Skills Review Handbook

Real Numbers

Operations with Signed Numbers

When adding signed numbers, you may find using a number line helpful. When subtracting signed numbers, remember that you can add the opposite because $a - b = a + (-b)$.

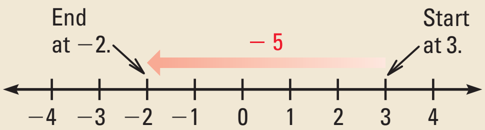
EXAMPLE Simplify the expression.

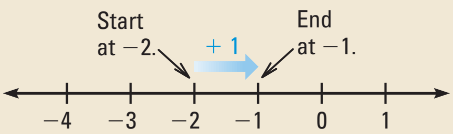
a. $3 + (-5)$ **b.** $(-2) - (-1)$

Solution

a. $3 + (-5) = -2$

b. $(-2) - (-1) = -2 + 1$
$= -1$





When multiplying and dividing signed numbers, use the following rules.

- Two positive numbers have a positive product or dividend.
- Two negative numbers have a positive product or dividend.
- A positive number and a negative number have a negative product or dividend.

EXAMPLE Perform the operation.

a. $3 \cdot 5$ **b.** $(-4) \cdot (-3)$ **c.** $(-6) \div (-2)$ **d.** $(-4) \div 2$ **e.** $2 \cdot (-5)$

Solution

a. $3 \cdot 5 = 15$ **b.** $(-4) \cdot (-3) = 12$ **c.** $(-6) \div (-2) = 3$ **d.** $(-4) \div 2 = -2$ **e.** $2 \cdot (-5) = -10$

Practice

Simplify the expression.

1. $1 + (-3)$ **2.** $3 + 12$ **3.** $(-4) + 4$ **4.** $(-8) + (-3)$

5. $7 + (-8)$ **6.** $(-3) + (-9)$ **7.** $(-4) + 10$ **8.** $4 + (-12)$

9. $6 + (-16)$ **10.** $(-18) + 2$ **11.** $(-13) + (-8)$ **12.** $(-3) + (-22)$

13. $8 - 2$ **14.** $(-5) - 8$ **15.** $1 - 5$ **16.** $0 - (-3)$

17. $7 - (-2)$ **18.** $(-8) - (-8)$ **19.** $6 - 11$ **20.** $(-1) - 4$

21. $(-2) - 1$ **22.** $(-11) - (-3)$ **23.** $12 - (-3)$ **24.** $(-11) - (-5)$

Simplify the expression.

25. $8 \cdot 3$ **26.** $(-7) \cdot 2$ **27.** $4 \cdot (-6)$ **28.** $(-3) \cdot (-3)$

29. $(-6) \cdot (-5)$ **30.** $2 \cdot (-3)$ **31.** $5 \cdot 5$ **32.** $(-9) \cdot 2$

33. $5 \div (-1)$ **34.** $(-9) \div (-3)$ **35.** $16 \div 2$ **36.** $(-4) \div 4$

37. $(-36) \div 9$ **38.** $21 \div (-7)$ **39.** $(-12) \div (-4)$ **40.** $(-36) \div (-3)$

Perform the indicated operation.

41. $(-5) \cdot 4$ **42.** $3 - (-4)$ **43.** $(-9) + 7$ **44.** $27 \div 3$

45. $(-30) \div 10$ **46.** $45 + (-5)$ **47.** $17 - (-12)$ **48.** $(-8) \cdot (-6)$

49. $14 \div (-2)$ **50.** $(-5) + 4$ **51.** $(-9) \cdot (-15)$ **52.** $(-20) - 12$

53. $(-42) - (-7)$ **54.** $7 \cdot (-3)$ **55.** $18 \div (-6)$ **56.** $(-13) + (-6)$

57. $(-11) + 18$ **58.** $12 \cdot (-8)$ **59.** $(-14) - (-7)$ **60.** $(-24) \div (-3)$

61. $63 \div (-7)$ **62.** $(-7) - (-26)$ **63.** $-12 \cdot (-11)$ **64.** $(-27) + (-15)$

CONVERTING DECIMALS, FRACTIONS, AND PERCENTS

Percent means "per hundred." It is a ratio (see page 910) that compares a number to 100.

EXAMPLE Write as a percent.

a. 0.3 **b.** $\frac{4}{5}$ **c.** 1.6

SOLUTION

a. $0.3 = \frac{3}{10} = \frac{30}{100} = 30\%$ **b.** $\frac{4}{5} = \frac{4 \cdot 20}{5 \cdot 20} = \frac{80}{100} = 80\%$ **c.** $1.6 = 1\frac{6}{10} = \frac{16}{10} = \frac{160}{100} = 160\%$

EXAMPLE Write as a decimal.

a. 66% **b.** $\frac{17}{25}$ **c.** 125%

SOLUTION

a. $66\% = \frac{66}{100} = 0.66$ **b.** $\frac{17}{25} = 17 \div 25 = 0.68$ **c.** $125\% = \frac{125}{100} = 1.25$

PRACTICE

Write as a percent.

1. 0.20 **2.** 0.15 **3.** 0.55 **4.** 1.34 **5.** 0.87

6. $\frac{9}{10}$ **7.** $\frac{2}{5}$ **8.** $\frac{15}{50}$ **9.** $\frac{3}{5}$ **10.** $\frac{21}{20}$

Write as a decimal.

11. 50% **12.** 120% **13.** 2% **14.** 85% **15.** 40%

16. $\frac{3}{5}$ **17.** $\frac{9}{25}$ **18.** $\frac{11}{20}$ **19.** $\frac{75}{50}$ **20.** $\frac{9}{10}$

CALCULATING PERCENTS

To calculate a percent of a number, write the percent as a fraction or decimal and multiply.

EXAMPLE **a.** Find 14% of 150. **b.** Find 80% of 200.

SOLUTION **a.** $14\% \text{ of } 150 = \frac{14}{100}(150) = 21$ **b.** $80\% \text{ of } 200 = 0.8 \cdot 200 = 160$

To find the percent one number is of another, divide.

EXAMPLE **a.** What percent is 2 of 8? **b.** What percent is 12 of 9?

SOLUTION **a.** $\frac{2}{8} = 0.25 = 25\%$ **b.** $\frac{12}{9} = 1.\overline{3} = 133\frac{1}{3}\%$

To find a percent increase or decrease, find the difference between the two numbers and divide by the first number.

EXAMPLE A television is marked down from \$500 to \$400. Find the percent increase or decrease.

SOLUTION Percent change in price $= \frac{\text{New price} - \text{Old price}}{\text{Old price}} = \frac{400 - 500}{500} = \frac{-100}{500} = \frac{-20}{100} = -20\%$

▶ The negative sign indicates that the percent change is a decrease. Therefore, the price of a television decreased 20%.

PRACTICE

Find the number.

1. 15% of 20
2. 50% of $\frac{2}{3}$
3. 10% of 3
4. 20% of $\frac{1}{2}$
5. 60% of 50
6. 12% of 18.5
7. 9% of 6
8. 2% of 100
9. 100% of 12
10. 25% of $\frac{3}{5}$
11. 1% of $\frac{3}{8}$
12. 85% of $\frac{1}{10}$
13. 5% of 0.5
14. 20% of 90
15. 10% of 0.84
16. 38% of 16
17. 200% of 7
18. 33% of 15
19. 0.5% of 1
20. 125% of 1.2

Find the answer.

21. What percent is 15 of 30?
22. What percent is 3 of 12?
23. What percent is 10 of 10?
24. What percent is 1 of 20?
25. What percent is 14 of 40?
26. What percent is 300 of 200?
27. What percent is 4 of 18?
28. What percent is 6 of 16?
29. What percent is 2 of 85?
30. What percent is 4 of 100?
31. What percent is 8 of 40?
32. What percent is 90 of 50?
33. What percent is 0.4 of 200?
34. What percent is 80 of 5?
35. What percent is 0.22 of 50?

Find the percent increase or decrease.

36. 50 votes increased to 200 votes

37. \$80 decreased to \$56

38. 300 fish increased to 360 fish

39. \$400 increased to \$600

40. 15 feet decreased to 12 feet

41. 4500 units sold increased to 4800 units sold

42. 100 students increased to 108 students

43. A 40 minute run decreased to a 35 minute run

FACTORS AND MULTIPLES

Factors are numbers or variable expressions that are multiplied together. A **prime number** is a whole number greater than 1 that has exactly two factors, itself and 1. To write the **prime factorization** of a number, write the number as a product of prime numbers.

Prime numbers less than 100
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

EXAMPLE Write the prime factorization of 24.

SOLUTION Use a tree diagram:

24

4×6 — Write 24 as a product.

$2 \times 2 \times 2 \times 3$ — Write 4 and 6 as products.

A **common factor** of two whole numbers is a whole number that is a factor of each number. The **greatest common factor (GCF)** of two whole numbers is the greatest whole number that is a factor of each number. The **least common multiple (LCM)** of two whole numbers is the smallest whole number (other than zero) that is a multiple of each number.

EXAMPLE What is the greatest common factor of 16 and 24?

SOLUTION

Method 1

Make a list of each number's factors.

16: **1**, **2**, **4**, **8**, 16
24: **1**, **2**, 3, **4**, 6, **8**, 12, 24

The common factors are 1, 2, 4, and 8. The GCF of 16 and 24 is 8.

Method 2

The GCF of two whole numbers is equal to the product of all common prime factors of the numbers.

$16 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 2$ — Prime factorization of 16
$24 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{2} \cdot 3$ — Prime factorization of 24

The common prime factors are 2, 2, and 2, so the GCF of 16 and 24 is $2 \cdot 2 \cdot 2$, or 8.

EXAMPLE What is the least common multiple of 16 and 24?

SOLUTION

Method 1

Make a list of each number's multiples.

Multiples of 16: 16, 32, **48**, 64, . . .
Multiples of 24: 24, **48**, 72, . . .

The LCM of 16 and 24 is 48.

Method 2

The LCM of two whole numbers is the product of the highest power of each prime number that appears in the factorization of either number.

$16 = \mathbf{2^4}$ — Prime factorization of 16
$24 = 2^3 \cdot \mathbf{3}$ — Prime factorization of 24

The LCM of 16 and 24 is $2^4 \cdot 3 = 48$.

The **least common denominator (LCD)** of two fractions is the least common multiple of the denominators. To add or subtract two fractions with unlike denominators, first write equivalent fractions using the LCD, then add or subtract the numerators.

