

# Table of Symbols

| Symbol   |  | Page | Symbol                     |   | Page        |
|--|--|------|----------------------------|---|-------------|
| $\dots$  | and so on                                  | 3    | $f(x) \rightarrow +\infty$ | $f$ of $x$ approaches positive infinity                         | 331         |
| $\approx$                                      | is approximately equal to                  | 3    | $\sqrt[n]{a}$              | $n$ th root of $a$  | 401         |
| $<$  | is less than                               | 4    | $f^{-1}$                   | inverse of function $f$   | 423         |
| $>$  | is greater than                            | 4    | $\bar{x}$                  | $x$ -bar; the mean of a data set                                | 445         |
| $\leq$   | is less than or equal to                   | 4    | $\sigma$                   | sigma; the standard deviation of a data set                     | 446         |
| $\geq$   | is greater than or equal to                | 4    | $e$                        | irrational number $\approx 2.718$                               | 480         |
| $\cdot$  | multiplication, times                      | 5    | $\log_b y$                 | base- $b$ logarithm of $y$                                      | 486         |
| $-a$   | opposite of $a$                            | 5    | $\log x$                   | base-10 logarithm of $x$  | 487         |
| $\frac{1}{a}$                                  | reciprocal of $a$ , $a \neq 0$             | 5    | $\ln x$                    | base- $e$ logarithm of $x$                                      | 487         |
| $\neq$   | not equal to                               | 5    | $\Sigma$                   | summation   | 653         |
| $\pi$  | pi; irrational number $\approx 3.14$       | 28   | $S_n$                      | sum of the first $n$ terms of an arithmetic or geometric series | 661,<br>668 |
| $ x $  | absolute value of $x$                      | 50   | $n!$                       | $n$ factorial; number of permutations of $n$ objects            | 703         |
| $(x, y)$                                       | ordered pair                               | 67   | ${}_n P_r$                 | number of permutations of $r$ objects from $n$ distinct objects | 703         |
| $f(x)$   | $f$ of $x$ , or the value of $f$ at $x$    | 69   | ${}_n C_r$                 | number of combinations of $r$ objects from $n$ distinct objects | 708         |
| $m$  | slope                                      | 75   | $P(A)$                     | probability of event $A$  | 716         |
| $x_1$  | $x$ sub 1                                  | 75   | $P(B A)$                   | probability of event $B$ given that event $A$ has occurred      | 732         |
| $\lceil x \rceil$                              | greatest integer less than or equal to $x$ | 115  | $\theta$                   | theta; name of an angle, or measure of an angle                 | 769         |
| $(x, y, z)$                                    | ordered triple                             | 170  | $\sin$                     | sine  | 769         |
| $f(x, y)$                                      | function of two variables                  | 171  | $\cos$                     | cosine  | 769         |
| $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | matrix                                     | 199  | $\tan$                     | tangent   | 769         |
| $ A $  | determinant of matrix $A$                  | 214  | $\csc$                     | cosecant  | 769         |
| $A^{-1}$                                       | inverse of matrix $A$                      | 223  | $\sec$                     | secant  | 769         |
| $\sqrt{a}$                                     | the nonnegative square root of $a$         | 264  | $\cot$                     | cotangent   | 769         |
| $i$  | imaginary unit equal to $\sqrt{-1}$        | 272  | $\sin^{-1}$                | inverse sine  | 792         |
| $ z $  | absolute value of complex number $z$       | 275  | $\cos^{-1}$                | inverse cosine  | 792         |
| $+\infty$                                      | positive infinity                          | 331  | $\tan^{-1}$                | inverse tangent   | 792         |
| $-\infty$                                      | negative infinity                          | 331  |                            |   |             |
| $x \rightarrow +\infty$                        | $x$ approaches positive infinity           | 331  |                            |   |             |

