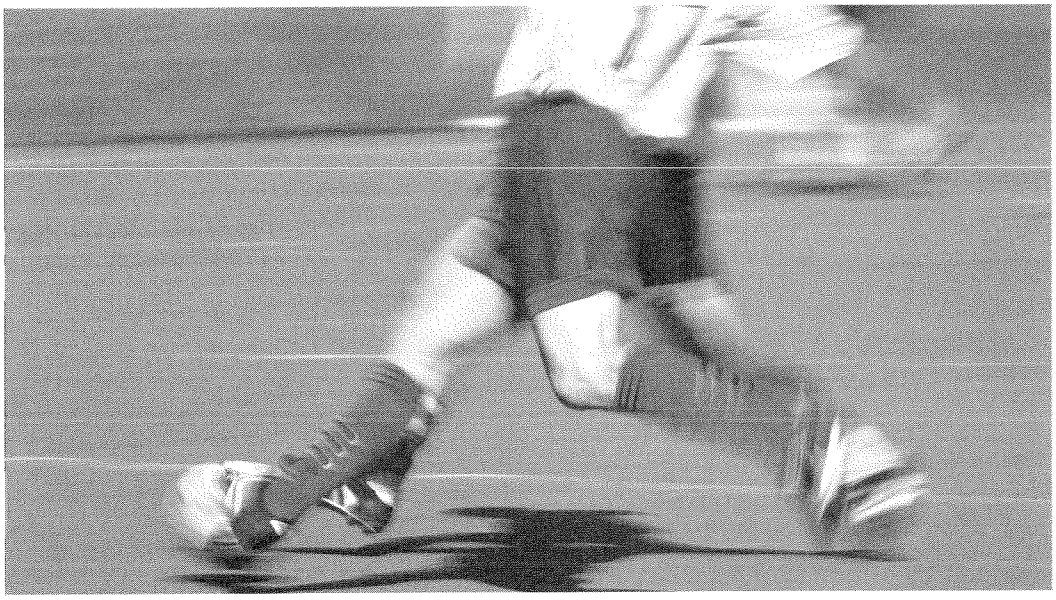
2

Fractions



Soccer, played with two teams of 11 players, is a popular sport in many parts of the world, particularly in Europe, Latin America, and Africa. In Lesson 2.3, you will answer questions about soccer teams in a local neighborhood sports organization.

2

2.1 Comic Strips

Dividing a Whole into Fractional Parts

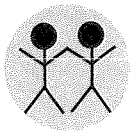
Objectives

In this lesson, you will:

- Use fractions to represent parts of a whole.

Key Terms

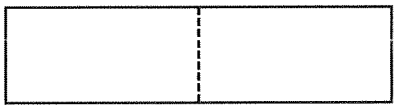
- fraction
- numerator
- denominator

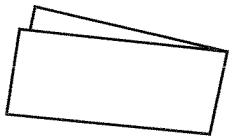


Problem 1

You decide to create a comic strip for your school's newspaper. To do this, you cut a strip of paper that is a little narrower than the width of a newspaper page. The strip represents one whole comic.

A. For your first comic, you want to have two frames. Your teacher will provide you with a strip of paper that represents one whole comic. Work with your partner to divide the strip into two parts of equal size by folding the strip like the one shown below. Do not measure the strip.





Note: Throughout this lesson, always fold the strip as shown above so that the fold decreases the length of the longest side.

B. Write a complete sentence that describes how you divided the strip into two equal parts.

C. How can you be sure that you have two parts that are exactly the same size? Use complete sentences to explain.

Investigate Problem 1

1. Math Path: Fractions

We can use a fraction to represent one or more parts of a whole.

A **fraction** is a number of the form $\frac{a}{b}$ where a is the **numerator** and b is the denominator. The **denominator** tells us how many equal parts the whole is divided into and the numerator tells us how many of these parts we have. The denominator of a fraction cannot be 0.

Write a fraction that represents one frame of the strip.

2

2. What is the denominator of the fraction from Question 1? Use complete sentences to explain what the denominator represents.

3. What is the numerator of the fraction from Question 1? Use complete sentences to explain what the numerator represents.

4. Label each part of the strip with this fraction.

Problem 2

A. Fold a new strip of paper. Without opening up the strip, fold the strip again. How many frames would be in your comic if you used this strip? How many equal parts do you have? Label each of the parts with the appropriate fraction.

B. Repeat the process in part (A) with a new strip of paper, but this time fold the strip a total of three times. How many frames would be in your comic if you used this strip? How many equal parts do you have? Label each of the parts with the appropriate fraction.

C. Repeat the process in part (B) with a new strip of paper, but this time fold the strip a total of four times. How many equal parts has this strip been divided into? Could you use this strip for frames of a comic? Use complete sentences to explain why or why not. Label each of the parts with the appropriate fraction.

Investigate Problem 2

1. How difficult was the process in Problem 2? Use complete sentences to describe the fractions that you can find using this process.

2. Arrange your strips in a column so that all of the left edges are lined up and the strips are ordered from the strip with the largest parts to the strip with the smallest parts. If you folded carefully, you will notice that some of the folds line up with each other. Use your fraction strips to complete each statement below.

   It takes _____ of the parts labeled as $\frac{1}{4}$ to make up one of the parts labeled as $\frac{1}{2}$.

   It takes _____ of the parts labeled as $\frac{1}{8}$ to make up two of the parts labeled as $\frac{1}{4}$.

   It takes _____ of the parts labeled as $\frac{1}{16}$ to make up three of the parts labeled as $\frac{1}{8}$.