EXAMPLE Add: $\frac{3}{16} + \frac{7}{24}$

SOLUTION The LCD is 48. Rewrite fractions: $\frac{3}{16} = \frac{3 \cdot 3}{16 \cdot 3} = \frac{9}{48}$ and $\frac{7}{24} = \frac{7 \cdot 2}{24 \cdot 2} = \frac{14}{48}$

Add the rewritten fractions: $\frac{9}{48} + \frac{14}{48} = \frac{23}{48}$

PRACTICE

Write the prime factorization of the number. If the number is prime, write *prime*.

1. 8 **2.** 100 **3.** 64 **4.** 21 **5.** 17

6. 9 **7.** 12 **8.** 56 **9.** 22 **10.** 50

11. 41 **12.** 30 **13.** 31 **14.** 46 **15.** 25

Give the greatest common factor (GCF) and least common multiple (LCM) of the pair of numbers.

16. 15, 25 **17.** 4, 7 **18.** 10, 15 **19.** 20, 6 **20.** 3, 13

21. 20, 40 **22.** 12, 9 **23.** 18, 24 **24.** 8, 10 **25.** 48, 36

26. 4, 6 **27.** 2, 3 **28.** 22, 11 **29.** 6, 18 **30.** 45, 25

Find the least common denominator.

31. $\frac{7}{48}, \frac{5}{24}$ **32.** $\frac{3}{7}, \frac{5}{6}$ **33.** $\frac{3}{13}, \frac{1}{2}$ **34.** $\frac{7}{2}, \frac{3}{4}$ **35.** $\frac{6}{10}, \frac{5}{2}$

36. $\frac{1}{3}, \frac{7}{8}$ **37.** $\frac{5}{6}, \frac{11}{12}$ **38.** $\frac{3}{10}, \frac{17}{30}$ **39.** $\frac{7}{15}, \frac{7}{12}$ **40.** $\frac{7}{4}, \frac{3}{5}$

41. $\frac{5}{6}, \frac{3}{4}, \frac{1}{2}$ **42.** $\frac{2}{3}, \frac{5}{12}, \frac{4}{9}$ **43.** $\frac{7}{8}, \frac{5}{6}, \frac{4}{5}$ **44.** $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ **45.** $\frac{8}{12}, \frac{6}{4}$

46. $\frac{2}{11}, \frac{4}{9}, \frac{5}{3}$ **47.** $\frac{5}{6}, \frac{11}{20}, \frac{14}{15}$ **48.** $\frac{4}{7}, \frac{3}{4}, \frac{1}{28}$ **49.** $\frac{1}{5}, \frac{5}{12}, \frac{1}{4}$ **50.** $\frac{5}{3}, \frac{1}{2}$

Perform the indicated operation(s). Simplify the result.

51. $\frac{3}{8} + \frac{5}{12}$ **52.** $\frac{7}{4} - \frac{1}{12}$ **53.** $-\frac{3}{4} - \frac{1}{2}$ **54.** $\frac{7}{8} - \frac{1}{2}$

55. $\frac{2}{5} - \frac{1}{3} - \frac{1}{6}$ **56.** $\frac{8}{9} + \frac{2}{3} + \frac{1}{2}$ **57.** $\frac{5}{16} + \frac{2}{5} - \frac{3}{10}$ **58.** $\frac{4}{9} + \frac{2}{3}$

59. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ **60.** $-\frac{5}{24} + \frac{2}{3} - \frac{1}{6}$ **61.** $\frac{9}{11} - \frac{5}{3} - \frac{5}{6}$ **62.** $-\frac{6}{7} - \frac{2}{14}$

63. $\frac{2}{6} - \frac{1}{3} + \frac{1}{2}$ **64.** $\frac{2}{8} - \frac{3}{4} + \frac{1}{2}$ **65.** $-\frac{3}{12} + \frac{4}{10} - \frac{1}{5}$ **66.** $\frac{1}{3} + \frac{5}{6} - \frac{2}{9}$

67. $\frac{3}{2} - \frac{4}{8} + \frac{1}{6}$ **68.** $\frac{1}{12} - \frac{5}{6} + \frac{4}{9}$ **69.** $\frac{7}{15} - \frac{4}{5} + \frac{2}{3}$ **70.** $\frac{1}{2} - \frac{8}{10} + \frac{5}{4}$

Writing Ratios and Solving Proportions

A *ratio* compares two numbers using division. If a and b are two quantities measured in the same units, then the **ratio of a to b** can be written in three ways:

a to b $\qquad a : b \qquad \frac{a}{b}$

EXAMPLE Write the ratio 4 to 3 in two other ways.

SOLUTION 4 : 3 $\quad \frac{4}{3}$

To write a ratio in lowest terms, divide out any common factors.

EXAMPLE Write the ratio 12 to 18 in lowest terms.

SOLUTION 6 is the greatest common factor, so divide each number by 6.

▶ In lowest terms, the ratio 12 to 18 is 2 to 3.

A **proportion** is an equation stating that two ratios are equivalent. If a proportion contains a variable, you can cross multiply to solve for the variable.

EXAMPLE Solve the proportion $\frac{5}{6} = \frac{10}{x}$.

SOLUTION

$\frac{5}{6} = \frac{10}{x}$ **Rewrite proportion.**

$5 \cdot x = 6 \cdot 10$ **Cross multiply.**

$5x = 60$ **Simplify.**

$x = 12$ **Solve for x.**

Practice

Write the ratio in two other ways.

1. 4 to 5 **2.** 1 : 1 **3.** 2 to 6 **4.** 3 : 5

5. $\frac{1}{5}$ **6.** 10 to 1 **7.** $\frac{8}{5}$ **8.** 5 : 4

9. 3 : 1 **10.** $\frac{2}{3}$ **11.** $\frac{3}{4}$ **12.** 6 to 3

Write the ratio in lowest terms.

13. 2 to 8 **14.** 5 : 10 **15.** 4 : 16 **16.** 80 to 100

17. $\frac{2}{10}$ **18.** $\frac{3}{27}$ **19.** 25 to 15 **20.** 9 : 3

21. $\frac{12}{20}$ **22.** 4 : 24 **23.** $\frac{24}{18}$ **24.** 20 : 35

Solve the proportion.

25. $\frac{x}{4} = \frac{2}{8}$ **26.** $\frac{5}{7} = \frac{a}{28}$ **27.** $\frac{6}{b} = \frac{3}{8}$ **28.** $\frac{20}{4} = \frac{10}{y}$

29. $\frac{2}{1} = \frac{6}{c}$ **30.** $\frac{5}{4} = \frac{x}{10}$ **31.** $\frac{3}{7} = \frac{9}{b}$ **32.** $\frac{36}{r} = \frac{12}{3}$

33. $\frac{p}{2} = \frac{5}{2}$ **34.** $\frac{8}{5} = \frac{w}{25}$ **35.** $\frac{9}{4} = \frac{k}{12}$ **36.** $\frac{n}{3} = \frac{3}{1}$

37. $\frac{2}{11} = \frac{x}{99}$ **38.** $\frac{80}{48} = \frac{10}{s}$ **39.** $\frac{z}{5} = \frac{8}{2}$ **40.** $\frac{1}{8} = \frac{5}{j}$

41. $\frac{16}{3} = \frac{2a}{6}$ **42.** $\frac{c}{15} = \frac{4}{3}$ **43.** $\frac{60}{40} = \frac{12}{m}$ **44.** $\frac{3x}{4} = \frac{27}{12}$

45. $\frac{2}{9} = \frac{y}{27}$ **46.** $\frac{13}{w} = \frac{39}{9}$ **47.** $\frac{x}{20} = \frac{3}{60}$ **48.** $\frac{x}{3} = \frac{40}{6}$

SIGNIFICANT DIGITS

Significant digits indicate how precisely a number is known. Use the following guidelines to determine the number of significant digits.

- All nonzero digits are significant.
- All zeros that appear between two nonzero digits are significant.
- For a decimal, all zeros that appear after the last nonzero digit are significant. For a whole number, you cannot tell whether any zeros after the last nonzero digit are significant, so you should assume that they are not significant (unless you know otherwise).

Sometimes calculations involve measurements that have various numbers of significant digits. In this case, a general rule is to carry all digits through the calculation and then round the result to the same number of significant digits as the measurement with the *fewest* number of significant digits.

EXAMPLE Add: 76.33 + 22.0 + 1500

SOLUTION Of the three numbers, 1500 has the fewest number of significant digits. Add all three numbers, then round the sum to two significant digits.

$$76.33 + 22.0 + 1500 = 1598.33$$
$$\approx 1600$$

EXAMPLE Perform the indicated operation.

a. 0.004 • 3.22 **b.** 374,039.8 ÷ 305

SOLUTION **a.** Since the zeros before the 4 in 0.004 are not significant, round the answer to one significant digit.

$$0.004 \cdot 3.22 = 0.01288$$
$$\approx 0.01$$

b. Since the zero between the 3 and the 5 in 305 is significant, round the answer to three significant digits.

$$374{,}039.8 \div 305 = 1226.36$$
$$\approx 1230$$

Note that some units, such as number of people, cannot be divided into fractional parts. In that case, use the significant digits of the other numbers to round the answer.

EXAMPLE A bill of $98.80 is divided among 8 people. How much does each person pay?

SOLUTION The number of people is exact, so the fact that it is a one-digit number is irrelevant. Use the significant digits for the money to round your answer.

$98.80 ÷ 8 = $12.35

▶ Each person should pay $12.35.

PRACTICE

Simplify the expression. Write your answer with the appropriate number of significant digits.

1. 8244 + 3.6
2. −25 − 3
3. 2.50 • 3.80
4. 0.95 ÷ 4.25
5. 30.82 − 2.6690
6. 16 ÷ 7
7. 700 + 20
8. 60 ÷ 24
9. 50 ÷ 4.5
10. 2.64 + 3.0008
11. 38.25 ÷ 52
12. 6 − 3.4
13. 5.0 − 1.8
14. 0.74 • 2.15
15. 25.000 ÷ 25
16. 13.36 + 40.58
17. 200 − 3.5
18. 40 ÷ 0.368
19. 14.85 + 5.00 + 4.8
20. 0.0036 + 0.017 + 0.0249
21. 23.89 − 2.5 − 3.74
22. 100 − 21 − 2.9 − 3.62
23. 27.5 • 9.8 • 0.332
24. 0.783 • 2.11 • 4.51
25. 2.48 • 16.4 ÷ 56.25
26. 42.6 • 2.05 ÷ 0.0068
27. 60 ÷ (52.4 • 20)
28. 388 • 16 • 108 • 27
29. 13,720 + 2800 − 513
30. (200 • 45) ÷ (36 • 15)

Perform the calculation. Write your answer with the appropriate number of significant digits.

