13.1

Running a Tree Farm

Relations and Functions

13

Students should be able to answer these questions after Lesson 13.1:

- What is a relation? What is a function?
- What is function notation?
- What are independent and dependent variables?
- What are the domain and range of a function?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Suppose that you are preparing for a big party, for which you plan to bake many cakes. For each cake, you will need two cups of flour.

Step 1 Write an equation to represent the number of cups of flour to be used in terms of the number of cakes.

Let f represent the number of cups of flour to be used and let a represent the number of cakes to be made. So, $f(a) = 2a$.

Step 2 Make an input-output table that shows the relationship between the input value, which is the number of cakes, and the output value, which is the number of cups of flour. Then choose values, such as 1, 2, 3, 4, and 5, for the number of cakes and evaluate the relationship to find the numbers of cups of flour needed.

Number of cakes, a	Function, $f = 2a$	Number of cups of flour, f
1	$f = 2(1)$	2
2	$f = 2(2)$	4
3	$f = 2(3)$	6
4	$f = 2(4)$	8
5	$f = 2(5)$	10

2. You plan to make small sandwiches to serve to your guests. You estimate that each guest will eat 4 small sandwiches. Write an equation to represent the number of sandwiches to be eaten in terms of the number of guests. Then make an input-output table to represent the situation.

13.2

Scaling a Cliff

Linear Functions

13

Students should be able to answer these questions after Lesson 13.2:

- How can you express a rule algebraically?
- How can you create a graph from a table?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Suppose that you earn $5 each week for doing chores around the house. Make a table for the total amount of money that you have earned over a certain period. Then create a graph to represent the values in the table.

Step 1 Make a table with one column to represent the time in weeks and one column to represent the amount of money earned in dollars. Complete the table by entering values, such as 1, 2, and 3, in the time column and then calculating the amount of money earned. The time is the independent variable and the amount of money is the dependent variable.

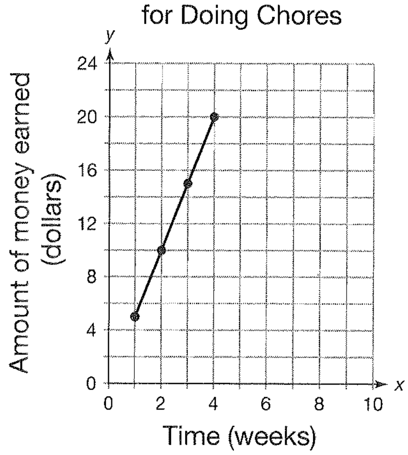
Time	Amount of Money Earned
weeks	**dollars**
1	5
2	10
3	15
4	20

Step 2 Write each row in the table as an ordered pair.

(1, 5), (2, 10), (3, 15), (4, 20)

Step 3 Make a graph. Be sure to label the axes.

Amount of Money Earned
for Doing Chores



2. Suppose that you earn $7 each week for delivering newspapers. Make a table for the total amount of money that you have earned over a certain period. Then create a graph to represent the values in the table.

13.3 Biking Along

Slope and Rates of Change

Students should be able to answer these questions after Lesson 13.3:

- How can you find the slope of a line as a ratio?
- How can you find the slope of a line as a rate of change?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Suppose that you laid a plank over a staircase. What would the slope of the plank be? How could you find it?

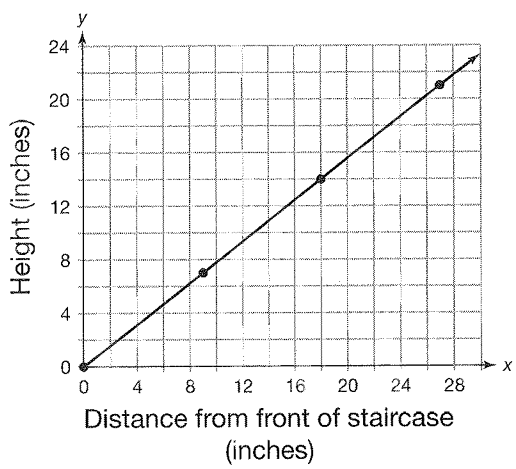
Step 1 Measure the vertical height from the floor to the top of the lowest step, or from the top of one step to the top of the next one. In many staircases, this distance is 7 inches.

Step 2 Measure the depth (from front to back) of one step. In many staircases, this distance is 9 inches.