3. Write two other sentences similar to those in Question 2 relating the parts of your fraction strips.

Problem 3 More Strips with Equal Parts

A. Divide a strip into exactly three equal parts by folding. Is this more or less difficult than dividing a strip into two equal parts? Explain your answer using complete sentences.

Label each part of the strip with a fraction.

B. Divide another strip into three equally sized parts by folding. Then divide each of these parts into two equal parts by folding. Label each part of the strip with a fraction.

C. Take a third strip and repeat the procedure you used in part (B). Then divide each of the parts of the strip into two equal parts by folding. Label each part of the strip with an appropriate fraction.

Investigate Problem 3

1. You have created fraction strips for many common fractions. You can create three additional strips that are useful.

Divide a strip into exactly five equal parts by folding. Label each part of the strip with a fraction.

Divide another strip into exactly five equal parts by folding. Then divide each of these parts into two equal parts by folding. Label each part of the strip with a fraction.

Divide a third strip and repeat the procedure you used in the previous step. Then divide each of the parts of the strip into two equal parts by folding. Label each part of the strip with a fraction.

2. Arrange your strips from Question 1 in a column so that all of the left edges are lined up and the strips are ordered from the strip with the largest parts to the strip with the smallest parts. Write as many sentences as you can that relate the sizes of your fraction pieces. We will be using these fraction strips throughout this chapter and throughout the course, so be sure to keep them.

3. Circles, squares, and rectangles can also be used to represent fractions. Represent each fraction as indicated.

Use a circle to represent $\frac{3}{4}$.

Use a rectangle to represent $\frac{6}{7}$.

Use a square to represent $\frac{1}{3}$.

Use a circle to represent $\frac{5}{8}$.

Investigate Problem 3

Use a square to represent $\frac{5}{12}$.

Use a square to represent $\frac{10}{11}$.

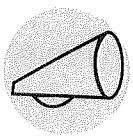
Use a rectangle to represent $\frac{7}{8}$.

Use a square to represent $\frac{4}{9}$.

Use a circle to represent $\frac{5}{7}$.

Use a circle to represent $\frac{11}{12}$.

4. Which fractions in Question 3 were more difficult to represent accurately? Which were easier? Use complete sentences to explain your reasoning.



2

2.2 Dividing Quesadillas

Dividing More Than One Whole into Parts

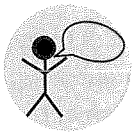
Objectives

In this lesson, you will:

- Use fractions to divide more than one whole into equally sized parts.
- Determine whether a solution is reasonable.

Key Terms

- reasonable solution



Problem 1

As part of your school's international foods festival, a classmate brought quesadillas that he made for the entire class. However, he only brought 21 quesadillas for the 28 students in your class. Because your class normally works in groups of four, your teacher suggests that you give the same number of quesadillas to each group of four students. How many quesadillas should each group receive? Each group must then decide how to divide their quesadillas equally among the group members.

A. Work together as a class to complete the following tasks.

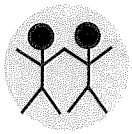
Read the problem carefully. What is a quesadilla? How many quesadillas should each group get?

B. Work by yourself to answer the following questions.

How would you divide the quesadillas equally among the group members? Use complete sentences to explain. Does each person get the same amount? How do you know?

Draw a diagram that represents the problem.

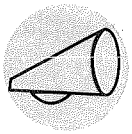
Investigate Problem 1



1. Explain your solution to your partner. Does your partner have the same solution? If not, do both solutions give the same answer? Do both solutions work? Choose one of the solutions to share with another partner team.

2. Form a group with another partner team. Take turns sharing the solution that you chose. Are both solutions the same? If not, do both solutions give the same answer? Do both solutions work? Choose a solution to share with the class. Use complete sentences to explain the solution that you chose.

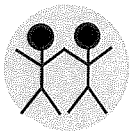
2



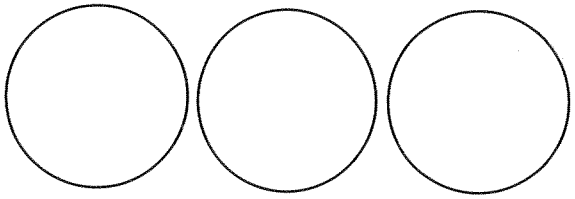
3. Share your solution with the class. Do all of the groups have the same solution? If not, do all of the solutions give the same answer? Do all of the solutions work? Use complete sentences to explain why or why not.

Problem 2

Several students suggest ways to divide the quesadillas equally. Paula wants to have at least one piece that is one half of a quesadilla, so she starts by dividing all of the quesadillas in half.



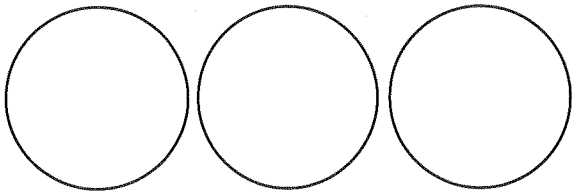
Below are three circles that represent a group's quesadillas. Work with a partner to divide each circle into halves. Label each person's portion on the diagram.



Will Paula's method work? Why or why not? If possible, explain what you need to do to make Paula's method work.
Use complete sentences.

Investigate Problem 2

1. Dwayne says that because each group has three quesadillas, he will divide each quesadilla into thirds. Divide each circle into thirds and label each person's portion on the diagram.

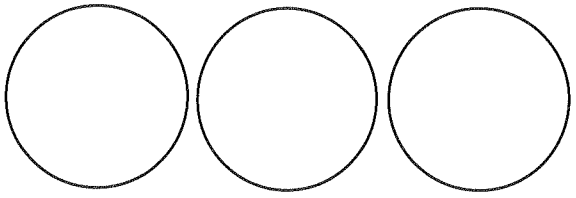


Will Dwayne's method work? Why or why not? If possible, explain what you need to do to make Dwayne's method work. Use complete sentences.