31. $1.50 per card • 5 cards
32. 324 pens ÷ 36 students
33. $39.95 per sweater • 6 sweaters
34. $.40 per orange • 10 oranges
35. 89 miles ÷ 6.8 gallons
36. 282 books ÷ 47 students
37. 101 gallons of milk + 8.75 gallons of milk − 6.9 gallons of milk
38. 210 pounds of sand − 16.25 pounds of sand − 1.5 pounds of sand
39. 20.3 milliliters of water + 1.08 milliliters of hydrochloric acid
40. 8.0 liters of juice − 5 liters of juice
41. 38,050 computers ÷ 52 computer stores
42. 3000 kilogram car + 65.50 kilogram passenger + 2.37 kilogram groceries
43. 13.2 milligrams of rice + 0.015 milligram of saffron + 1.25 milligrams of salt
44. 325 milligrams of Vitamin C + 5.50 milligrams of Vitamin C − 24.3 milligrams of Vitamin C
45. 7.55 inches of rain in March + 12.25 inches of rain in April + 6.08 inches of rain in May

Scientific Notation

Numbers written in scientific notation have the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer. Recall that $10^0 = 1$.

EXAMPLE Write each number in scientific notation.

a. 721,000,000 **b.** 0.001046

Solution **a.** Move the decimal point 8 places to the left.

$721{,}000{,}000 = 7.21 \times 10^8$

b. Move the decimal point three places to the right.

$0.001046 = 1.046 \times 10^{-3}$

EXAMPLE Write each number in standard form.

a. 5.23×10^7 **b.** 2.600×10^{-4}

Solution **a.** Move the decimal point 7 places to the right.

$5.23 \times 10^7 = 52{,}300{,}000$

b. Move the decimal point 4 places to the left. Note that 4 significant digits are kept.

$2.600 \times 10^{-4} = 0.0002600$

Practice

Write each number in scientific notation.

1. 0.4 **2.** 0.34 **3.** 0.09 **4.** 30.58

5. 4 **6.** 0.0000000025 **7.** 0.0000926 **8.** 4,983,200,000

9. 211.111 **10.** 4193 **11.** 0.005 **12.** 21,040

13. 98,400 **14.** 0.00002 **15.** 204.89 **16.** 295

17. 0.00037 **18.** 0.2000 **19.** 59.8 **20.** 5,000,000

21. 23,085,600 **22.** 0.0000004 **23.** 0.000100 **24.** 0.101001

Write each number in standard form.

25. 9×10^2 **26.** 2.52×10^{-1} **27.** 3.1×10^3

28. 6×10^5 **29.** 2.90×10^{-1} **30.** 9.1×10^0

31. 1.001×10^4 **32.** 5.273×10^{-3} **33.** 7.926×10^6

34. 8.13×10^{-1} **35.** 3.84×10^{-4} **36.** 4.6000×10^8

37. 3.7×10^{-5} **38.** 1.11×10^{-2} **39.** 4.9831×10^{-3}

40. 7.05×10^{-7} **41.** 3.9502×10^5 **42.** 1.0063×10^0

43. 2.64095×10^3 **44.** 3.03×10^{-7} **45.** 4.55×10^{-4}

46. 5.0×10^3 **47.** 5.9438×10^{-2} **48.** 6.105×10^{-6}

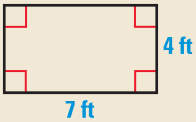
GEOMETRY

PERIMETER, AREA, AND VOLUME

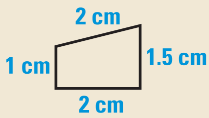
The **perimeter** of a two-dimensional figure is the sum of the lengths of the edges, or the distance around the figure.

EXAMPLE Find the perimeter of the figure.

a.



b.



SOLUTION

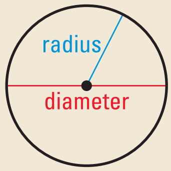
a. $4 + 4 + 7 + 7 = 22$

▶ The perimeter is 22 feet.

b. $2 + 1.5 + 2 + 1 = 6.5$

▶ The perimeter is 6.5 cm.

The perimeter of a circle, called its **circumference**, is the distance around the circle. The formula for circumference is $C = 2\pi r$ where r is the **radius**. Because the **diameter** is twice the radius, the formula for circumference can also be written as $C = \pi d$ where d is the diameter.



EXAMPLE Find the circumference of the circle with radius 3 inches.

SOLUTION $C = 2\pi r = 2\pi(3) = 6\pi \approx 6(3.14) = 18.84$

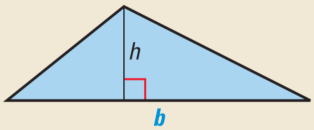
▶ The circumference is 6π inches or about 18.84 inches.

EXAMPLE Find the circumference of the circle with diameter 4 meters.

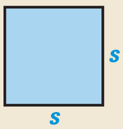
SOLUTION $C = \pi d = \pi(4) = 4\pi \approx 4(3.14) = 12.56$

▶ The circumference is 4π meters or about 12.56 meters.

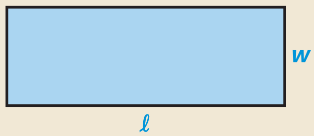
The **area** of a two-dimensional figure is the number of square units enclosed within the boundary of the figure.



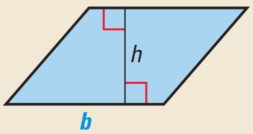
Area of a triangle: $A = \frac{1}{2}bh$



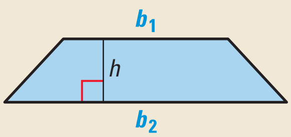
Area of a square: $A = s^2$



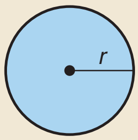
Area of a rectangle: $A = \ell w$



Area of a parallelogram: $A = bh$



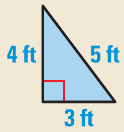
Area of a trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$



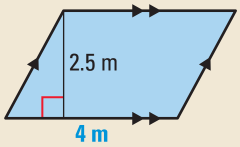
Area of a circle: $A = \pi r^2$

EXAMPLE Find the area of the figure.

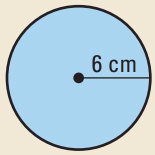
a.



b.



c.



SOLUTION

a. $A = \frac{1}{2}bh$
$= \frac{1}{2}(3)(4)$
$= 6$ square feet

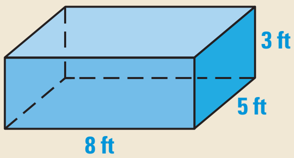
b. $A = bh$
$= (4)(2.5)$
$= 10$ square meters

c. $A = \pi r^2$
$= \pi(6)^2$
$= 36\pi$
≈ 113 square centimeters

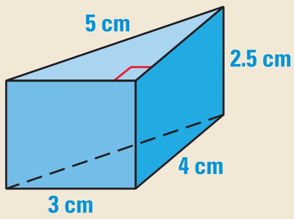
A **prism** is a three-dimensional figure with two congruent faces, called *bases*, that lie in parallel planes. The **surface area** of a prism is the sum of the areas of all the faces of the prism. Surface area is measured in square units.

EXAMPLE Find the surface area of the prism or cylinder.

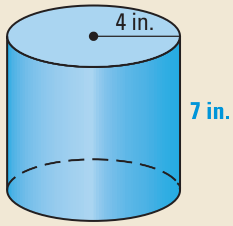
a.



b.



c.



SOLUTION

a. A rectangular prism has three pairs of identical rectangular faces.

Surface area $= 2(8 \cdot 5) + 2(8 \cdot 3) + 2(5 \cdot 3)$
$= 80 + 48 + 30$
$= 158$

▶ The prism has a surface area of 158 square feet.

b. Surface area = area of bases + area of faces
$= 2\left[\frac{1}{2}(3)(4)\right] + (2.5)(3) + (2.5)(4) + (2.5)(5)$
$= 12 + 7.5 + 10 + 12.5$
$= 42$

▶ The prism has a surface area of 42 square centimeters.

c. Surface area = area of bases + (circumference)(height)
$= 2(\pi r^2) + (2\pi r)(h)$
$= 2[\pi(4)^2] + [2\pi(4)](7)$
$= 32\pi + 56\pi$
$= 88\pi$
≈ 276

▶ The cylinder has a surface area of about 276 square inches.

The **volume** of a solid is a measure of how much it will hold and is measured in cubic units. The volume of a prism is calculated by multiplying the area of the base by the height.

EXAMPLE Find the volume of the three solids in the previous example.

SOLUTION

a. Volume $= (8 \cdot 5) \cdot 3$
$= 120$

▶ The prism has a volume of 120 ft^3.

b. Volume $= \left(\frac{1}{2} \cdot 3 \cdot 4\right) \cdot 2.5$
$= 15$

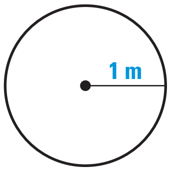
▶ The prism has a volume of 15 cm^3.

c. Volume $= \left[\pi(4)^2\right] \cdot 7$
$= 112\pi$
≈ 352

▶ The cylinder has a volume of about 352 $in.^3$

PRACTICE

Find the perimeter or circumference of the figure.

1.


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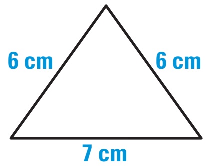

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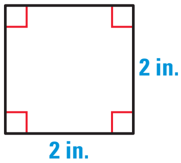

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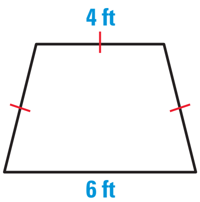

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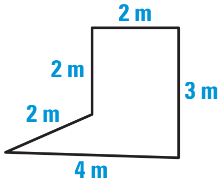
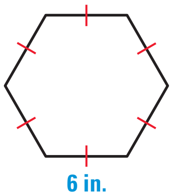


6.


7. A square 3 ft on each side

8. A circle with diameter 10 in.

9. A rectangle with sides of 4 cm and 7 cm

10. A regular pentagon with side length 2.5 m

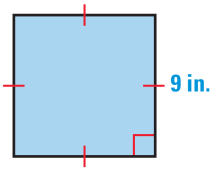
11. A triangle with sides of length 8 cm, 3 cm, and 7 cm

12. A parallelogram with sides 3.5 m and 5.8 m

13. A circle with a diameter of 22 in.

14. A rectangle with sides of 5 ft and 8 ft

Find the area of the figure.