Step 3 Make a table of the horizontal distance from the first step in the staircase and the vertical height of the step from the bottom of the staircase.

Horizontal distance from front of staircase	Vertical height from bottom of staircase
inches	inches
0	0
9	7
18	14
27	21

Step 4 Make a graph.



Step 5 Draw a straight line through the points. The vertical change between each point and the next point is 7 inches. The horizontal change between each point and the next point is 9 inches. So, the slope is $\frac{7}{9}$.

2. Find the slope of a ramp that has a horizontal change of 48 inches and a vertical change of 4 inches.

13.4

Let's Have a Pool Party!

Finding Slope and y-Intercepts

13

Students should be able to answer these questions after Lesson 13.4:

- How can you find x- and y-intercepts of a line?
- How can you find the slope of a line?

Directions

Read Question 1 and its solution. Then complete Questions 2 through 4.

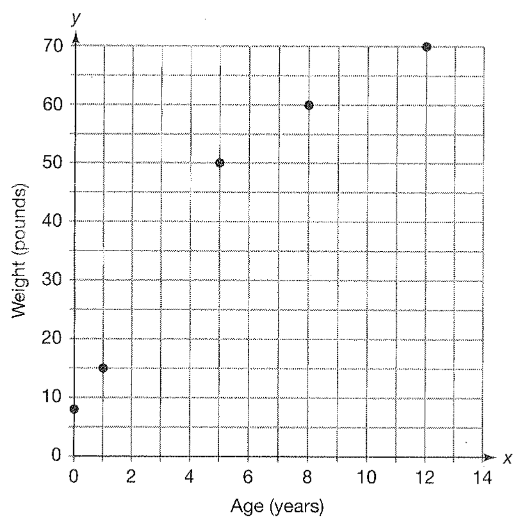
1. The table at the right shows the age of a child and the weight of the child at that age. Graph the data in this table.

Age	Weight
years	pounds
0	8
1	15
5	50
8	60
12	70

Step 1 Write the data in the chart as a set of 5 ordered pairs.

(0, 8), (1, 15), (5, 50), (8, 60), (12, 70)

Step 2 Plot the points in the coordinate plane below.



2. Is the graph linear? If so, what is the slope?

3. Is there a y-intercept? If so, what is it? What in the child's life does the y-intercept represent?

4. Is there an x-intercept? If so, what is it? What in the child's life does the x-intercept represent?

13.5 What's for Lunch?

Using Slope and Intercepts to Graph Lines

13

Students should be able to answer these questions after Lesson 13.5:

- How can you use slope and intercepts to draw a graph?
- How can you write an equation in slope-intercept form?
- How can you plot the graph of the equation?

Directions

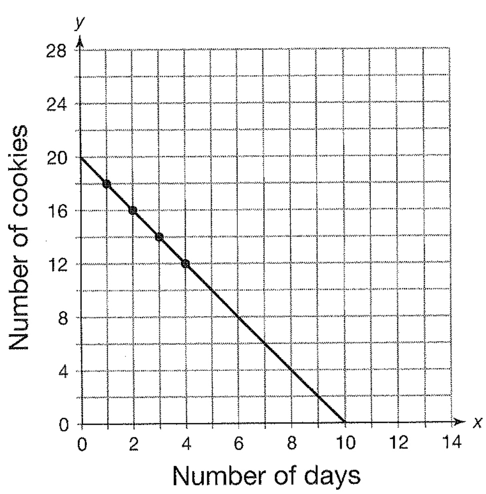
Read Question 1 and its solution. Then complete Question 2.

1. Suppose that there are 20 cookies in a cookie jar and that you and your friend each take one cookie a day. Let x represent the number of days since there were 20 cookies in the jar. Let y represent the number of cookies in the jar. Then an equation to represent the situation is $y = 20 - 2x$. Graph the equation.

Step 1 The equation is written in the form $y = mx + b$, which is slope-intercept form. In slope-intercept form, m represents the slope and b represents the y-intercept. So, the slope of the line $y = 20 - 2x$ is -2 and the y-intercept is 20.

Step 2 Plot the y-intercept. The slope, -2, tells you that you can move 2 units down and 1 unit to the right to find the next value on the graph. Continue this process until you have several points on the graph.

Step 3 Draw a straight line through the points.