2

For Dwayne's method, name the different fractions and the number of each fraction that represents how much of the quesadillas each person should receive. Is this the same amount as in your group's solution? How do you know? Use complete sentences to explain.

2. Clifton wants to divide each quesadilla into eighths because he says that each person will get more pieces. Divide each circle into eighths and label each person's portion on the diagram.

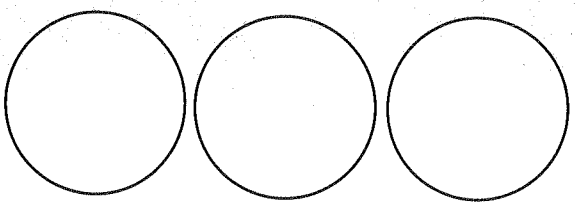


Will Clifton's method work? Why or why not? If possible, explain what you need to do to make Clifton's method work. Use complete sentences.

For Clifton's method, name the different fractions and the number of each fraction that represents how much of the quesadillas each person should receive. Is this the same amount as in your group's solution? How do you know? Use complete sentences to explain.

Investigate Problem 2

3. Juanita decides that she will first divide all of the quesadillas into any number of equal-sized pieces. Then she will give each person the same number of pieces up until the last piece or last few pieces. Then she will divide the last piece or last few pieces into four equal parts. For example, she wants to divide the quesadillas into fifths. Divide each circle into fifths and label each person's portion on the diagram.



Will Juanita's method work? Why or why not? If possible, explain to your partner what you need to do to make Juanita's method work.

For Juanita's method, name the different fractions and the number of each fraction that represents how much of the quesadillas each person should receive.

4. In Lesson 2.1, you learned that a fraction is used to describe equal parts of one whole. Use complete sentences to explain what a fraction describes in this lesson.

5. Math Path: Reasonable Solutions

Often a solution will not make sense in the context of a problem. This means that the solution is not a **reasonable solution.** For instance, because there are more students than quesadillas, each student should receive less than one quesadilla. So, if your solution indicated that each person should receive more than one quesadilla, you would know that you did something wrong.

Determine whether the following solution is reasonable. Use complete sentences to explain why or why not.

For the international foods festival, a teacher brings 225 biscotti for each of the 150 students in your grade. Each student receives $\frac{2}{3}$ of a biscotti.

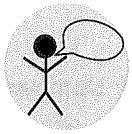
2.3 No "I" in Team

Dividing Groups into Fractional Parts

Objectives

In this lesson, you will:

- Use fractions to represent portions of a whole.



Problem 1

You are a member of a local neighborhood sports organization. The organization sponsors 7 baseball teams each for boys and girls, 6 basketball teams each for boys and girls, 10 soccer teams for girls, and 6 football teams for boys.

A. Work together as a class to complete the following tasks.

Read the problem carefully. What information is known?

B. Work by yourself to answer the following questions.

How many total teams does the organization sponsor?

How many boys' teams?

How many girls' teams?

How many baseball teams?

How many basketball teams?

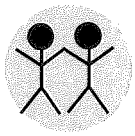
How many soccer teams?

How many football teams?

Draw a diagram that represents the teams.

2

Investigate Problem 1



1. What fraction of the total number of teams are girls' teams?

   What fraction are boys' teams?

   What fraction are soccer teams?

   What fraction are basketball teams?

   What fraction are football teams?

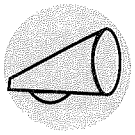
   What fraction are baseball teams?

2. What fraction of the total number of teams are not basketball teams?

   What fraction are not soccer teams?

3. Form a group with another partner team. Compare your answers to Questions 1 and 2. Are your answers the same? Take turns explaining how you got your answers and why you think they are correct.

4. As a group, write two other questions like Questions 1 and 2. Write the answers to the questions.



5. Exchange your questions with another group. Answer the questions. Then compare your answers with the other group's answers.

6. We used fractions in this problem in a slightly different way. Use complete sentences to answer each question about the fractions.

   What does the denominator of any of the fractions represent?

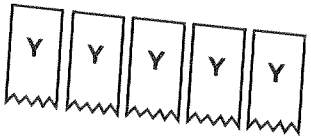
   What does the numerator represent?

   What is one whole?

Problem 2

Jennifer and Trevor are on the school swim team and have both won ribbons. Jennifer has 5 yellow ribbons, 8 blue ribbons, and a handful of red ribbons. Trevor has 3 blue ribbons, 4 yellow ribbons, and a handful of red ribbons.

A. The picture below shows Jennifer's yellow ribbons, which represent one fifth of her total number of ribbons. Complete the picture to show all of Jennifer's ribbons.



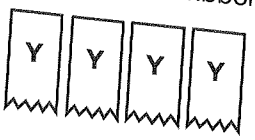
B. How many red ribbons does Jennifer have?

What fraction of her total number of ribbons are red ribbons?

What fraction of her total number of ribbons are blue ribbons?

Investigate Problem 2

1. Trevor's yellow ribbons are shown below. Draw his blue ribbons.



2. How many total ribbons does Trevor have if together the yellow and blue ribbons are one third of his total number of ribbons?

What fraction of his total number of ribbons are red ribbons?

3. Jennifer and Trevor put all of their ribbons together in one display.

What fraction of the ribbons are Jennifer's ribbons?

What fraction are Trevor's ribbons?

What fraction are yellow ribbons?

What fraction are blue ribbons?

What fraction are red ribbons?