15.


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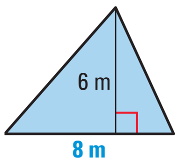

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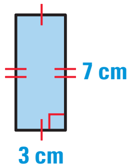

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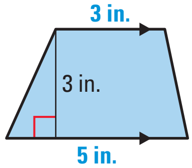


19. A trapezoid with bases 4 in. and 8 in. and height 4 in.

20. A square with side length 7 ft

21. A circle with radius 0.5 in.

22. A parallelogram with height 6 m and base 9 m

23. A 2 mi by 6 mi rectangle

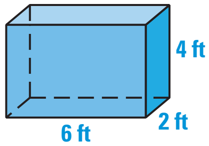
24. A circle with radius 9 mm

25. A triangle with a base of 5 in. and height of 4 in.

26. A circle with a radius of 10 ft

Find the surface area of the prism or cylinder.

27.



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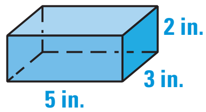

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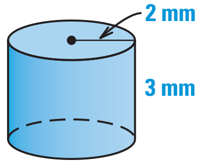

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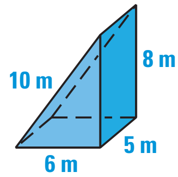


31. A cylinder with radius 2 in. and height 14 in.

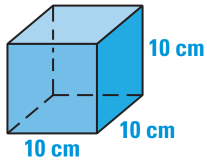
32. A cube with side length 3 cm

33. A rectangular prism 4 cm by 6 cm by 12 cm

34. A cylinder with a radius of 50 ft and height of 200 ft

Find the volume of the prism or cylinder.

35.



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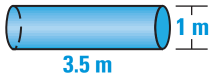

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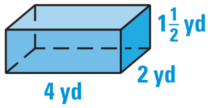
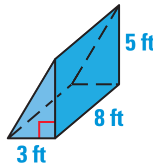


38.



39. A rectangular prism 1 ft by 1 ft by 5 ft

40. A cube 8 in. on each side

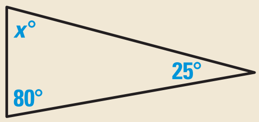
41. A cylinder with diameter 9 in. and height 2 in.

42. A rectangular prism with base 12.8 m^2 and height 3 m

TRIANGLE RELATIONSHIPS

The sum of the angles of a triangle is 180°.

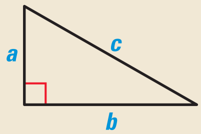
EXAMPLE Find the value of x.



SOLUTION $180° = 25° + 80° + x$

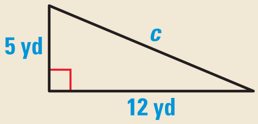
$x = 75°$

The **Pythagorean theorem** states that in a right triangle with legs of length a and b and hypotenuse of length c, $c^2 = a^2 + b^2$.

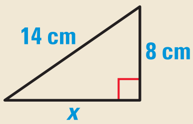


EXAMPLE Find the length of the unknown side.

a.



b.



SOLUTION

a.
$5^2 + 12^2 = c^2$
$25 + 144 = c^2$
$169 = c^2$
$c = 13$

▶ The hypotenuse is 13 yards long.

b.
$8^2 + x^2 = 14^2$
$64 + x^2 = 196$
$x^2 = 132$
$x \approx 11.5$

▶ The length of the second leg is about 11.5 centimeters.

The sum of the lengths of the two shorter sides of a triangle must be greater than the length of the third side.

EXAMPLE Can you form a triangle with the given side lengths? Write *yes* or *no*.

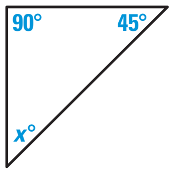
a. 2, 3, 8 **b.** 9, 11, 19 **c.** 3, 18, 21

SOLUTION

a. No, because $2 + 3 < 8$. **b.** Yes, because $9 + 11 > 19$. **c.** No, because $18 + 3 = 21$.

PRACTICE

Find the value of *x*.

1.


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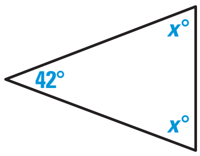

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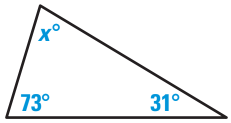

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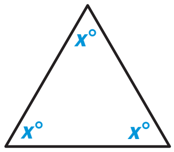


5. A triangle with angles $x°$, 5°, and 10°

6. A triangle with angles $x°$, 48°, and 22°

7. A triangle with angles $x°$, 38°, and 82°

8. A triangle with angles $x°$, 25°, and 63°

Can a triangle have the following angle measures? Write *yes* or *no*.

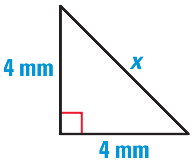
9. 60°, 60°, 60° **10.** 136°, 19°, 45° **11.** 112°,15°, 43° **12.** 45°, 67°, 68°

13. 47°, 90°, 23° **14.** 59°, 60°, 61° **15.** 31°, 78°, 91° **16.** 25°, 30°, 125°

17. 160°, 5°, 5° **18.** 55°, 75°, 60° **19.** 40°, 50°, 90° **20.** 113°, 14°, 53°

21. 17°, 52°, 111° **22.** 20°, 140°, 20° **23.** 70°, 60°, 50° **24.** 43°, 56°, 101°

Find the length of the unknown side.

25.


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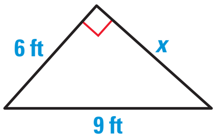

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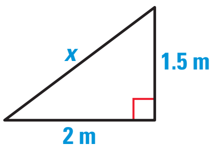

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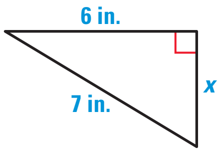

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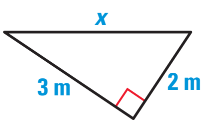

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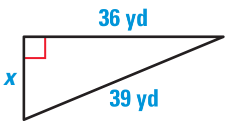

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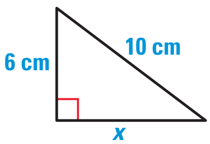

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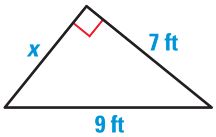


33. A right triangle with two sides 5 in. long

34. A right triangle with one side 8 ft and hypotenuse 10 ft

Can you form a triangle with the given side lengths? Write *yes* or *no*.

35. 8, 3, 7 **36.** 10, 10, 10 **37.** 16, 5, 11 **38.** 3, 6, 8

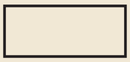
39. 4, 4, 5 **40.** 2, 5, 2 **41.** 6, 5, 4 **42.** 17, 9, 8

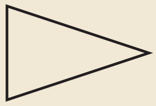
43. 85, 19, 51 **44.** 12, 7, 4 **45.** 10, 24, 26 **46.** 46, 22, 17

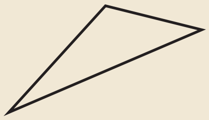
SYMMETRY

A figure has **line symmetry** if it can be divided by a line into two parts, each of which is the mirror image of the other. The line that divides the figure into two parts is called the **line of symmetry**.

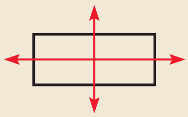
EXAMPLE Identify the lines of symmetry in the figure.

a. 

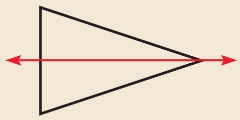
b. 

c. 

SOLUTION **a.** This figure has a vertical line of symmetry and a horizontal line of symmetry.



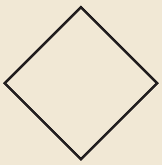
b. This figure has a horizontal line of symmetry.

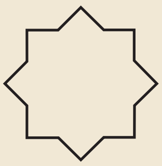


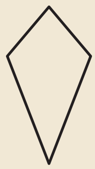
c. This figure is not symmetric. It has no line of symmetry.

A figure has **rotational symmetry** if it coincides with itself after rotating 180° or less, either clockwise or counterclockwise, about a point. The point of rotation is usually the center of the figure.

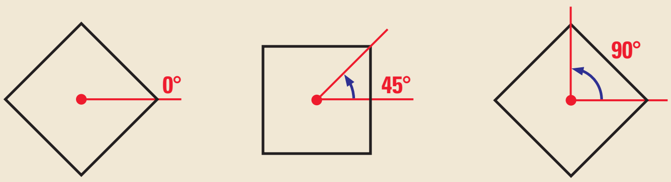
EXAMPLE Identify any rotational symmetry in the figure.

a. 

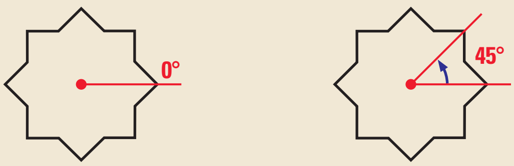
b. 

c. 

SOLUTION **a.** This figure has rotational symmetry. It will coincide with itself after being rotated 90° or 180° in either direction. Notice that the point of rotation is located at the center of the figure.

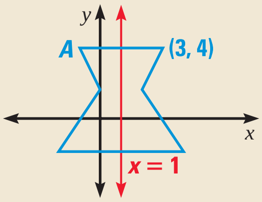


b. This figure has rotational symmetry. It will coincide with itself after being rotated 45°, 90°, 135°, or 180° in either direction. The point of rotation is located at the center of the figure.



c. This figure has no rotational symmetry.

EXAMPLE The figure at the right has line symmetry. Find the coordinates of point *A*.

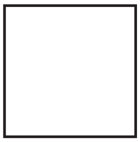


SOLUTION The line of symmetry is $x = 1$. Point (3, 4) is 2 units to the right of the line of symmetry so *A* must be an equal distance to the left. Therefore, *A* is at $(-1, 4)$.

PRACTICE

State whether the figure has line symmetry or rotational symmetry. Then identify the line(s) of symmetry or the angle of rotation.

1.