# Table of Measures

| Time                              |                             |
|-----------------------------------|-----------------------------|
| 60 seconds (sec) = 1 minute (min) |                             |
| 60 minutes = 1 hour (h)           |                             |
| 24 hours = 1 day                  |                             |
| 7 days = 1 week                   |                             |
| 4 weeks (approx.) = 1 month       |                             |
|                                   | 365 days                    |
|                                   | 52 weeks (approx.) = 1 year |
|                                   | 12 months                   |
|                                   | 10 years = 1 decade         |
|                                   | 100 years = 1 century       |

| Metric  | United States Customary  |
|---|--|
| <p><b>Length</b></p> <p>10 millimeters (mm) = 1 centimeter (cm)</p> <p><math>\frac{100 \text{ cm}}{1000 \text{ mm}} = 1 \text{ meter (m)}</math></p> <p>1000 m = 1 kilometer (km)</p>                               | <p><b>Length</b></p> <p>12 inches (in.) = 1 foot (ft)</p> <p><math>\frac{36 \text{ in.}}{3 \text{ ft}} = 1 \text{ yard (yd)}</math></p> <p><math>\frac{5280 \text{ ft}}{1760 \text{ yd}} = 1 \text{ mile (mi)}</math></p>                      |
| <p><b>Area</b></p> <p>100 square millimeters (mm<sup>2</sup>) = 1 square centimeter (cm<sup>2</sup>)</p> <p>10,000 cm<sup>2</sup> = 1 square meter (m<sup>2</sup>)</p> <p>10,000 m<sup>2</sup> = 1 hectare (ha)</p> | <p><b>Area</b></p> <p>144 square inches (in.<sup>2</sup>) = 1 square foot (ft<sup>2</sup>)</p> <p>9 ft<sup>2</sup> = 1 square yard (yd<sup>2</sup>)</p> <p><math>\frac{43,560 \text{ ft}^2}{4840 \text{ yd}^2} = 1 \text{ acre (A)}</math></p> |
| <p><b>Volume</b></p> <p>1000 cubic millimeters (mm<sup>3</sup>) = 1 cubic centimeter (cm<sup>3</sup>)</p> <p>1,000,000 cm<sup>3</sup> = 1 cubic meter (m<sup>3</sup>)</p>   | <p><b>Volume</b></p> <p>1728 cubic inches (in.<sup>3</sup>) = 1 cubic foot (ft<sup>3</sup>)</p> <p>27 ft<sup>3</sup> = 1 cubic yard (yd<sup>3</sup>)</p>   |
| <p><b>Liquid Capacity</b></p> <p><math>\frac{1000 \text{ milliliters (mL)}}{10 \text{ deciliters (dL)}} = 1 \text{ liter (L)}</math></p> <p>1000 L = 1 kiloliter (kL)</p>   | <p><b>Liquid Capacity</b></p> <p>8 fluid ounces (fl oz) = 1 cup (c)</p> <p>2 c = 1 pint (pt)</p> <p>2 pt = 1 quart (qt)</p> <p>4 qt = 1 gallon (gal)</p>   |
| <p><b>Mass</b></p> <p>1000 milligrams (mg) = 1 gram (g)</p> <p>1000 g = 1 kilogram (kg)</p> <p>1000 kg = 1 metric ton (t)</p>   | <p><b>Weight</b></p> <p>16 ounces (oz) = 1 pound (lb)</p> <p>2000 lb = 1 ton (t)</p>   |
| <p><b>Temperature</b>    <b>Degrees Celsius (°C)</b></p> <p>0°C = freezing point of water</p> <p>37°C = normal body temperature</p> <p>100°C = boiling point of water</p>   | <p><b>Temperature</b>    <b>Degrees Fahrenheit (°F)</b></p> <p>32°F = freezing point of water</p> <p>98.6°F = normal body temperature</p> <p>212°F = boiling point of water</p>  |

# Table of Formulas

| Formulas from Coordinate Geometry    |  |
|--------------------------------------|--|
| Slope of a line                      | $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $m$ is the slope of the nonvertical line through $(x_1, y_1)$ and $(x_2, y_2)$   |
| Parallel and perpendicular lines     | If line $l_1$ has slope $m_1$ and line $l_2$ has slope $m_2$ , then:<br>$l_1 \parallel l_2$ if and only if $m_1 = m_2$<br>$l_1 \perp l_2$ if and only if $m_1 = -\frac{1}{m_2}$ or $m_1 m_2 = -1$  |
| Distance formula                     | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ where $d$ is the distance between points $(x_1, y_1)$ and $(x_2, y_2)$  |
| Midpoint formula                     | $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment joining points $(x_1, y_1)$ and $(x_2, y_2)$  |
| Formulas from Matrix Algebra         |  |
| Determinant of a $2 \times 2$ matrix | $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$  |
| Determinant of a $3 \times 3$ matrix | $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$   |
| Area of a triangle                   | The area of a triangle with vertices $(x_1, y_1)$ , $(x_2, y_2)$ , and $(x_3, y_3)$ is given by<br>$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ where the appropriate sign ( $\pm$ ) should be chosen to yield a positive value.  |
| Cramer's rule                        | Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the coefficient matrix of this linear system:<br>$ax + by = e$ $cx + dy = f$ If $\det A \neq 0$ , then the system has exactly one solution. The solution is:<br>$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$ Cramer's rule can be extended to a linear system of 3 equations in 3 variables. |
| Inverse of a $2 \times 2$ matrix     | The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is<br>$A^{-1} = \frac{1}{ A } \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided $ad - cb \neq 0$ .  |