Investigate Problem 1

Use your fraction strips from Lesson 2.1 to answer the following questions.

1. Compare the fraction strip that represents $\frac{1}{2}$ to the fraction strip that represents $\frac{1}{12}$. Write complete sentences that describe what you observe.

2. Compare the fraction strip that represents $\frac{1}{8}$ to the fraction strip that represents $\frac{1}{12}$. Write complete sentences that describe what you observe.

3. How does your observation in Question 2 relate to your observation in Question 1?

4. What part of the fraction strip that represents $\frac{1}{12}$ is the same size as two parts of the fraction strip that represents $\frac{1}{8}$?

5. **Math Path: Equations and Equivalent Fractions**

   Two fractions that represent the same amount or quantity are called **equivalent fractions.** Equivalent fractions are indicated by writing an equals sign, =, between them as shown below.

   $$\frac{1}{3} = \frac{2}{6}$$

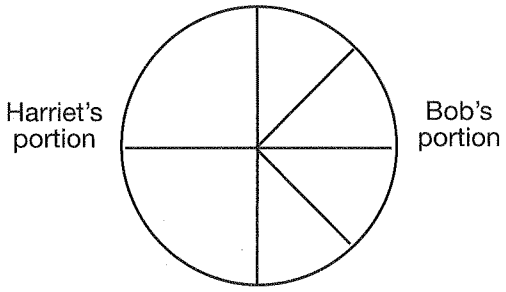
   The equals sign was developed to show the relationship between quantities that are the same but not identical. A mathematical sentence that contains an equals sign is called an **equation.** For example, in Lesson 1.2, you learned that $6 \times 2 = 2 \times 6$, where both 6×2 and 2×6 are different representations of the same quantity, but are not exactly identical.

   Write equivalent fractions for the fractions you found in Questions 1 and 2. Express your answers as equations.

Investigate Problem 1

6. Use the diagram below to write equivalent fractions that represent Harriet's and Bob's portions of the pizza.

_____ = _____



Problem 2

Harriet and Bob decide to have a pizza party. They invite all of their friends and order six pizzas. They asked the pizzeria to slice each pizza in a different way. Here's what they noticed.

$\frac{1}{2}$ of a pizza is the same amount as $\frac{6}{12}$ of a pizza, so $\frac{1}{2} = \frac{6}{12}$.

$\frac{2}{3}$ of a pizza is the same amount as $\frac{8}{12}$ of a pizza, so $\frac{2}{3} = \frac{8}{12}$.

$\frac{3}{4}$ of a pizza is the same amount as $\frac{12}{16}$ of a pizza, so $\frac{3}{4} = \frac{12}{16}$.

A. What do you notice about the numerators and denominators of the equivalent fractions? Write your ideas using complete sentences.

B. What do you need to do to both the numerator and the denominator of a fraction in order to write another fraction that is equivalent? Write your answer using a complete sentence.

Investigate Problem 2

1. Fill in the blanks so that each pair of fractions are equivalent. If possible, use your fraction strips to check your work.

$\frac{2}{3} = \frac{\square}{6}$ $\quad \frac{1}{3} = \frac{\square}{9}$ $\quad \frac{2}{5} = \frac{\square}{15}$

$\frac{5}{6} = \frac{15}{\square}$ $\quad \frac{\square}{16} = \frac{5}{8}$ $\quad \frac{7}{8} = \frac{\square}{32}$

2. Fill in the blanks so that each equality is true. If possible, use your fraction strips to check your work.

$\frac{3}{4} = \frac{3 \times \square}{4 \times \square} = \frac{9}{12}$ $\quad \frac{5}{6} = \frac{5 \times \square}{6 \times \square} = \frac{\square}{12}$

$\frac{5}{8} = \frac{5 \times \square}{8 \times \square} = \frac{15}{\square}$ $\quad \frac{7}{40} = \frac{7 \times \square}{40 \times \square} = \frac{\square}{40}$

3. Math Path: Multiplicative Identity

In Question 2, you multiplied the numerator and denominator of a fraction by the same number, such as 2. This is the same as multiplying the given fraction by a fraction with the same numerator and denominator, such as $\frac{2}{2}$. Any fraction whose numerator and denominator are the same number is equivalent to 1. For instance, the fraction $\frac{2}{2}$ is the same as 1. So, in Question 2, you were just multiplying each fraction by the number 1, the multiplicative identity. (You learned about the multiplicative identity in Lesson 1.4).

Use your fraction strips to compare the equivalent fractions $\frac{3}{4}$ and $\frac{9}{12}$. How many twelfths are the same as one fourth?

How does this help you determine what number to multiply the numerator 3 and denominator 4 by to find the equivalent fraction $\frac{9}{12}$? Use complete sentences to explain.

4. Do you think that it is always possible to find a fraction with a denominator of eight that is equivalent to a given fraction? Use complete sentences and an example to explain your answer.

2.5 When Twelfths Are Eighths

Simplifying Fractions

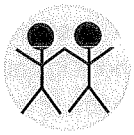
Objectives

In this lesson, you will:

- Write fractions in simplest form.

Key Terms

- simplest form
- simplest terms
- completely simplified



Problem 1

Your aunt has made two pans of cornbread for dinner. Your younger cousins, Tobias and Henry, are having their usual argument. Tobias says that he got more of the cornbread than Henry got. Both pans of cornbread are the same size. The first pan was cut into 12 pieces and Tobias got 9 pieces. The second pan was cut into 8 pieces and Henry got 6 pieces.

A. Because you are becoming a fraction expert, you help them to settle the argument. Write a paragraph to explain who got more and why. Use your fraction strips along with any other representations that you think will be helpful.