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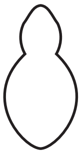

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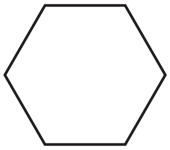

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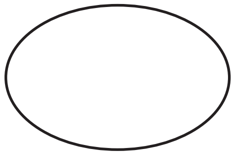

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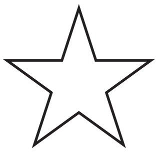

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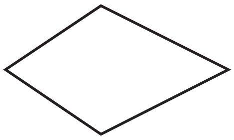

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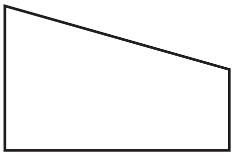

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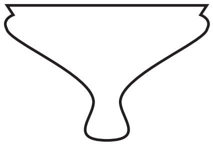
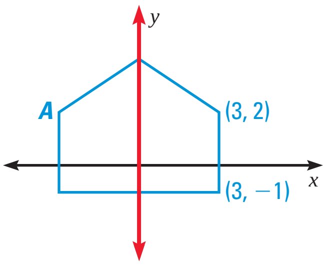


The figure is symmetric. The line of symmetry is shown in red. Find the coordinates of point *A*.

9.



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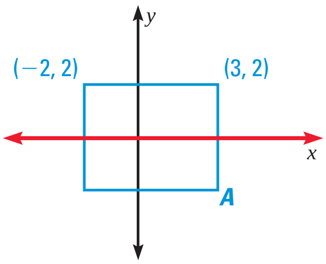

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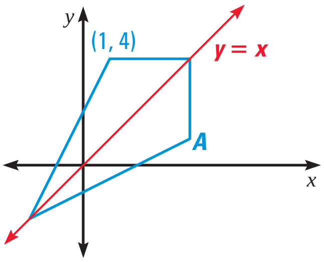

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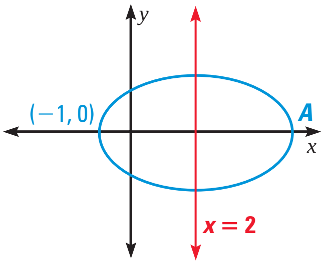

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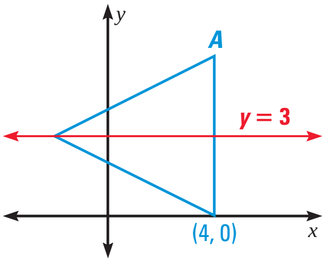

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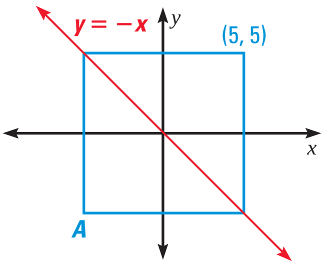
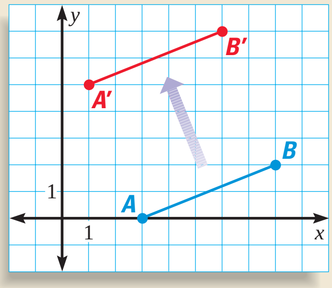


TRANSFORMATIONS

A **transformation** is a change made to the size or position of a figure. A **translation** is a transformation that slides every point of a figure the same distance in the same direction while preserving its size and orientation.

EXAMPLE Translate $\overline{AB}$ 2 units to the left and 5 units up.



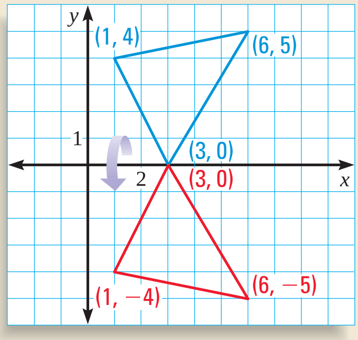
SOLUTION To shift $\overline{AB}$ left 2 units, subtract 2 from each x-coordinate. To shift $\overline{AB}$ up 5 units, add 5 to each y-coordinate. You can describe this transformation as $\overline{AB}$ *is mapped onto* $\overline{A'B'}$, written symbolically as $\overline{AB} \to \overline{A'B'}$. In particular:

$$A(3, 0) \to A'(1, 5)$$
$$B(8, 2) \to B'(6, 7)$$

A **reflection** is a transformation in which each point of a figure has an image that is the same distance from the **line of reflection** as the original point but on the opposite side. A reflection preserves the size of a figure but not its orientation.

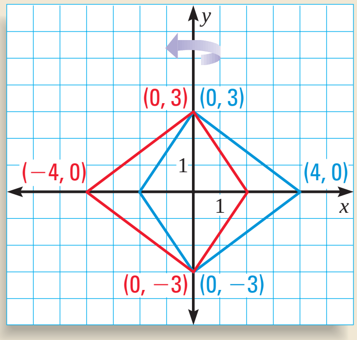
EXAMPLE **a.** Reflect the blue triangle over the x-axis. **b.** Reflect the blue kite over the y-axis.

SOLUTION **a.**



Change each y-coordinate to its opposite.

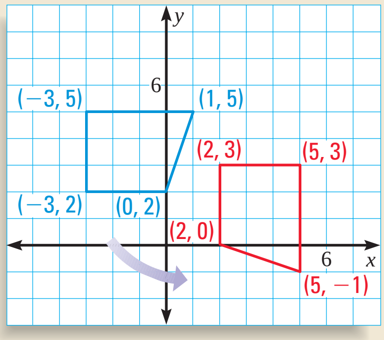
b.



Change each x-coordinate to its opposite.

A **rotation** is a transformation in which every point moves along a circular path around a fixed point. Rotations preserve both size and orientation.

EXAMPLE Rotate the blue quadrilateral 270° about the origin.



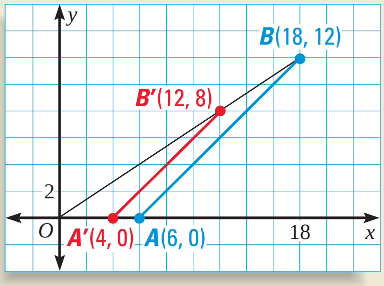
SOLUTION When a point (x, y) is rotated counterclockwise around the origin use the following patterns:

90° rotation: $A(x, y) \to A'(-y, x)$
180° rotation: $A(x, y) \to A'(-x, -y)$
270° rotation: $A(x, y) \to A'(y, -x)$

In this case, use the pattern for a 270° rotation about the origin. So, $(-3, 5) \to (5, 3)$, $(-3, 2) \to (2, 3)$, $(0, 2) \to (2, 0)$, and $(1, 5) \to (5, -1)$.

A **dilation** is a transformation in which every point of a figure is multiplied by a **scale factor** to create a similar image (see page 923). Dilations preserve the orientation of the original figure while enlarging or reducing the size.

EXAMPLE Dilate $\overline{AB}$ by a scale factor of $\frac{2}{3}$.



SOLUTION Multiply the coordinates of A and B by the scale factor $\frac{2}{3}$. Then graph points A' and B'.

$$A' = \left(\frac{2}{3} \cdot 6, \frac{2}{3} \cdot 0\right) = (4, 0)$$

$$B' = \left(\frac{2}{3} \cdot 18, \frac{2}{3} \cdot 12\right) = (12, 8)$$

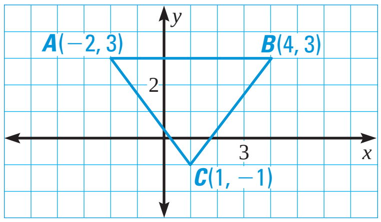
$\overline{A'B'}$ is $\frac{2}{3}$ as long as $\overline{AB}$.

PRACTICE

Give the coordinates of $A(3, -6)$ after the following transformations. For rotations, rotate about the origin.

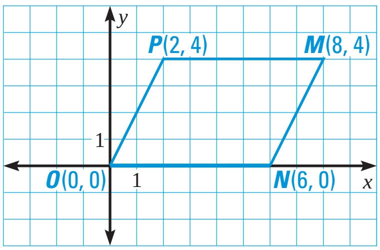
1. Reflect in y-axis.
2. Dilate by $\frac{2}{3}$.
3. Dilate by $\frac{3}{2}$.
4. Translate left 6 units.
5. Rotate 90°.
6. Translate up 6 units.
7. Reflect in x-axis.
8. Rotate 180°.
9. Translate left 3 units and up 5 units.
10. Translate right 2 units and up 1 unit.

Transform $\triangle ABC$. Graph the result. For rotations, rotate about the origin.



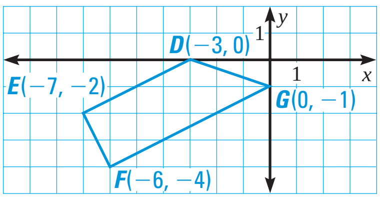
11. Translate down 1 unit.
12. Translate left 3 units.
13. Rotate 180°.
14. Dilate by $\frac{1}{4}$.
15. Reflect in the x-axis.
16. Translate right 2 units.
17. Dilate by 3.
18. Rotate 90°.

Transform *MNOP*. Graph the result. For rotations, rotate about the origin.



19. Dilate by $\frac{1}{2}$.
20. Translate left 1 unit and down 2 units.
21. Rotate 270°.
22. Translate down 4 units.
23. Reflect in the x-axis.
24. Dilate by $\frac{5}{2}$.
25. Reflect in the y-axis.
26. Reflect in the line $y = x$.

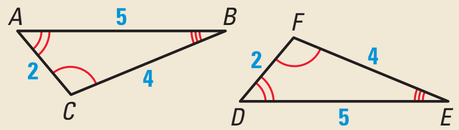
Transform *DEFG*. Graph the result. For rotations, rotate about the origin.



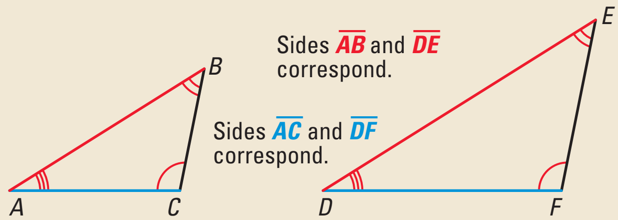
27. Translate up 4 units.
28. Reflect in the y-axis.
29. Rotate 90°.
30. Translate right 9 units.
31. Translate left 5 units.
32. Dilate by 4.

SIMILAR FIGURES

Two figures are **congruent** if they are exactly the same shape and the same size. Triangles *ABC* and *DEF* are congruent. Corresponding angles are marked with the same symbol.



Two figures are **similar** if corresponding angles are congruent and the lengths of corresponding sides are in proportion.



The ratios of the lengths of corresponding sides of similar figures are equal. For example, in the similar triangles above, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.

EXAMPLE The two polygons are similar. Find the values of *x* and *y*.

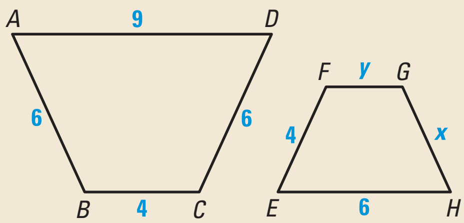
SOLUTION Write and solve a proportion to find each unknown length.