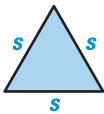
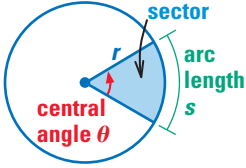
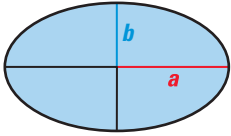
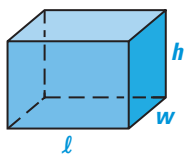
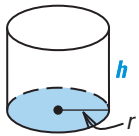
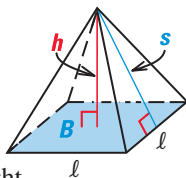
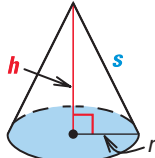
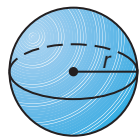
| Formulas and Theorems from Algebra              |  |
|---|--|
| Special product patterns                        | <p><b>Sum and difference:</b> <math>(a + b)(a - b) = a^2 - b^2</math></p> <p><b>Square of a binomial:</b> <math>(a + b)^2 = a^2 + 2ab + b^2</math><br/> <math>(a - b)^2 = a^2 - 2ab + b^2</math></p> <p><b>Cube of a binomial:</b> <math>(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3</math><br/> <math>(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3</math></p>  |
| Special factoring patterns                      | <p>Each of the patterns above can be read from right to left as a factoring pattern. In addition, there are two other special factoring patterns:</p> <p><b>Sum of two cubes:</b> <math>a^3 + b^3 = (a + b)(a^2 - ab + b^2)</math></p> <p><b>Difference of two cubes:</b> <math>a^3 - b^3 = (a - b)(a^2 + ab + b^2)</math></p>   |
| Quadratic formula                               | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>where <math>x</math> is a solution of <math>ax^2 + bx + c = 0</math> and <math>a, b,</math> and <math>c</math> are real numbers such that <math>a \neq 0</math></p>  |
| Discriminant of a quadratic equation            | <p>The expression <math>b^2 - 4ac</math> is called the discriminant of the associated equation <math>ax^2 + bx + c = 0</math>. The value of the discriminant can be positive, zero, or negative, which corresponds to an equation having two real solutions, one real solution, or two imaginary solutions, respectively.</p>  |
| Remainder theorem                               | <p>If a polynomial <math>f(x)</math> is divided by <math>x - k</math>, then the remainder is <math>r = f(k)</math>.</p>  |
| Factor theorem                                  | <p>A polynomial <math>f(x)</math> has a factor <math>x - k</math> if and only if <math>f(k) = 0</math>.</p>  |
| Rational zero theorem                           | <p>If <math>f(x) = a_nx^n + \cdots + a_1x + a_0</math> has <i>integer</i> coefficients, then every rational zero of <math>f</math> has this form:</p> $\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$  |
| Fundamental theorem of algebra                  | <p>If <math>f(x)</math> is a polynomial of degree <math>n</math> where <math>n &gt; 0</math>, then the equation <math>f(x) = 0</math> has at least one solution in the set of complex numbers.</p>   |
| Formulas from Statistics                        |  |
| Mean of a data set                              | $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$ <p>where <math>\bar{x}</math> (read “x-bar”) is the mean of the data <math>x_1, x_2, \dots, x_n</math></p>  |
| Standard deviation of a data set                | $\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$ <p>where <math>\sigma</math> (read “sigma”) is the standard deviation of the data <math>x_1, x_2, \dots, x_n</math></p>   |
| Areas under a normal curve                      | <p>The mean <math>\bar{x}</math> and standard deviation <math>\sigma</math> of a normal distribution determine the following areas under the corresponding normal curve.</p> <ul style="list-style-type: none"> <li>• The total area under the curve is 1.</li> <li>• 68% of the area lies within 1 standard deviation of the mean.</li> <li>• 95% of the area lies within 2 standard deviations of the mean.</li> <li>• 99.7% of the area lies within 3 standard deviations of the mean.</li> </ul> |
| Normal approximation of a binomial distribution | <p>Consider the binomial distribution consisting of <math>n</math> trials with a probability <math>p</math> of success on each trial. If <math>np \geq 5</math> and <math>n(1 - p) \geq 5</math>, then the binomial distribution can be approximated by a normal distribution with a mean of <math>\bar{x} = np</math> and a standard deviation of <math>\sigma = \sqrt{np(1 - p)}</math>.</p>   |