B. Just when you think you have them convinced, in walks your other cousin, Anita, who has a third pan of cornbread that is the same size, but cut into 4 pieces. Anita took 3 of the pieces. She now joins the discussion, saying that she actually has the most because her pieces are the biggest. Write a short paragraph to explain which of the three cousins, if any, got the most cornbread.

Investigate Problem 1

1. In Problem 1, you were again thinking about equivalent fractions. Fill in the blanks so that each pair of fractions is equivalent.

   Use your fraction strips that represent $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$.

   $\frac{8}{16} = \frac{\square}{8}$ $\quad\quad$ $\frac{2}{8} = \frac{\square}{4}$ $\quad\quad$ $\frac{12}{16} = \frac{\square}{4}$

2. In Question 1, the numerator and denominator of each fraction on the right of the equals sign is smaller than the numerator and the denominator of each fraction on the left of the equals sign. Are the equivalent fractions on the right smaller? Use complete sentences to explain why or why not.

3. Whenever we find an equivalent fraction whose numerator and denominator are smaller numbers than the original fraction's numerator and denominator, the new fraction is "simpler."
   We can say
   $\frac{1}{2}$ is simpler than $\frac{2}{4}$.

   $\frac{1}{2}$ is a simplified form of $\frac{2}{4}$.

   Notice that sometimes a simplified form can be simplified further. Which of the fractions in Question 1 can be simplified further? What is the simpler form?

4. Math Path: Simplest Form

When a fraction cannot be simplified further, we say that the fraction is in **simplest form** (or in **simplest terms,** or that it is **completely simplified**). This process is sometimes called *reducing a fraction to simplest terms.*

Fill in the blanks to write each fraction in simplest form.

$\frac{6}{8} = \frac{\square}{4}$ $\quad\quad$ $\frac{12}{15} = \frac{\square}{5}$ $\quad\quad$ $\frac{18}{24} = \frac{\square}{4}$

In each case, what did you do to the original numerator to get the new numerator? Use complete sentences to explain.

Investigate Problem 1

5. Fill in the blanks so that each equality is true. Use your fraction strips to verify your answers.

$\frac{9}{12} = \frac{9 \div \square}{12 \div \square} = \frac{3}{4}$ $\quad \frac{10}{16} = \frac{10 \div \square}{16 \div \square} = \frac{\square}{\square}$ $\quad \frac{18}{24} = \frac{18 \div \square}{24 \div \square} = \frac{\square}{\square}$

Problem 2 Correct or Not?

Kasha says that she simplified the following fraction completely.

$$\frac{8}{24} = \frac{8 \div 2}{24 \div 2} = \frac{4}{12}$$

A. Did Kasha simplify the fraction completely? Use complete sentences to explain your answer.

B. Determine whether each fraction is written in simplest form. If not, completely simplify the fraction.

$\frac{18}{27} = \frac{18 \div 3}{27 \div 3} = \frac{6}{9}$ $\quad \frac{21}{27} = \frac{21 \div 3}{27 \div 3} = \frac{7}{9}$

$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$ $\quad \frac{16}{20} = \frac{16 \div 2}{20 \div 2} = \frac{8}{10}$

C. Determine which of the following fractions are in simplest terms.

$\frac{10}{12}$ $\quad \frac{9}{11}$ $\quad \frac{10}{25}$ $\quad \frac{11}{121}$ $\quad \frac{17}{29}$ $\quad \frac{44}{55}$

D. Recall that the factors of a number are those numbers that evenly divide the number with no remainder. How can you use the factors of a number to determine when you need to simplify a fraction? Use complete sentences to explain your reasoning.

Investigate Problem 2

1. Jim simplified $\frac{24}{28}$ by first dividing both the numerator and denominator by 2 and then dividing the numerator and the denominator of the result, $\frac{12}{14}$, by 2 to get the fraction $\frac{6}{7}$.

$$\frac{24}{28} = \frac{24 \div 2}{28 \div 2} = \frac{12}{14} = \frac{12 \div 2}{14 \div 2} = \frac{6}{7}$$

Sylvia said that this was wrong. She simplified $\frac{24}{28}$ by dividing both the numerator and denominator by 4 to get the fraction $\frac{6}{7}$.

$$\frac{24}{28} = \frac{24 \div 4}{28 \div 4} = \frac{6}{7}$$

Who is correct, Sylvia or Jim? Explain your answer using complete sentences.

2. Juan said that he found another method for simplifying $\frac{24}{28}$. He wrote the prime factorizations of the numerator and denominator and then divided out common prime factors.

$$\frac{24}{28} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 3}{\cancel{2} \times \cancel{2} \times 7} = \frac{6}{7}$$

Use Juan's method to simplify each fraction.

$\frac{12}{18} =$ _____ $=$ ____ $\qquad$ $\frac{15}{25} =$ _____ $=$ ____

$\frac{25}{30} =$ _____ $=$ ____ $\qquad$ $\frac{15}{24} =$ _____ $=$ ____

$\frac{24}{30} =$ _____ $=$ ____ $\qquad$ $\frac{42}{56} =$ _____ $=$ ____

Problem 3 Celeste's Fundamental Rule for Simplifying Fractions in One Step

Celeste looked at Juan's method and recognized that 2 × 2 is 4 and 4 is the greatest common factor (GCF) of 24 and 28. She proposed Celeste's Fundamental Rule for Simplifying Fractions in One Step. Here's the rule:

To write a fraction in simplest form in one step, divide the numerator and the denominator by their GCF.

Does Celeste's rule work? Use complete sentences to explain why or why not.