2. Graph the linear equation $y = \frac{1}{2}x + 3$. What is the slope of the line? What is the y-intercept?

13.6 Healthy Relationships

Finding Lines of Best Fit

13

Students should be able to answer these questions after Lesson 13.6:

- What is a scatter plot?
- How can you find a line of best fit for a set of data?

Directions

Read Question 1 and its solution. Then complete Questions 2 through 4.

1. The table at the right shows the growth of a tree over a period of 10 years. Can you determine an approximate linear relationship between tree age and tree height?

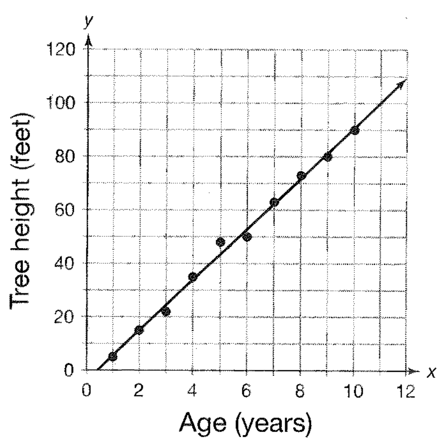
Age	Tree Height
years	**feet**
1	5
2	15
3	22
4	35
5	48
6	50
7	63
8	73
9	80
10	90

Step 1 Arrange the data into ordered pairs. Let the x-coordinate represent the age and let the y-coordinate represent the height.

(1, 5), (2, 15), (3, 22), (4, 35), (5, 48), (6, 50), (7, 63), (8, 73), (9, 80), (10, 90)

Step 2 Plot the ordered pairs in a coordinate plane. Label the horizontal and vertical axes.

Step 3 Draw a line of best fit, which is a line that is very close to most of the points.



2. What is the slope of the line of best fit?

3. What is the y-intercept of the line of best fit?

4. Write the equation of the line of best fit.

14.1 Is It a Bird or a Plane?

Rational Numbers

Students should be able to answer these questions after Lesson 14.1:

- What are rational numbers?
- How can you perform operations with these sets of numbers?

Directions

Read Question 1 and its solution. Then complete Questions 2 and 3.

14

1. Perform the indicated operations.

Multiply $-4\frac{2}{3}$ and $-6\frac{7}{8}$. Then divide the answer by $2\frac{1}{10}$.

Step 1 Write the multiplication problem. Then rewrite the mixed numbers as improper fractions.

$$-4\frac{2}{3} \times \left(-6\frac{7}{8}\right) = -\frac{14}{3} \times \left(-\frac{55}{8}\right)$$

Step 2 Perform the multiplication. Rewrite and simplify your answer. Because both factors are negative, the product is positive.

$$-\frac{14}{3} \times \left(-\frac{55}{8}\right) = \frac{770}{24} = 32\frac{2}{24} = 32\frac{1}{12}$$

Step 3 Now divide the product by $2\frac{1}{10}$. First rewrite the mixed numbers as improper fractions.

$$32\frac{1}{12} \div 2\frac{1}{10} = \frac{385}{12} \div \frac{21}{10}$$

Step 4 Perform the division.

$$\frac{385}{12} \div \frac{21}{10} = \frac{385}{12} \times \frac{10}{21} = \frac{3850}{252}$$

Step 5 Write your answer as a simplified mixed number.

$$\frac{3850}{252} = 15\frac{70}{252} = 15\frac{5}{18}$$

> **Rational Numbers**
>
> Rational numbers are numbers that can be written in the form $\frac{a}{b}$, where a and b are both integers and b is not equal to 0. You can add, subtract, multiply, and divide rational numbers in much the same way as integers.

2. Find the product to the nearest ten-thousandth.

$4.65 \times (-2.35)$

3. Find the quotient to the nearest thousandth.

$$\frac{-3.3}{4.1}$$

14.2 How Many Times?

Powers of Rational Numbers

Students should be able to answer these questions after Lesson 14.2:

- How can you find powers of rational numbers?
- How can you multiply and divide powers of rational numbers?

Directions

14

Read Question 1 and its solution. Then complete Questions 2 and 3.

1. Perform the indicated operations.

Multiply $\left(-\frac{2}{3}\right)^{-4}$ and $\left(-\frac{2}{3}\right)^{6}$. Then divide the answer by $\left(-\frac{2}{3}\right)^{3}$.