| Formulas for Sequences and Series               |  |
|---|--|
| Explicit rule for an arithmetic sequence        | The $n$ th term of an arithmetic sequence with first term $a_1$ and common difference $d$ is:<br>$a_n = a_1 + (n - 1)d$  |
| Explicit rule for a geometric sequence          | The $n$ th term of a geometric sequence with first term $a_1$ and common ratio $r$ is:<br>$a_n = a_1 r^{n-1}$  |
| Sum of a finite arithmetic series               | The sum of the first $n$ terms of an arithmetic series is:<br>$S_n = n \left( \frac{a_1 + a_n}{2} \right)$   |
| Sum of a finite geometric series                | The sum of the first $n$ terms of a geometric series with common ratio $r \neq 1$ is:<br>$S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)$  |
| Sum of an infinite geometric series             | The sum of an infinite geometric series with first term $a_1$ and common ratio $r$ is<br>$S = \frac{a_1}{1 - r}$<br>provided $ r  < 1$ . If $ r  \geq 1$ , the series has no sum.  |
| Formulas for sums of special series             | 1. $\sum_{i=1}^n 1 = n$ 2. $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ 3. $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$  |
| Formulas from Combinatorics                     |  |
| Fundamental counting principle                  | If one event can occur in $m$ ways and another event can occur in $n$ ways, then the number of ways that <i>both</i> events can occur is $m \cdot n$ .   |
| Permutations of $n$ objects taken $r$ at a time | The number of permutations of $r$ objects taken from a group of $n$ distinct objects is denoted by ${}_n P_r$ and is given by:<br>${}_n P_r = \frac{n!}{(n-r)!}$   |
| Permutations with repetition                    | The number of distinguishable permutations of $n$ objects where one object is repeated $q_1$ times, another is repeated $q_2$ times, and so on is:<br>$\frac{n!}{q_1! \cdot q_2! \cdot \dots \cdot q_k!}$  |
| Combinations of $n$ objects taken $r$ at a time | The number of combinations of $r$ objects taken from a group of $n$ distinct objects is denoted by ${}_n C_r$ and is given by:<br>${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$  |
| Pascal's triangle                               | If you arrange the values of ${}_n C_r$ in a triangular pattern in which each row corresponds to a value of $n$ , you get what is called Pascal's triangle.<br>$\begin{array}{cccccccc} & & & & & & & 1 \\ & & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & 1 & 3 & 3 & 1 \\ & 1 & 4 & 6 & 4 & 1 \end{array}$ |
| Binomial theorem                                | The binomial expansion of $(a + b)^n$ for any positive integer $n$ is:<br>$(a + b)^n = {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n$<br>$= \sum_{r=0}^n {}_n C_r a^{n-r} b^r$   |