$\frac{AB}{EF} = \frac{CD}{GH}$ and $\frac{AD}{EH} = \frac{BC}{FG}$

$\frac{6}{4} = \frac{6}{x}$ $\quad$ $\frac{9}{6} = \frac{4}{y}$

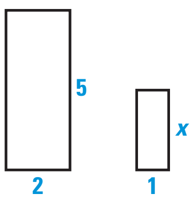
$6x = 24$ $\quad$ $9y = 24$

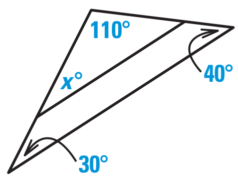
$x = 4$ $\quad$ $y = 2\frac{2}{3}$

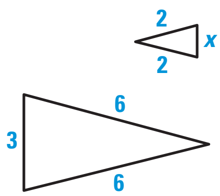


PRACTICE

The two polygons are similar. Find the value of *x*.

1.


2.


3.


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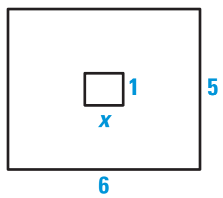

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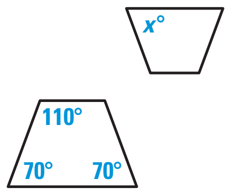

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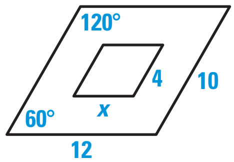

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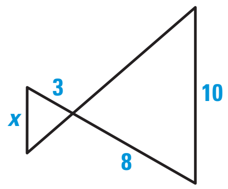

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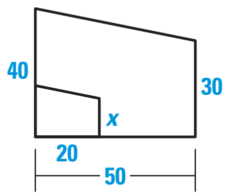


Logical Reasoning

LOGICAL ARGUMENT

A logical argument has two given statements, called **premises**, and a statement, called a **conclusion**, that follows them.

If a figure is a rhombus, then it is a parallelogram.	**Premise**
JKLM is a rhombus.	**Premise**
Therefore, JKLM is a parallelogram.	**Conclusion**

There are five types of logical arguments that can be made using these statements. Arguments that use these patterns correctly will have a **valid conclusion**. Arguments that use these patterns incorrectly will have an **invalid conclusion**. The letters p and q are often used to write an argument symbolically. In the examples below, p and q are given the following meanings.

p: a figure is a rhombus
q: a figure is a parallelogram

Direct Argument
If p is true, then q is true.
p is true.
Therefore, q is true.

Example:
If *JKLM* is a rhombus, then it is a parallelogram.
JKLM is a rhombus.
Therefore, *JKLM* is a parallelogram.

Indirect Argument
If p is true, then q is true.
q is not true.
Therefore, p is not true.

Example:
If *JKLM* is a rhombus, then it is a parallelogram.
JKLM is not a parallelogram.
Therefore, *JKLM* is not a rhombus.

Chain Rule
If p is true, then q is true.
If q is true, then r is true.
Therefore, if p, then r.

Example:
If *JKLM* is a rhombus, then it is a parallelogram.
If *JKLM* is a parallelogram, then it is a quadrilateral.
Therefore, if *JKLM* is a rhombus, then it is a quadrilateral.

***Or* Rule**
p is true or q is true.
p is not true.
Therefore, q is true.

Example:
JKLM is a rhombus or a parallelogram.
JKLM is not a rhombus.
Therefore, *JKLM* is a parallelogram.

***And* Rule**
p and q are not both true.
But q is true.
Therefore, p is not true.

Example:
JKLM is not both a rhombus and a parallelogram.
JKLM is a parallelogram.
Therefore, *JKLM* is not a rhombus.

EXAMPLE Use logical reasoning to decide whether the conclusion is *valid* or *invalid*. State the type of logical argument used to arrive at the conclusion.

a. If $x = 2$, then $3x - 1 = 5$. $3x - 1 \neq 5$. Therefore, $x \neq 2$.

b. If *ABCD* is a square, then it is a rectangle. *ABCD* is a rectangle. Therefore, *ABCD* is a square.

SOLUTION **a.** The conclusion is valid. This is an example of indirect argument.

b. The conclusion is invalid. This does not follow the pattern for a direct argument.

A compound statement has two or more parts joined by *or* or *and*. For an *and* statement to be true, each part must be true. For an *or* statement to be true, at least one part must be true.

EXAMPLE Tell whether the compound statement is *true* or *false*.

a. $2 < 3$ and $1 < 2$
b. $8 > 7$ and $7 > 9$
c. $-2 < 1$ or $-1 < 1$
d. $2 > 3$ or $1 > 2$

SOLUTION

a. True; both parts are true.
b. False; only one part is true, but both must be true for *and*.
c. True; at least one part is true, as required for *or*.
d. False; no part is true.

PRACTICE

Use logical reasoning to decide whether the conclusion is *valid* or *invalid*. State the type of logical argument used to arrive at the conclusion.

1. If Harold can drive, he has a license.
If Harold has a license, he is at least 16.
Therefore, if Harold is at least 16, he can drive.

2. If triangle *ABC* is equilateral, it is also isosceles.
Triangle *ABC* is equilateral.
Therefore, triangle *ABC* is isosceles.

3. John is in his room or John is in the kitchen.
John is not in the kitchen.
Therefore, John is in his room.

4. If Grace is 19, her twin brother Michael is also 19.
Michael is 17.
Therefore, Grace is not 19.

5. If $x = 4$, then $y = 1$.
If $y = 1$, then $z = 8$.
Therefore, if $x = 4$, then $z = 8$.

6. If an animal has a backbone, it is a vertebrate.
A horse has a backbone.
Therefore, a horse is a vertebrate.

7. If $x = 2$, then $2x = 4$.
$2x = 6$.
Therefore, $x = 2$.

8. If an apple is a Granny Smith, it is green.
The apple is green.
Therefore, the apple is a Granny Smith.

9. It is impossible for my watch to be right and that we are late. We are late.
Therefore, my watch cannot be right.

10. Triangle *ABC* is equilateral or scalene.
Triangle *ABC* is not scalene.
Therefore, triangle *ABC* is not equilateral.

State whether each compound statement is *true* or *false*.

11. $1 < 2$ and $8 \geq 5$

12. $5 < 1$ or $3 < 4$

13. $-6 \geq -6$ or $3 < -3$

14. $2 \leq 5$ and $2 \leq 1$

15. $-8 < 5$ and $-5 < 8$

16. $3 < 1$ or $3 < -1$

17. $4 = 4$ or $4 = 5$ or $4 = -4$

18. $-1 < 1$ and $1 \geq 0$ and $-1 < 0$

19. $8 < 9$ and $9 < 14$ and $14 < 20$

20. $3 > 4$ or $3 > 7$ or $3 > 6$

21. $-10 < -8$ and $-7 < -4$ and $7 > 4$

22. $-15 < -35$ or $0 \geq 1$ or $26 \geq 26$

23. $159 \leq 100$ or $100 < 159$

24. $47 \leq 48$ and $48 < 49$ and $49 > 47$

25. $95 \neq 95$ or $95 > -96$ or $95 > 94$

26. $5 \cdot 6 = 30$ or $-6 \cdot 5 = 30$

If-Then Statements

The **conditional** statement "if p, then q" has a **hypothesis** p and a **conclusion** q.

EXAMPLE Identify the hypothesis and conclusion.

a. $y = 6$ when $x = 5$.

b. Raspberries are a red fruit.

SOLUTION Rewrite the statement as an if-then statement.

a. If $x = 5$, then $y = 6$.
Hypothesis: $x = 5$
Conclusion: $y = 6$

b. If a fruit is a raspberry, then the fruit is red.
Hypothesis: a fruit is a raspberry
Conclusion: the fruit is red

The **converse** of the **conditional** statement "if p, then q" is "if q, then p."

EXAMPLE Give the converse of each statement. State whether the converse is *true* or *false*.

a. If $x = 8$, then $2x = 16$.

b. If a fruit is a raspberry, then the fruit is red.

SOLUTION Reverse the hypothesis and conclusion of each statement.

a. If $2x = 16$, then $x = 8$. True

b. If a fruit is red, then the fruit is a raspberry. False

When a conditional statement and its converse are combined by "if and only if," the resulting statement is called a **biconditional statement**. The biconditional "p *if and only if* q" is true only when the conditional "if p, then q" and its converse "if q, then p" are *both* true.

EXAMPLE Tell whether the statement is *true* or *false*. If false, tell why.

a. A parallelogram is a rectangle if and only if it has four right angles.

b. A triangle is an equilateral triangle if and only if it has two equal sides.

SOLUTION

a. True

b. False. It is true that an equilateral triangle has two equal sides (it has three), but not that a triangle with two equal sides must be equilateral (for example, a 5-5-2 triangle).

Practice

Rewrite the statement as an if-then statement.

1. The rain in Spain falls on the plain.
2. A rhombus is a parallelogram with four equal sides.
3. $3x^2 = 48$ when $x = 4$.
4. The area of a square is given by the formula $A = s^2$.
5. You can go out tonight if you finish cleaning.
6. Luis will earn $50 for baby-sitting 12 hours.
7. $y = 16$ when $x = 3$.
8. Corresponding angles of similar figures are congruent.
9. A square is a rectangle with four equal sides.
10. He earns a bonus for sales over $10,000 each month.
11. The graph of $y = x^2$ is a parabola.
12. The circumference of a circle is π times the diameter.

Give the converse of each statement. State whether the converse is *true* or *false*.

13. If $x = 4$, then $x^2 = 16$.

14. If you live in Ohio, then you live in the United States.

15. If a line is vertical, then its slope is undefined.

16. If an animal is a pigeon, then it is a bird.

17. If a figure has two pairs of opposite congruent sides, then it is a parallelogram.

18. If you add two odd numbers, then the answer will be an even number.

19. If you are in Minnesota in January, then you will be cold.

20. If an animal is a dog, then it has four legs.

21. If Margot won the election, then she got more votes than her opponent.

22. If a triangle has three sides of different lengths, then it is a scalene triangle.

23. If a convex polygon has five equal sides, then it is a regular pentagon.

24. If $x = 3$, then $x - 2 = 1$.

Determine whether the statement is *true* or *false*. If false, tell why.

25. A figure is a square if and only if it has four equal sides.

26. Eric will win the election if and only if he receives 60% of the vote.

27. $x^2 = 25$ if and only if $x = -5$.