2

Investigate Problem 3

1. Use Celeste's rule to simplify each fraction.

$\frac{24}{30} = \frac{24 \div}{30 \div} = \frac{\quad}{\quad}$ GCF:

$\frac{56}{72} = \frac{56 \div}{72 \div} = \frac{\quad}{\quad}$ GCF:

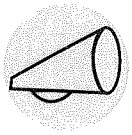
$\frac{72}{90} = \frac{72 \div}{90 \div} = \frac{\quad}{\quad}$ GCF:

$\frac{44}{55} = \frac{44 \div}{55 \div} = \frac{\quad}{\quad}$ GCF:

$\frac{36}{60} = \frac{36 \div}{60 \div} = \frac{\quad}{\quad}$ GCF:

$\frac{100}{150} = \frac{100 \div}{150 \div} = \frac{\quad}{\quad}$ GCF:

2. In this lesson, you have found several methods for simplifying fractions completely. Explain how you know when a fraction is completely simplified. Use complete sentences.



2

2.6 When Bigger Means Smaller

Comparing and Ordering Fractions

Objectives

In this lesson, you will:

- Compare and order fractions.

Key Terms

- least common denominator
- less than
- greater than

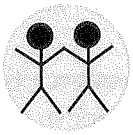
Problem 1

Your friends Tasha and Courtney are twins. They each have identical tropical fish tanks in their room, except that the number of fish in each tank is different. Tasha has 9 tetras and 5 of them are blue. Courtney has 9 tetras and 4 of them are blue.

A. Tasha and Courtney are arguing how someone knows when one fraction is greater than another.

Tasha says that the sizes of the numbers in the numerator and denominator are not important, but rather it is the quantity that the fraction represents that is important.

Courtney says that the size of the numerator tells you how big the fraction is. Who is correct? How can you explain to both of them how you know whether one fraction is greater than another? Use the fraction of blue fish in each of their fish tanks in your explanation.



B. Work with your partner to decide which fraction in each pair is greater. Circle the greater fraction.

$\frac{5}{8}$ and $\frac{7}{8}$ $\quad\frac{5}{12}$ and $\frac{7}{12}$ $\quad\frac{5}{8}$ and $\frac{1}{2}$ $\quad\frac{5}{6}$ and $\frac{11}{12}$

Write complete sentences to explain how you knew which of the fractions was greater in each pair.

Investigate Problem 1

1. We can use our knowledge of common fractions like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{4}$ to help us compare fractions. For example, to compare the fractions $\frac{5}{8}$ and $\frac{11}{12}$ you know that $\frac{5}{8}$ is close to $\frac{4}{8}$, which equals $\frac{1}{2}$, and $\frac{11}{12}$ is close to 1. Because $\frac{1}{2}$ is less than 1, you can estimate that $\frac{5}{8}$ is less than $\frac{11}{12}$.

Use this method to compare the following fraction pairs. Circle the greater fraction. Write complete sentences to explain how you knew which of the fractions was greater in each pair.

$\frac{5}{12}$ and $\frac{10}{11}$

$\frac{5}{6}$ and $\frac{7}{16}$

$\frac{7}{24}$ and $\frac{1}{6}$

$\frac{11}{15}$ and $\frac{17}{20}$

2. The method in Question 1 works pretty well for familiar fractions with relatively small denominators. Study the following fractions, and try to circle the greater fraction in each pair.

$\frac{5}{24}$ and $\frac{7}{30}$ $\quad$ $\frac{7}{20}$ and $\frac{11}{30}$ $\quad$ $\frac{11}{25}$ and $\frac{17}{32}$ $\quad$ $\frac{13}{18}$ and $\frac{17}{24}$

3. As you can see, using the method in Question 1 becomes more difficult when the fractions have larger denominators, so we need to find a method that works for all fractions. Begin by comparing fractions using your fraction strips. Write all of the fractions that can be represented using the strips that represent $\frac{1}{4}$ and $\frac{1}{8}$.

Then order the fractions from greatest to smallest. Why was this easy to do? Use complete sentences to explain.

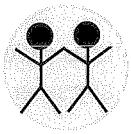
Investigate Problem 1

4. List all of the fractions with denominators of 9 and 12 in order without using your fraction strips. Then check your answer using the fraction strips.

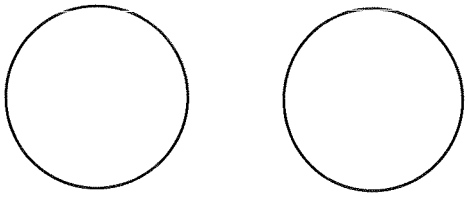
Why was this more difficult than ordering the fractions in Question 3? Use complete sentences to explain.

2

Problem 2 A Better Way



A. Use the circles below to compare the fractions $\frac{3}{4}$ and $\frac{5}{6}$.



Write complete sentences to explain how you compared the fractions.

B. Here is a much more efficient method for comparing fractions. Take out your fraction strips for $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$. Line up the strips so that the strip for $\frac{1}{12}$ is between the other two strips. How many twelfths are there in $\frac{3}{4}$? How many twelfths are there in $\frac{5}{6}$?

Investigate Problem 2

1. Knowing how many twelfths there are in each fraction makes it easy to compare $\frac{3}{4}$ and $\frac{5}{6}$. Recall from Lesson 1.3 that 12 is a common multiple of both 4 and 6. In fact, 12 is the least common multiple (LCM) of 4 and 6. Use complete sentences to explain why.