Step 1 Write the multiplication problem.

$$\left(-\frac{2}{3}\right)^{-4}\left(-\frac{2}{3}\right)^{6} = \_\_\_\_\_$$

Step 2 Solve using the formula for multiplying rational numbers.

$$a^b a^c = a^{b+c}$$

$$\left(-\frac{2}{3}\right)^{-4}\left(-\frac{2}{3}\right)^{6} = \left(-\frac{2}{3}\right)^{-4+6} = \left(-\frac{2}{3}\right)^{2}$$

Step 3 Write the division problem. Solve using the formula for dividing rational numbers.

$$\frac{a^b}{a^c} = a^{b-c}, a \neq 0.$$

$$\frac{\left(-\frac{2}{3}\right)^{2}}{\left(-\frac{2}{3}\right)^{3}} = \left(-\frac{2}{3}\right)^{-1}$$

Step 4 Find the value of the power.

$$\left(-\frac{2}{3}\right)^{-1} = -\frac{3}{2} = -1\frac{1}{2}$$

2. Find the product.

$$\left(-\frac{5}{6}\right)^{4}\left(-\frac{5}{6}\right)^{-5}$$

3. Find the quotient.

$$\frac{(-5)^6}{(-5)^{-2}}$$

Multiplying and Dividing Rational Numbers

The formula for multiplying rational numbers is

$$a^b a^c = a^{b+c}.$$

The formula for dividing rational numbers is

$$\frac{a^b}{a^c} = a^{b-c}, a \neq 0.$$

14.3 Sew What?

Irrational Numbers

Students should be able to answer these questions after Lesson 14.3:

- What is a repeating decimal?
- What is bar notation?
- What is an irrational number?

Directions

14

Read Question 1 and its solution. Then complete Questions 2 and 3.

1. Write the fraction that represents the repeating decimal 0.3030... .

Step 1 Write an equation by setting the decimal equal to a variable that will represent the fraction.

$w = 0.3030\ldots$

Step 2 Next, multiply both sides of the equation by a power of 10. The decimal starts repeating after two decimal places, so the exponent on the power of 10 is 2.

$100w = 30.30\ldots$

Step 3 Subtract the first equation from the second.

$$\begin{array}{r} 100w = 30.3030\ldots \\ -w = 0.3030\ldots \\ \hline 99w = 30 \end{array}$$

Step 4 Solve the equation by dividing each side by 99. Then simplify.

$$w = \frac{30}{99} = \frac{10}{33}$$

> **Repeating Decimals and Irrational Numbers**
>
> A repeating decimal is a decimal with digits that repeat in sets of one or more. An irrational number is a number that cannot be written in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0.

2. Write the fraction that represents the repeating decimal 0.77... .

3. Write the fraction that represents the repeating decimal $0.\overline{45}$.

14.4 Worth 1000 Words

Real Numbers and Their Properties

Students should be able to answer these questions after Lesson 14.4:

- How can you classify the different kinds of numbers in the real number system?
- What are the properties of real numbers?

Directions

Read Question 1 and its solution. Then answer Questions 2 and 3.

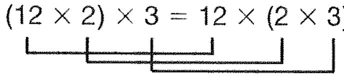
1. Identify the property of real numbers that is represented by the statement below. Then find the value represented by the statement.

$(12 \times 2) \times 3 = 12 \times (2 \times 3)$

Step 1 Look for clues about the operations that are being performed. The multiplication signs tell you that the property represented is related to multiplication.

Step 2 Look for more clues. There are three terms on each side of the equation.

$(12 \times 2) \times 3 = 12 \times (2 \times 3)$



Step 3 Notice that the grouping of the terms is different on each side of the equation.

Step 4 The Associative Property of Multiplication states that the way in which the terms of a product are grouped does not change the product. So, the statement $(12 \times 2) \times 3 = 12 \times (2 \times 3)$ represents the Associative Property of Multiplication.

Step 5 To find the value represented by the statement, choose one side of the equation and evaluate the expression.

$(12 \times 2) \times 3 = 24 \times 3 = 72$

or $12 \times (2 \times 3) = 12 \times 6 = 72$

2. Identify the property of real numbers that is represented by the statement. Then find the value of x.

$6\frac{5}{8} + (-6.625) = x$

3. Identify the property of real numbers that is represented by the statement. Then find the value of x.

If $a = 3 \cdot 6$ and $3 \cdot 6 = \frac{54}{3}$, then $a = \frac{54}{x}$.