| Formulas from Probability                                   |  |
|---|--|
| Theoretical probability of an event                         | When all outcomes are equally likely, the theoretical probability that an event $A$ will occur is:<br>$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$   |
| Probability of compound events                              | If $A$ and $B$ are two events, then the probability of $A$ or $B$ is:<br>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ If $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ is:<br>$P(A \text{ or } B) = P(A) + P(B)$  |
| Probability of the complement of an event                   | The probability of the complement of event $A$ , denoted $A'$ , is:<br>$P(A') = 1 - P(A)$  |
| Probability of independent events                           | If $A$ and $B$ are independent, then the probability that both $A$ and $B$ occur is:<br>$P(A \text{ and } B) = P(A) \cdot P(B)$  |
| Probability of dependent events                             | If $A$ and $B$ are dependent, then the probability that both $A$ and $B$ occur is:<br>$P(A \text{ and } B) = P(A) \cdot P(B   A)$  |
| Binomial probabilities                                      | For a binomial experiment consisting of $n$ trials where the probability of success on each trial is $p$ , the probability of exactly $k$ successes is:<br>$P(k \text{ successes}) = {}_n C_k p^k (1 - p)^{n - k}$   |
| Formulas and Identities from Trigonometry                   |  |
| Conversion between degrees and radians                      | To rewrite a degree measure in radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$ .<br>To rewrite a radian measure in degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$ .   |
| Definition of trigonometric functions                       | Let $\theta$ be an angle in standard position and $(x, y)$ be any point (except the origin) on the terminal side of $\theta$ . Let $r = \sqrt{x^2 + y^2}$ . Then the six trigonometric functions of $\theta$ are:<br>$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}, x \neq 0$ $\csc \theta = \frac{r}{y}, y \neq 0 \qquad \sec \theta = \frac{r}{x}, x \neq 0 \qquad \cot \theta = \frac{x}{y}, y \neq 0$ |
| Law of sines  | If $\triangle ABC$ has sides of length $a$ , $b$ , and $c$ , then:<br>$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$   |
| Area of a triangle (given two sides and the included angle) | If $\triangle ABC$ has sides of length $a$ , $b$ , and $c$ , then the area is:<br>$\text{Area} = \frac{1}{2}bc \sin A \qquad \text{Area} = \frac{1}{2}ac \sin B \qquad \text{Area} = \frac{1}{2}ab \sin C$   |
| Law of cosines  | If $\triangle ABC$ has sides of length $a$ , $b$ , and $c$ , then:<br>$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$   |

| Formulas and Identities from Trigonometry (continued) |  |
|---|--|
| Heron's area formula                                  | The area of the triangle with sides of length $a$ , $b$ , and $c$ is<br>$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ .   |
| Reciprocal identities                                 | $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\cot \theta = \frac{1}{\tan \theta}$  |
| Tangent and cotangent identities                      | $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$  |
| Pythagorean identities                                | $\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$  |
| Cofunction identities                                 | $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$  |
| Negative angle identities                             | $\sin(-\theta) = -\sin \theta$ $\cos(-\theta) = \cos \theta$ $\tan(-\theta) = -\tan \theta$  |
| Sum formulas  | $\begin{aligned} \sin(u+v) &= \sin u \cos v + \cos u \sin v \\ \cos(u+v) &= \cos u \cos v - \sin u \sin v \\ \tan(u+v) &= \frac{\tan u + \tan v}{1 - \tan u \tan v} \end{aligned}$   |
| Difference formulas                                   | $\begin{aligned} \sin(u-v) &= \sin u \cos v - \cos u \sin v \\ \cos(u-v) &= \cos u \cos v + \sin u \sin v \\ \tan(u-v) &= \frac{\tan u - \tan v}{1 + \tan u \tan v} \end{aligned}$   |
| Double-angle formulas                                 | $\begin{aligned} \cos 2u &= \cos^2 u - \sin^2 u & \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= 2 \cos^2 u - 1 & \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \\ \cos 2u &= 1 - 2 \sin^2 u \end{aligned}$   |
| Half-angle formulas                                   | $\begin{aligned} \sin \frac{u}{2} &= \pm \sqrt{\frac{1 - \cos u}{2}} & \tan \frac{u}{2} &= \frac{1 - \cos u}{\sin u} \\ \cos \frac{u}{2} &= \pm \sqrt{\frac{1 + \cos u}{2}} & \tan \frac{u}{2} &= \frac{\sin u}{1 + \cos u} \end{aligned}$<br>The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.   |
| Formulas from Mathematical Modeling                   |  |
| Projectile motion                                     | <b>Height as a function of time:</b> $h = -16t^2 + v_0t + h_0$ where $h$ is the height (in feet) of the object $t$ seconds after launch, $h_0$ is the object's initial height, and $v_0$ is the object's initial vertical velocity (in feet per second)<br><b>Parametric equations for a projectile's path:</b> $x = (v \cos \theta)t + x_0$ and $y = -\frac{1}{2}gt^2 + (v \sin \theta)t + y_0$ where $\theta$ is the angle at which the projectile is launched, $v$ is the initial speed, and $(x_0, y_0)$ is the projectile's location at time $t = 0$ . (The constant $g$ is the acceleration due to gravity; its value is 32 ft/sec <sup>2</sup> or 9.8 m/sec <sup>2</sup> .) |
| Compound interest                                     | <b>Compounded <math>n</math> times per year:</b> $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where $A$ is the amount in the account after $t$ years, $P$ is the initial deposit (called the principal), and $r$ is the annual interest rate (expressed as a decimal)<br><b>Compounded continuously:</b> $A = Pe^{rt}$ where $A$ is the amount in the account after $t$ years, $P$ is the initial deposit (called the principal), and $r$ is the annual interest rate (expressed as a decimal)   |