Investigate Problem 2

2. Math Path: Least Common Denominator

When we are working with fractions, you can use the LCM of the denominators to help you compare fractions. The LCM of two or more denominators is called the **least common denominator.** For example, 12 is the LCD of $\frac{3}{4}$ and $\frac{5}{6}$ because 12 is the LCM of 4 and 6. You can compare the fractions by writing fractions equivalent to $\frac{3}{4}$ and $\frac{5}{6}$ such that their denominators are 12.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

Now it is easy to see that $\frac{9}{12}$ is less than $\frac{10}{12}$ because we are comparing parts of the whole that is divided into the same number of parts, twelfths. So, you know that $\frac{3}{4}$ is less than $\frac{5}{6}$.

Find the LCD of fractions. Then use the LCD to rewrite each fraction. Circle the original fraction that is greater.

$\frac{7}{8}, \frac{5}{6}$ $\quad \frac{7}{8} = \frac{7 \times}{8 \times} = \_\_\_$ $\quad \frac{5}{6} = \frac{5 \times}{6 \times} = \_\_\_$

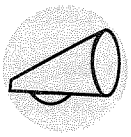
$\frac{7}{9}, \frac{5}{6}$ $\quad \frac{7}{9} = \frac{7 \times}{9 \times} = \_\_\_$ $\quad \frac{5}{6} = \frac{5 \times}{6 \times} = \_\_\_$

$\frac{1}{6}, \frac{2}{15}$ $\quad \frac{1}{6} = \frac{1 \times}{6 \times} = \_\_\_$ $\quad \frac{2}{15} = \frac{2 \times}{15 \times} = \_\_\_$

$\frac{5}{8}, \frac{2}{3}$ $\quad \frac{5}{8} = \frac{5 \times}{8 \times} = \_\_\_$ $\quad \frac{2}{3} = \frac{2 \times}{3 \times} = \_\_\_$

$\frac{7}{20}, \frac{11}{30}$ $\quad \frac{7}{20} = \frac{7 \times}{20 \times} = \_\_\_$ $\quad \frac{11}{30} = \frac{11 \times}{30 \times} = \_\_\_$

3. You can use the symbols **< (less than)** and **> (greater than)** to write a statement that compares two numbers. For example, you can say that $\frac{3}{4}$ is smaller than $\frac{5}{6}$ by writing $\frac{3}{4} < \frac{5}{6}$. Write a statement using < or > that compares each fraction pair in Question 2.



Looking Back at Chapter 2

Key Terms

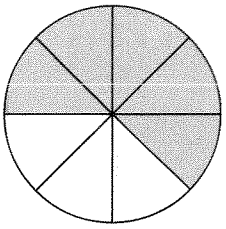
fraction ○ p. 40
numerator ○ p. 40
denominator ○ p. 40
reasonable solution ○ p. 48
equivalent fractions ○ p. 54
equation ○ p. 54
simplest form ○ p. 58
simplest terms ○ p. 58
completely simplified ○ p. 58
least common denominator ○ p. 66
less than ○ p. 66
greater than ○ p. 66

Summary

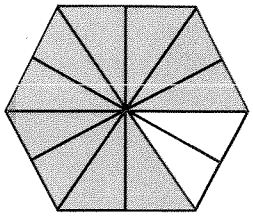
Writing Fractions (p. 40)

To write a fraction from a model, find the total number of parts. This number is the denominator. Then find the number of shaded parts. This number is the numerator.

Examples



$$\frac{\text{numerator}}{\text{denominator}} = \frac{5}{8}$$



$$\frac{\text{numerator}}{\text{denominator}} = \frac{10}{12}$$

Using a Model to Represent Fractions (p. 42)

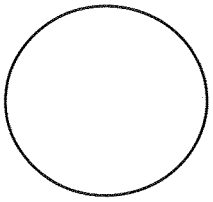
To use a circle to represent the fraction $\frac{a}{b}$, first divide the circle into b equal sections.

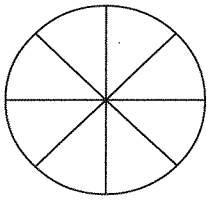
Then shade a sections of the circle.

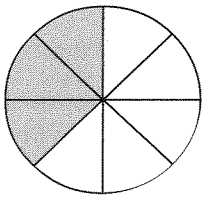
Example

To use a circle to represent $\frac{3}{8}$, first divide the circle into 8 equal sections.

Then shade 3 of the sections.





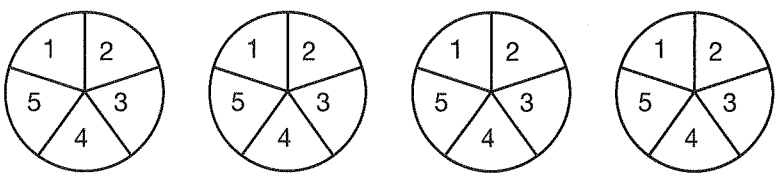


Dividing More Than One Whole into Parts (p. 45)

To divide more than one whole into n equal parts, first draw a diagram that represents all of the wholes. Then, divide each whole into n equal parts.

Example

To divide 4 pans of quiche for 5 family members, draw a diagram of the 4 pans of quiche. Then, divide each quiche into 5 equal parts. Each family member gets $\frac{4}{5}$ pan of quiche.



Determining Whether an Answer is Reasonable (p. 48)

To determine if an answer is a reasonable solution to a problem, think about the answer in the context of the problem.

Example

Dara is working in a computer lab. The lab has 50 computers and 36 keyboards. Dara decides that each computer gets $\frac{18}{25}$ keyboard.