14.5 The House That Math Built

The Distributive Property

Students should be able to answer these questions after Lesson 14.5:

- What is the distributive property?
- How is the distributive property used?

Directions

Read Question 1 and its solution. Then complete Questions 2 and 3.

14

1. Use the distributive property to evaluate the expression in two different ways.

   $9(14 - 9) =$ ______

   Step 1 Start by rewriting the expression. Then evaluate the expression inside the parentheses.

   $9(14 - 9) = 9(5)$

   Step 2 Multiply.

   $9(5) = 45$

   Step 3 Now, start by using the distributive property.

   $9(14 - 9) = 9(14) - 9(9)$

   Step 4 Multiply.

   $9(14) - 9(9) = (126) - (81)$

   Step 5 Subtract.

   $126 - 81 = 45$

> ### The Distributive Property
> The distributive property states that certain operations can be distributed over others. *The Distributive Property of Multiplication Over Addition* says that you can multiply a number and a sum by first multiplying the number by each addend of the sum and then adding the products. Multiplication also distributes over subtraction in the same way. Division distributes over addition and subtraction.

2. Use the distributive property to evaluate the expression in two different ways.

   $\frac{20 + 12}{4}$

3. Use the distributive property to evaluate the expression in two different ways.

   $-4(8x + x)$

15.1

Worms and Ants

Graphing in Four Quadrants

Students should be able to answer this question after Lesson 15.1:

- How can you graph a linear function if you know the coordinates of two points on it?

Directions

Read and complete Question 1. Then complete Questions 2 and 3.

1. To convert from a temperature expressed in degrees Celsius, C, to a temperature expressed in degrees Fahrenheit, F, use the equation $F = \frac{9}{5}C + 32$. Both Fahrenheit and Celsius temperatures can be negative, so the graph will lie in more than Quadrant I.

   Let y represent degrees Fahrenheit and let x represent degrees Celsius. On a sheet of graph paper, graph the equation $y = \frac{9}{5}x + 32$.

15

Step 1 Make a table of values.

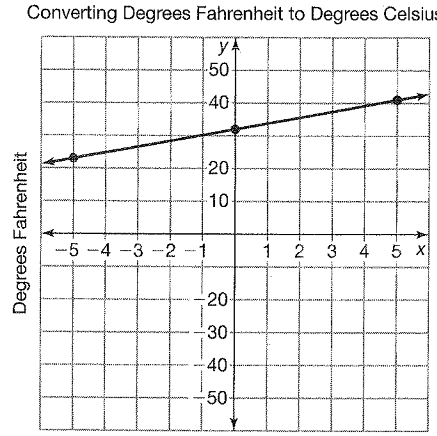
x	y
−5	23
0	32
5	41

Step 2 Determine an interval for each axis.

Step 3 Plot the points from the table of values.

Step 4 Draw a straight line through the points.

Converting Degrees Fahrenheit to Degrees Celsius



Degrees Celsius

2. What is the y-intercept? What does the y-intercept represent?

3. What is the slope?

15.2 Maps and Models

Scale Drawings and Scale Models

Students should be able to answer these questions after Lesson 15.2:

- What are scale drawings and scale models?
- How can you enlarge a representation or make it smaller?
- How can you use a scale drawing or a scale model?

Directions

Read and complete Question 1.

1. Suppose that for a science project you are considering making a scale model of the solar system. You are given the data shown in the table.

   Do you think a scale model of the solar system can be built?

Solar System Body	Distance from Sun (km)	Diameter (km)
Sun		1,390,000
Mercury	58,000,000	4900
Venus	110,000,000	12,000
Earth	150,000,000	13,000
Mars	230,000,000	6800
Jupiter	780,000,000	140,000
Saturn	1,400,000,000	120,000
Uranus	2,900,000,000	51,000
Neptune	4,500,000,000	50,000

Sources: National Air and Space Museum website, http://www.nasm.si.edu/ceps/etp/ and NASA's Jet Propulsion Lab's web site: http://www.jpl.nasa.gov/solar_system/sun/sun_index.html

(Note: The diameter of a sphere is the distance from one point on its surface, through the center, to another point on its surface.)

Step 1 Decide on a scale factor.

Step 2 To determine whether your scale will work, test the most extreme data in the table.

Step 3 What conclusion can you draw?