## Formulas from Geometry

|  |   |   |
|--|---|---|
| Basic geometric figures                              | See page 914 for area formulas for basic two-dimensional geometric figures.   |   |
| Area of an equilateral triangle                      | Area = $\frac{\sqrt{3}}{4}s^2$ where $s$ is the length of a side  |    |
| Arc length and area of a sector                      | Arc length = $r\theta$ where $r$ is the radius and $\theta$ is the radian measure of the central angle that intercepts the arc<br>Area = $\frac{1}{2}r^2\theta$   |    |
| Area of an ellipse                                   | Area = $\pi ab$ where $a$ and $b$ are half the lengths of the major and minor axes of the ellipse   |    |
| Volume and surface area of a right rectangular prism | Volume = $\ell wh$ where $\ell$ is the length, $w$ is the width, and $h$ is the height<br>Surface area = $2(\ell w + wh + \ell h)$  |    |
| Volume and surface area of a right cylinder          | Volume = $\pi r^2 h$ where $r$ is the base radius and $h$ is the height<br>Lateral surface area = $2\pi rh$<br>Surface area = $2\pi r^2 + 2\pi rh$  |  |
| Volume and surface area of a right regular pyramid   | Volume = $\frac{1}{3}Bh$ where $B$ is the area of the base and $h$ is the height<br>Lateral surface area = $\frac{1}{2}nls$ where $n$ is the number of sides on the base, $l$ is the length of a side of the base, and $s$ is the slant height<br>Surface area = $B + \frac{1}{2}nls$ |  |
| Volume and surface area of a right circular cone     | Volume = $\frac{1}{3}\pi r^2 h$ where $r$ is the base radius and $h$ is the height<br>Lateral surface area = $\pi rs$ where $s$ is the slant height<br>Surface area = $\pi r^2 + \pi rs$  |  |
| Volume and surface area of a sphere                  | Volume = $\frac{4}{3}\pi r^3$ where $r$ is the radius<br>Surface area = $4\pi r^2$  |  |



# Table of Properties

| Properties of Real Numbers                           |   |
|--|---|
|  | Let $a$ , $b$ , and $c$ be real numbers.  |
| Closure Property                                     | <b>Addition</b><br>$a + b$ is a real number.  |
| Commutative Property                                 | $a + b = b + a$   |
| Associative Property                                 | $(a + b) + c = a + (b + c)$   |
| Identity Property                                    | $a + 0 = a$ , $0 + a = a$   |
| Inverse Property                                     | $a + (-a) = 0$  |
| Distributive Property                                | The distributive property involves both addition and multiplication:<br>$a(b + c) = ab + ac$  |
| Zero Product Property                                | Let $A$ and $B$ be real numbers or algebraic expressions. If $AB = 0$ , then $A = 0$ or $B = 0$ .   |
|  |   |
| Properties of Matrices                               |   |
|  | Let $A$ , $B$ , and $C$ be matrices, and let $c$ be a scalar.   |
| Associative Property of Addition                     | $(A + B) + C = A + (B + C)$   |
| Commutative Property of Addition                     | $A + B = B + A$   |
| Distributive Property of Addition                    | $c(A + B) = cA + cB$  |
| Distributive Property of Subtraction                 | $c(A - B) = cA - cB$  |
| Associative Property of Matrix Multiplication        | $(AB)C = A(BC)$   |
| Left Distributive Property of Matrix Multiplication  | $A(B + C) = AB + AC$  |
| Right Distributive Property of Matrix Multiplication | $(A + B)C = AC + BC$  |
| Associative Property of Scalar Multiplication        | $c(AB) = (cA)B = A(cB)$   |
| Multiplicative Identity                              | An $n \times n$ matrix with 1's on the main diagonal and 0's elsewhere is an identity matrix, denoted $I$ . For any $n \times n$ matrix $A$ , $AI = IA = A$ . |
| Inverse Matrices                                     | If the determinant of an $n \times n$ matrix $A$ is nonzero, then $A$ has an inverse, denoted $A^{-1}$ , such that $AA^{-1} = A^{-1}A = I$ .                  |
| Properties of Exponents                              |   |
|  | Let $a$ and $b$ be real numbers, and let $m$ and $n$ be integers.   |
| Product of Powers Property                           | $a^m \cdot a^n = a^{m+n}$   |
| Power of a Power Property                            | $(a^m)^n = a^{mn}$  |
| Power of a Product Property                          | $(ab)^m = a^m b^m$  |
| Negative Exponent Property                           | $a^{-m} = \frac{1}{a^m}$ , $a \neq 0$   |
| Zero Exponent Property                               | $a^0 = 1$ , $a \neq 0$  |
| Quotient of Powers Property                          | $\frac{a^m}{a^n} = a^{m-n}$ , $a \neq 0$  |
| Power of a Quotient Property                         | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ , $b \neq 0$   |