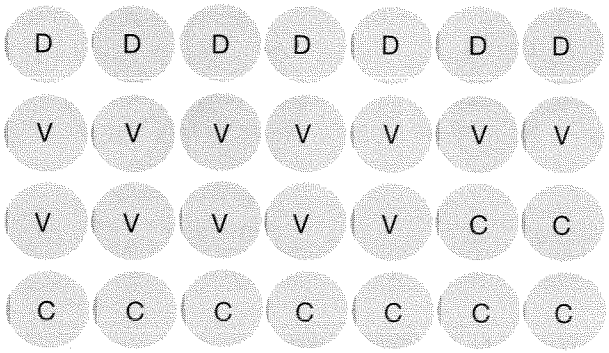
The solution is *not* reasonable because each computer must have 1 whole keyboard to function properly. It doesn't make sense to use a part of a keyboard with a computer.

Dividing Groups into Fractional Parts (p. 49)

To find the fraction of items that you have out of a group of items, first find the number of items you have out of the group. This number is the numerator. Then find the total number of items in the group. This number is the denominator.

Example

A flower garden has 7 daisies, 12 violets, and 9 carnations. First draw a diagram of the situation.



To write the fraction of the total flowers that are carnations, write the number of carnations as the numerator and the total number of flowers as the denominator. Carnations make up $\frac{9}{28}$ of the garden. To write the number of violets as a fraction of the total number of flowers, write the number of violets as the numerator and the total number of flowers as the denominator: $\frac{12}{28}$.

To write the number of daisies as a fraction of the total number of flowers, write the number of daisies as the numerator and the total number of flowers as the denominator: $\frac{7}{28}$.

Writing Equivalent Fractions (p. 54)

To write a fraction that is equivalent to a given fraction, multiply the numerator and denominator by the same number (that is, multiply the given fraction by the multiplicative identity, 1).

Examples

$\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}$ $\quad \frac{4}{9} = \frac{4 \times 7}{9 \times 7} = \frac{28}{63}$ $\quad \frac{16}{21} = \frac{16 \times 5}{21 \times 5} = \frac{80}{105}$

Writing Fractions in Simplest Form (p. 58)

To completely simplify a fraction, divide the numerator and the denominator by their GCF.

Examples

$\frac{54}{171} = \frac{54 \div 9}{171 \div 9} = \frac{6}{19}$

$\frac{112}{132} = \frac{112 \div 4}{132 \div 4} = \frac{28}{33}$

$\frac{96}{384} = \frac{96 \div 96}{384 \div 96} = \frac{1}{4}$

$\frac{350}{1050} = \frac{350 \div 350}{1050 \div 350} = \frac{1}{3}$

Comparing Fractions (p. 66)

To compare two fractions, use the least common denominator (LCD) to rewrite the fractions so that they have the same denominator. Then use <, >, or = to write a statement that compares the fractions.

Examples

$\frac{5}{6}$ and $\frac{4}{7}$

LCD: 42

$\frac{5 \times 7}{6 \times 7} = \frac{35}{42}$

$\frac{4 \times 6}{7 \times 6} = \frac{24}{42}$

$\frac{5}{6} > \frac{4}{7}$

$\frac{7}{8}$ and $\frac{11}{12}$

LCD: 24

$\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

$\frac{11 \times 2}{12 \times 2} = \frac{22}{24}$

$\frac{7}{8} < \frac{11}{12}$

$\frac{1}{9}$ and $\frac{2}{5}$

LCD: 45

$\frac{1 \times 5}{9 \times 5} = \frac{5}{45}$

$\frac{2 \times 9}{5 \times 9} = \frac{18}{45}$

$\frac{1}{9} < \frac{2}{5}$

Ordering Fractions (p. 66)

To order fractions, use the least common denominator (LCD) to rewrite the fractions so that they have the same denominator. Then write the fractions in order from least to greatest.

Example

The LCD of $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}$, and $\frac{5}{12}$ is 24.

$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$ $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$ $\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$ $\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$

The fractions in order from least to greatest are $\frac{5}{12}, \frac{2}{3}, \frac{5}{6}$, and $\frac{7}{8}$.

Looking Ahead to Chapter 3

Focus In Chapter 3, you will work with fractions and mixed numbers. You will add, subtract, multiply, and divide fractions and mixed numbers. You will write fractions as mixed numbers and mixed numbers as fractions. You will also learn to work with measurements in customary units and convert measurements.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 3.

Perform the indicated operation.

1. $138 + 587$
2. $964 - 26$
3. 14×52
4. 39×23
5. $1020 \div 68$
6. $2108 \div 31$

Write a fraction that is equivalent to the given fraction.

7. $\frac{5}{9}$
8. $\frac{17}{21}$
9. $\frac{25}{56}$

Write the fraction in simplest form.

10. $\frac{18}{27}$
11. $\frac{24}{64}$
12. $\frac{36}{108}$

Read the problem scenario below.

You ride a bicycle with tires that are 26 inches in diameter. You pedal at a rate of 50 revolutions per minute. You can travel about 340 feet in one minute.

13. Which units are used to measure distance?
14. Which units are used to measure time?

Key Terms

like fractions ○ p. 75
unlike fractions ○ p. 75
least common denominator ○ p. 78
improper fractions ○ p. 82
mixed number ○ p. 82
U.S. customary system ○ p. 85
metric system ○ p. 85
remainder ○ p. 89
multiplicative identity ○ p. 91
multiplicative inverse ○ p. 91
reciprocal ○ p. 91
customary units of measure ○ p. 101
length ○ p. 101
inch ○ p. 101
foot ○ p. 101
yard ○ p. 101
mile ○ p. 101
capacity ○ p. 101
fluid ounce ○ p. 101
cup ○ p. 101
pint ○ p. 101
quart ○ p. 101
gallon ○ p. 101
weight ○ p. 101
ounce ○ p. 101
pound ○ p. 101
ton ○ p. 101