15.3

Designer Mathematics

Sliding and Spinning

Students should be able to answer these questions after Lesson 15.3:

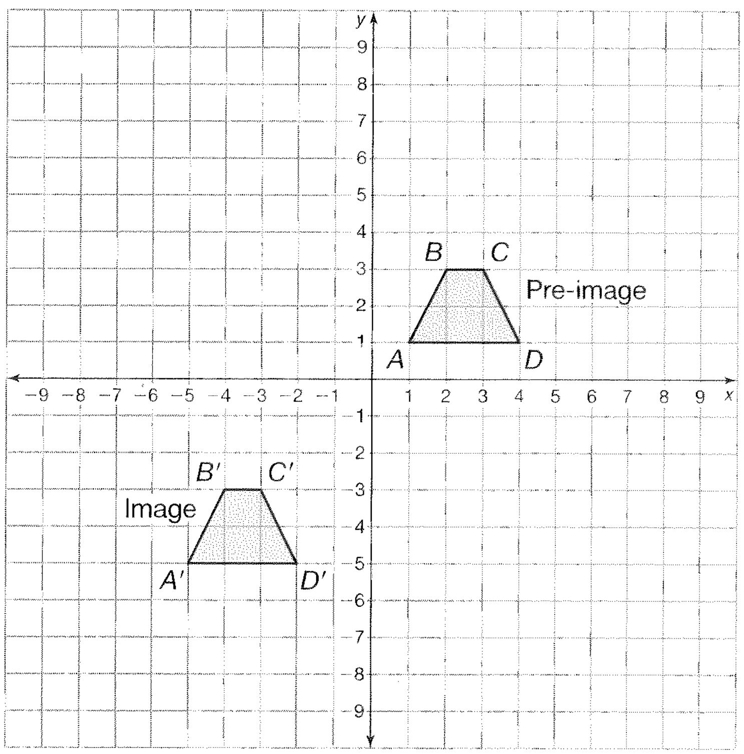
- What is a translation? How do you translate a figure in the coordinate plane?
- What is a rotation? How do you rotate a figure in the coordinate plane?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Draw a trapezoid with vertices $A(1, 1)$, $B(2, 3)$, $C(3, 3)$, and $D(4, 1)$. Translate the trapezoid -6 units vertically and -6 units horizontally. What are the vertices of the image?

Step 1 Draw trapezoid *ABCD*. This figure is the pre-image.



Step 2 To represent a translation of -6 units vertically and -6 units horizontally, move the trapezoid 6 units down and 6 units to the left.

Step 3 Write the vertices of the image.

$A'(-5, -5)$, $B'(-4, -3)$, $C'(-3, -3)$, and $D'(-2, -5)$

2. Draw a trapezoid with vertices $D(1, 3)$, $E(2, 1)$, $F(4, 1)$, and $G(5, 3)$. Rotate the trapezoid $+90°$ about the origin. What are the vertices of the image?

15.4

Secret Codes

Flipping, Stretching, and Shrinking

Students should be able to answer these questions after Lesson 15.4:

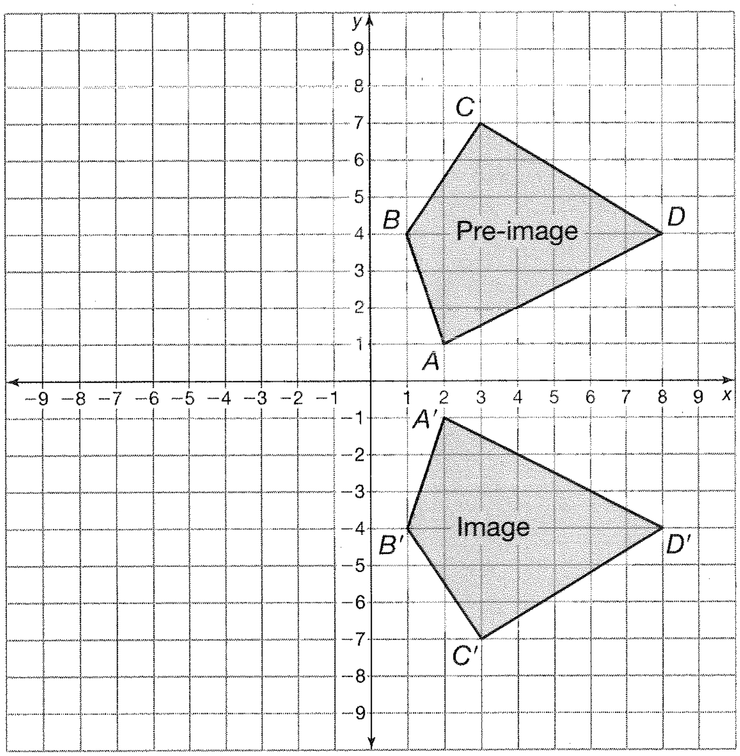
- What is a reflection? How do you create a reflected image from a pre-image?
- What is a dilation? How do you dilate an image?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Draw a quadrilateral with vertices $A(2, 1)$, $B(1, 4)$, $C(3, 7)$, and $D(8, 4)$. Reflect the quadrilateral in the x-axis. What are the vertices of the image?