28. A quadrilateral is a trapezoid if and only if it has exactly one pair of parallel opposite sides.

29. $2x + 6 = 6$ if and only if $x = 0$.

30. An animal is a cat if and only if it is a mammal.

31. Corresponding angles of figures are congruent if and only if the figures are similar.

32. A triangle is isosceles if and only if it has two equal angles.

33. Your team wins at basketball if and only if your team scores more points than your opponents.

34. $x^3 = 8$ if and only if $x = 2$.

35. You are north of the equator if and only if you are in the northern hemisphere.

36. A polygon is a decagon if and only if it has 10 sides.

COUNTEREXAMPLES

A **counterexample** disproves a logical statement.

EXAMPLE Is it true that when $|a| < |b|$, $a < b$?

SOLUTION No, because when $a = 1$ and $b = -2$, $|a| < |b|$, but $a > b$.

PRACTICE

Determine whether each statement is *true* or *false*. If false, give a counterexample.

1. If a quadrilateral is a parallelogram, then it is a rectangle.
2. If Joe has \$5, then he earned it mowing lawns.
3. If the last digit of a number is 6, then it is divisible by 3.
4. If a triangle is equilateral, then it is equiangular.
5. If a triangle contains a 90° angle, then it contains another 90° angle.
6. If a parallelogram has four right angles, then it is a square.
7. If an animal is black, then it is a dog.
8. If you live in California, then you live in the Pacific time zone.
9. If two lines are perpendicular, then they form right angles.
10. If two lines in the same plane are intersected by a transversal and alternate interior angles are equal in measure, then the lines are parallel.
11. If $a > 0$, then $3a - 4 > 0$.
12. If $a < b$, then $2a < 2b$.
13. If $c = d$, then $c - 2 = d - 2$.
14. If $x > 0$, then $x^2 > x$.
15. If $x \leq 0$, then $x^2 \geq x$.
16. If $x \leq 0$, then $2x^2 \geq -4x$.

JUSTIFY REASONING

Algebraic reasoning can be justified using the postulates of algebra.

Postulates of Algebra	Statement of Postulate	Example
ADDITION/SUBTRACTION PROPERTY OF EQUALITY	If the same number is added to (or subtracted from) equal numbers, then the sums (differences) are equal.	$x - 2 = 4$ $x - 2 + 2 = 4 + 2$
MULTIPLICATION/DIVISION PROPERTY OF EQUALITY	If equal numbers are multiplied by (or divided by) the same number, then the products (quotients) are equal.	$3x = -9$ $\frac{3x}{3} = \frac{-9}{3}$
SUBSTITUTION PROPERTY	If values are equal, then one value may be substituted for the other.	$x = y - 1$ and $x = 2$ $2 = y - 1$
DISTRIBUTIVE PROPERTY	$a(b + c) = ab + ac$	$3(2x - 1) = 3(2x) + 3(-1)$

You may also use algebraic definitions, such as the definition of raising to a power, to justify algebraic reasoning.

EXAMPLE Solve the equation $2x + 5 = 3$ and justify each step.

SOLUTION

$2x + 5 = 3$ — **Given**

$2x = -2$ — **Subtraction property of equality**

$x = -1$ — **Division property of equality**

PRACTICE

Identify the property that justifies the statement.

1. If $2x = 8$, then $x = 4$.

2. If $4x - 1 = 7$, then $4x = 8$.

3. If $\frac{4}{5}x = 8$, then $4x = 40$.

4. If $x^2 = 36$, then $x = 6$ or $x = -6$.

5. If $3x = 9$, then $3x + 2 = 11$.

6. If $3x^2 - 6 = 21$, then $x^2 - 2 = 7$.

7. If $x = 4$, then $3x = 12$.

8. If $x(2x - 3) = 7$, then $2x^2 - 3x = 7$.

9. If $\sqrt{x} = 4$, then $x = 16$.

10. If $-5x = 0$, then $x = 0$.

11. If $4x + 7 = 9$, then $4x = 2$.

12. If $x^5 - 1 = 0$, then $x^5 = 1$.

13. If $\frac{1}{6}x = \frac{2}{3}$, then $x = 4$.

14. If $\sqrt{x} = \frac{3}{4}$, then $x = \frac{9}{16}$.

15. If $6 = x - 1$, then $x = 7$.

16. If $4(2x + 2) = 12$, then $2x + 2 = 3$.

17. If $\frac{2x}{9} = 3$, then $2x = 27$.

18. If $2 + x = 5$, then $1 + x = 4$.

19. If $2(3 - 5x) = 1$, then $6 - 10x = 1$.

20. If $6x = 1$, then $6x - 3 = -2$.

Solve each equation for *x* and justify each step of the solution.

21. $9x = 27$

22. $8 + x = 8$

23. $\frac{x}{2} + 5 = 0$

24. $2(3 - x) = 1$

25. $\frac{5x}{2} - 2 = -5$

26. $3x - 8 = 13$

27. $\frac{3x}{4} = 6$

28. $6 = \frac{1}{2}(x - 2)$

Problem Solving

TRANSLATING PHRASES INTO ALGEBRAIC EXPRESSIONS

To solve a problem algebraically, you often must translate a phrase into an algebraic expression.

EXAMPLE Write the given phrase as an algebraic expression.

a. 3 less than a number **b.** twice a number **c.** the quotient of x and y

SOLUTION

a. "Less than" indicates subtraction:

$x - 3$

b. "Twice" indicates multiplication by 2:

$2x$

c. "Quotient of" indicates division:

$\frac{x}{y}$

EXAMPLE Write an algebraic expression to answer the question.

a. You buy x pounds of apples at \$1.49 per pound. How much do you spend?

b. You have x dollars and spend \$17.38 on dinner. How much do you have left?

SOLUTION

a. The amount you spent is the product of the number of pounds of apples and the price per pound:

$1.49x$

b. The amount you have left is the difference between the amount you had before dinner and the cost of the dinner:

$x - 17.38$

PRACTICE

Write the given phrase as an algebraic expression.

1. 8 more than a number

2. 3 times a number

3. a number minus 49

4. a number divided by 100

5. a number multiplied by 7

6. one sixth of a number

7. $\frac{3}{4}$ of a number

8. $\frac{7}{8}$ more than a number

9. 90% of a number

10. 5 times a number

11. 4 less than a number

12. the cube of a number

13. 3 more than a number, all divided by 2

14. the square root of the product of 8 and a number

Write an expression to answer the question.

15. A triangle has base b and height 6. What is its area?

16. Violet pays $50 for a membership fee and $25 per dress rental. How much does she spend altogether for x rentals?

17. Serge has $67.39. He spends x dollars for a new book. How much does he have left?

18. How much is 4.5% sales tax on an item that costs x dollars?

19. You bicycle at x miles per hour for 2.5 hours. How far do you go?

20. You buy s sandwiches at $4.25 apiece and d drinks at $1 apiece. How much do you spend all together?

ADDITIONAL PROBLEM SOLVING STRATEGIES

When solving mathematical or real-life problems, you will find that different strategies are appropriate for different types of problems. Refer to Lesson 1.5 for examples of problem solving based on the strategies *use a verbal model*, *draw a diagram*, *look for a pattern*, and *guess, check, and revise*.

Make a List or Table

An organized list or table is helpful when enumerating possibilities.

EXAMPLE Andrea, Brian, and Colleen are going to have their picture taken. In how many ways can the three friends line up for the picture?

SOLUTION You may find a tree diagram useful in listing the possibilities.

PERSON ON LEFT	PERSON IN MIDDLE	PERSON ON RIGHT	POSSIBLE ARRANGEMENT
Andrea	Brian	Colleen	Andrea, Brian, Colleen
	Colleen	Brian	Andrea, Colleen, Brian
Brian	Andrea	Colleen	Brian, Andrea, Colleen
	Colleen	Andrea	Brian, Colleen, Andrea
Colleen	Andrea	Brian	Colleen, Andrea, Brian
	Brian	Andrea	Colleen, Brian, Andrea

▶ There are six ways that the three friends can line up for the picture.

Use a Formula

You may be given a formula or know one that applies to the situation.

EXAMPLE Carol biked 25 kilometers in 2 hours. What was her average speed?

SOLUTION The formula for speed is $s = \frac{d}{t}$ where s is speed, d is distance, and t is time.

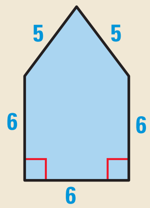
$$s = \frac{d}{t} = \frac{25 \text{ km}}{2 \text{ h}} = 12.5$$

▶ Carol's average speed was 12.5 kilometers per hour.

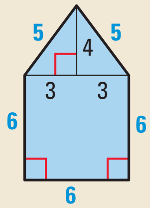
Break into Simpler Parts

You may want to break a difficult problem into more easily managed parts or cases. Be sure the parts or cases are *mutually exclusive* (that is, they do not overlap) and *collectively exhaustive* (that is, they cover all the possibilities).

EXAMPLE Find the area of the pentagon shown at the right.



SOLUTION Break the figure into a square and two right triangles as shown at the bottom right. Using the Pythagorean theorem, you find that the length of the leg shared by the two triangles is 4 units.



Area of pentagon = area of square + area of triangles

$$= 6^2 + 2\left[\frac{1}{2}(3)(4)\right]$$

$$= 36 + 12$$

$$= 48$$

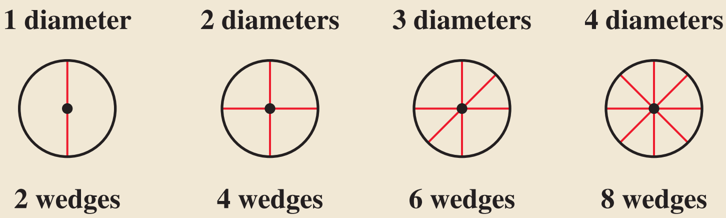
▶ The area of the pentagon is 48 square units.

Solve a Simpler Problem

You may try solving simpler problems and looking for a pattern in their solutions.

EXAMPLE When 15 diameters are drawn in a circle, into how many wedges is the circle divided?

SOLUTION Look for a pattern in the number of wedges when 1, 2, 3, and 4 diameters are drawn in a circle.



▶ The number of wedges is always twice the number of diameters. So, when 15 diameters are drawn in a circle, the circle is divided into 30 wedges.

PRACTICE

Use a list or table to solve the problem.