## Properties of Radicals and Rational Exponents

Number of Real  $n$ th Roots

Let  $n$  be an integer greater than 1, and let  $a$  be a real number.

- If  $n$  is odd, then  $a$  has one real  $n$ th root:  $\sqrt[n]{a} = a^{1/n}$
- If  $n$  is even and  $a > 0$ , then  $a$  has two real  $n$ th roots:  $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If  $n$  is even and  $a = 0$ , then  $a$  has one  $n$ th root:  $\sqrt[n]{0} = 0^{1/n} = 0$
- If  $n$  is even and  $a < 0$ , then  $a$  has no real  $n$ th roots.

Radicals and Rational Exponents

Let  $a^{1/n}$  be an  $n$ th root of  $a$ , and let  $m$  be a positive integer.

- $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$
- $a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$

Properties of Rational Exponents

All of the properties of exponents listed on the previous page apply to rational exponents as well as integer exponents.

Product and Quotient Properties of Radicals

Let  $n$  be an integer greater than 1, and let  $a$  and  $b$  be positive real numbers. Then:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \text{and} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

## Properties of Logarithms

Logarithms and Exponents

Let  $b, c, x, y, u,$  and  $v$  be positive real numbers such that  $b \neq 1$  and  $c \neq 1$ .

$$\log_b y = x \text{ if and only if } b^x = y$$

Special Logarithm Values

$$\log_b 1 = 0 \text{ because } b^0 = 1 \quad \text{and} \quad \log_b b = 1 \text{ because } b^1 = b$$

Common and Natural Logarithms

$$\log_{10} x = \log x \quad \text{and} \quad \log_e x = \ln x$$

Product Property of Logarithms

$$\log_b uv = \log_b u + \log_b v$$

Quotient Property of Logarithms

$$\log_b \frac{u}{v} = \log_b u - \log_b v$$

Power Property of Logarithms

$$\log_b u^n = n \log_b u$$

Change of Base

$$\log_c u = \frac{\log_b u}{\log_b c}$$

## Properties of Functions

Operations on Functions

Let  $f$  and  $g$  be any two functions. A new function  $h$  can be defined using any of the following operations:

**Addition:**  $h(x) = f(x) + g(x)$

**Subtraction:**  $h(x) = f(x) - g(x)$

**Multiplication:**  $h(x) = f(x) \cdot g(x)$

**Division:**  $h(x) = \frac{f(x)}{g(x)}$

**Composition:**  $h(x) = f(g(x))$

For addition, subtraction, multiplication, and division, the domain of  $h$  consists of the  $x$ -values that are in the domains of both  $f$  and  $g$ . Additionally, the domain of a quotient does not include  $x$ -values for which  $g(x) = 0$ .

For composition, the domain of  $h$  is the set of all  $x$ -values such that  $x$  is in the domain of  $g$  and  $g(x)$  is in the domain of  $f$ .

Inverse Functions

Functions  $f$  and  $g$  are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$