Step 1 Draw quadrilateral $ABCD$. This is the pre-image.



Step 2 To graph a reflection in a coordinate plane, flip the pre-image over a line of reflection.

Step 3 Write the vertices of the image.

$A'(2, -1)$, $B'(1, -4)$, $C'(3, -7)$, and $D'(8, -4)$

2. Draw a quadrilateral with vertices $D(1, 1)$, $E(0, 2)$, $F(1, 3)$, and $G(3, 1)$. Dilate the quadrilateral by a scale factor of 2 using the origin as the center of dilation. What are the vertices of the image?

A Stitch in Time

Multiple Transformations

Students should be able to answer these questions after Lesson 15.5:

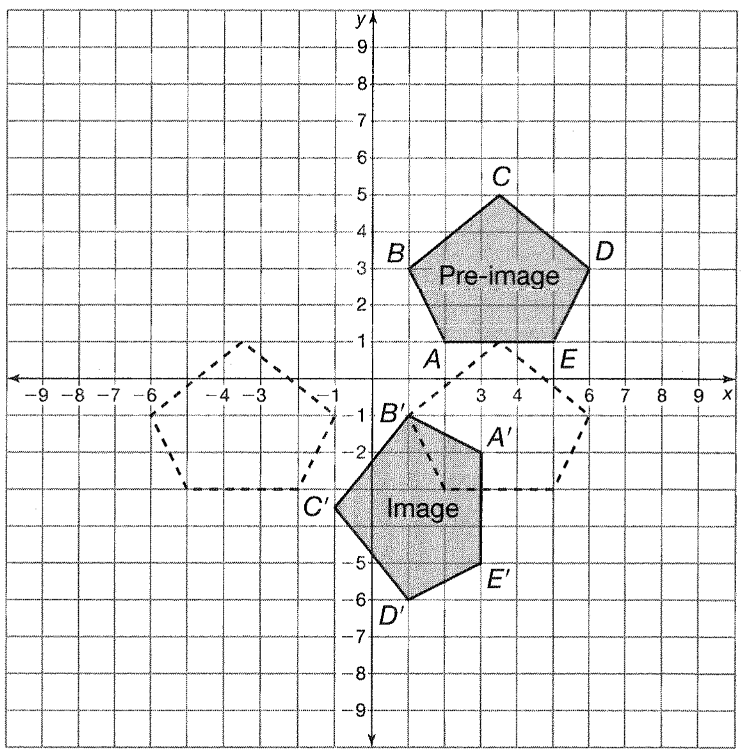
- How do you perform multiple transformations?
- When is the sequence of transformations important and when is it not?

Directions

Read Question 1 and its solution. Then complete Question 2.

1. Draw a pentagon with vertices $A(2, 1)$, $B(1, 3)$, $C(3.5, 5)$, $D(6, 3)$, and $E(5, 1)$. Perform a vertical translation of −4 units. Then perform a reflection over the y-axis. Finally, perform a rotation of −90° about the origin.

Step 1 Draw figure $ABCDE$. Then translate the figure −4 units vertically.



Step 2 Reflect figure $ABCDE$ over the y-axis.

Step 3 Rotate figure $ABCDE$ −90° about the origin.

Step 4 Write the vertices of the final figure.

$A'(3, -2)$, $B'(1, -1)$, $C'(-1, -3.5)$, $D'(1, -6)$ and $E'(3, -5)$

2. Draw a square with vertices $J(2, 2)$, $K(2, 4)$, $L(4, 4)$, and $M(4, 2)$. Dilate the square by a scale factor of $\frac{1}{2}$ using the origin as the center of dilation. Then perform a reflection over the x-axis. Finally, perform a rotation +90° about the origin. What are the vertices of the image?