measurements in Larger Metric Units to Smaller Metric Units (p. 134)

To convert a measurement in a larger metric unit to a smaller metric unit, multiply by an appropriate multiple of 10.

Examples

$$54 \text{ kilometers} = 54 \text{ kilometers} \times \frac{100{,}000 \text{ centimeters}}{1 \text{ kilometer}} = 5{,}400{,}000 \text{ centimeters}$$

$$7.2 \text{ hectograms} = 7.2 \text{ hectograms} \times \frac{100 \text{ grams}}{1 \text{ hectogram}} = 720 \text{ grams}$$

Converting Measurements in Smaller Metric Units to Larger Metric Units (p. 134)

To convert a measurement in a smaller metric unit to a larger metric unit, divide by an appropriate multiple of 10.

Examples

$$8245.3 \text{ millimeters} = 8245.3 \text{ millimeters} \times \frac{1 \text{ hectometers}}{100{,}000 \text{ millimeter}} = 0.082453 \text{ hectometers}$$

$$782.54 \text{ centigrams} = 782.54 \text{ centigrams} \times \frac{1 \text{ dekagram}}{1000 \text{ centigrams}} = 0.78254 \text{ dekagrams}$$

Choosing an Appropriate Unit of Measure (p. 136)

Example

To choose an appropriate unit of measure, use the real-life examples from Lesson 4.7 (p. 136) to approximate the measure.

Item	Metric Unit	Customary Unit
height of a tree	meter	foot or yard
mass of a pin	milligram	ounce
volume of liquid in a cup	dl or cl	cup or oz

Looking Ahead to Chapter 5

Focus In Chapter 5, you will work with ratios, rates, and proportions. You will write ratios, rates, unit rates, and proportions. You will compare ratios, compare rates, and solve proportions.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 5.

Write a fraction that is equivalent to the given fraction.

1. $\frac{3}{8}$
2. $\frac{16}{24}$
3. $\frac{27}{36}$
4. $\frac{15}{22}$
5. $\frac{9}{11}$
6. $\frac{5}{12}$

Fill in the blank with the correct number.

7. $\underline{?} + 3 = 28$
8. $5 \times \underline{?} = 35$
9. $\frac{?}{9} = 7$

Read the problem scenario below.

Sofia, Marian, Brianna, and Cassandra are comparing the number of dance CDs each person has in her CD collection. Three-fifths of Sofia's CD collection are dance CDs. Five-sixths of Marian's CD collection are dance CDs. Brianna's dance CDs make up $\frac{2}{9}$ of her CD collection. Cassandra's dance CDs make up $\frac{1}{3}$ of her CD collection.

10. Order the fractions from least to greatest.
11. Who has the greatest fraction of dance CDs in her collection?
12. Who has the least fraction of dance CDs in her collection?

5

Key Terms

ratio p. 145
rate p. 150
proportion p. 150
means p. 151
extremes p. 151
unit rate p. 156
variable p. 160