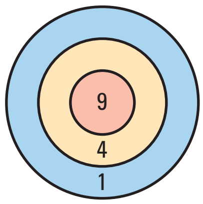
1. A frozen yogurt stand offers walnuts, peanuts, sprinkles, chocolate chips, and toffee bits as toppings. How many combinations of two different toppings are possible?
2. Kyle packs 4 pairs of pants and 6 shirts for a trip. How many different outfits are possible?
3. At a small theater, tickets for adults cost \$12 and tickets for children cost \$8. At one performance ticket sales were \$480. How many people may have attended the performance?

Use a formula to solve the problem.

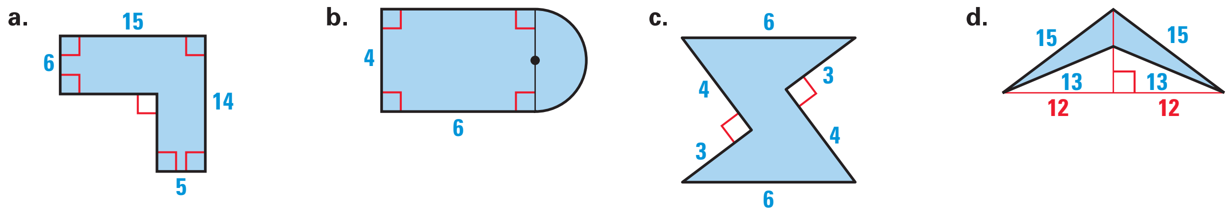
4. What is the area of a trapezoid with base lengths 7 inches and 11 inches and height 3 inches?
5. The formula $s = 32t$ gives the speed s (in feet per second) of an object falling without air resistance after t seconds have elapsed. How fast is a rock falling after 4 seconds?
6. You took two trips in your new car. The first trip covered 136 miles and used 6.4 gallons of gas. The second trip covered 285 miles and used 12.5 gallons of gas. On which trip did you get better gas mileage?

Solve the problem by breaking it into simpler parts or cases.

7. You throw three darts at the target shown. All three darts hit the target, and your score is the sum of the points that correspond to the regions where the darts land. What are the possibilities for your score?

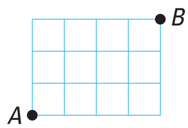


8. A painter is stenciling numbers, starting with 1, on the parking spaces in a parking lot. If there are 200 spaces, how many times does the painter stencil the digit 7?
9. In a best-of-five series, the first team to win three games wins the series. How many ways are there for a team to win a best-of-five series?
10. Find the area of the figure.



Solve the problem by first solving simpler problems.

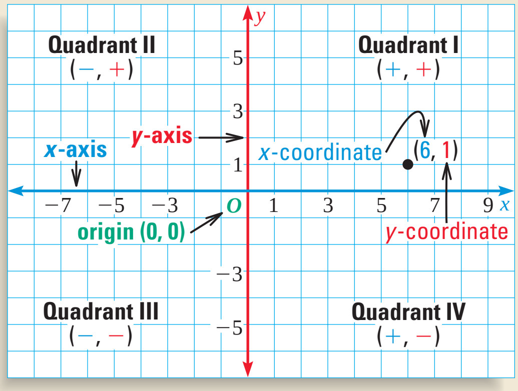
11. By moving only up and to the right, find the number of paths that lead from point A to point B on the grid shown.



12. How many diagonals does a convex polygon with 12 sides have?
13. How many squares of any integral size can you draw on an 8×8 grid?
14. Without using a calculator, find the value of $(100{,}000{,}001)^2$.

Graphing
POINTS IN THE COORDINATE PLANE

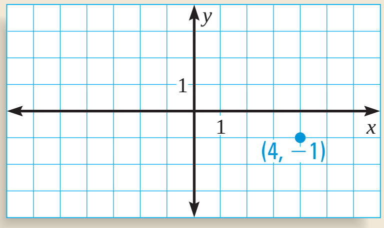
A **coordinate plane** is divided into four regions by the *x*-axis and *y*-axis. Each region is called a **quadrant**. A point in a coordinate plane can be represented by an **ordered pair** of numbers. The ***x*-coordinate** gives the horizontal position of the point. The ***y*-coordinate** gives the vertical position of the point. You can tell which quadrant a point is in by looking at the signs of its coordinates.



EXAMPLE Graph the point (4, −1). What quadrant is it in?

SOLUTION To graph the point, start at the origin and move 4 units to the right and 1 unit down.

(4, −1) is in Quadrant IV.



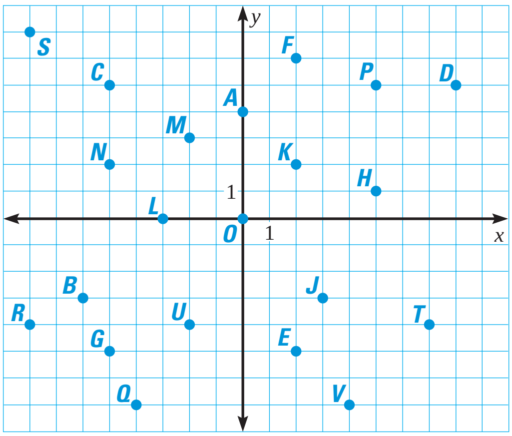
PRACTICE

Graph the point in a coordinate plane.

1. *A*(3, 4)	**2.** *B*(0, −4)	**3.** *C*(3, −1)	**4.** *D*(−4, 5)
5. *E*(−1, −1)	**6.** *F*(1, 1)	**7.** *G*(−6, −6)	**8.** *H*(0, 2)
9. *J*(1, 5)	**10.** *K*(−1, 0)	**11.** *L*(5, −2)	**12.** *M*(−2, −4)
13. *N*(0, 5)	**14.** *P*(−3, 2)	**15.** *Q*(3, 0)	**16.** *R*(−1, −3)

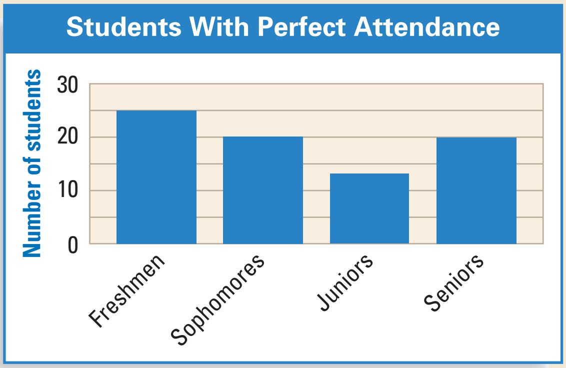
Give the coordinates and quadrant of each of the following points.

17. *A*	**18.** *B*	**19.** *C*
20. *D*	**21.** *E*	**22.** *F*
23. *G*	**24.** *H*	**25.** *J*
26. *K*	**27.** *L*	**28.** *M*
29. *N*	**30.** *O*	**31.** *P*
32. *Q*	**33.** *R*	**34.** *S*
35. *T*	**36.** *U*	**37.** *V*



BAR, CIRCLE, AND LINE GRAPHS

A **bar graph** is used to represent data that fall into distinct categories.



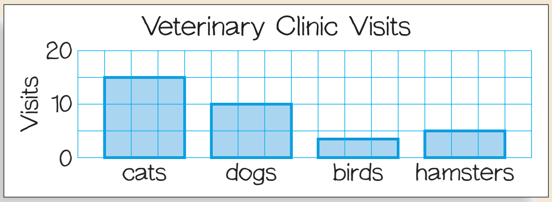
EXAMPLE According to the graph, how many juniors have perfect attendance?

SOLUTION The third bar represents juniors. Approximately 13 juniors have perfect attendance.

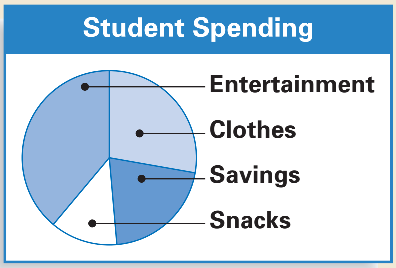
EXAMPLE Make a bar graph of the number of patients of a veterinary clinic. Represent each category with a bar.

Patients	Cats	Dogs	Birds	Hamsters
Visits	15	10	3	5

SOLUTION



A **circle graph** is used to show parts of a whole.



EXAMPLE According to the graph, for what category do students spend the most money? the least money?

SOLUTION Students spend the most money on entertainment and the least money on snacks.

EXAMPLE Make a circle graph of the favorite books that students read in English class this year.

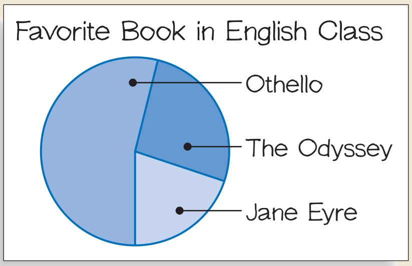
Favorite Book	Students
Othello	27
The Odyssey	13
Jane Eyre	10

SOLUTION There are 50 students altogether. Find the percent who chose each book, and use this to find the measure of the central angle for each category. Then draw the circle graph.

OTHELLO $\frac{27}{50} = 54\%$ and $0.54 \cdot 360° \approx 194°$

THE ODYSSEY $\frac{13}{50} = 26\%$ and $0.26 \cdot 360° \approx 94°$

JANE EYRE $\frac{10}{50} = 20\%$ and $0.20 \cdot 360° = 72°$

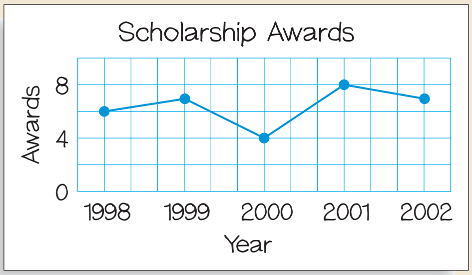


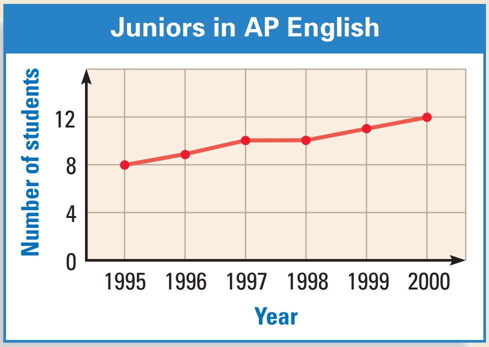
A **line graph** is often used to show change over time.

EXAMPLE Make a line graph of the number of scholarship awards given.

Year	1998	1999	2000	2001	2002
Awards	6	7	4	8	7

SOLUTION Graph the data in the table. Connect the data points from year to year.



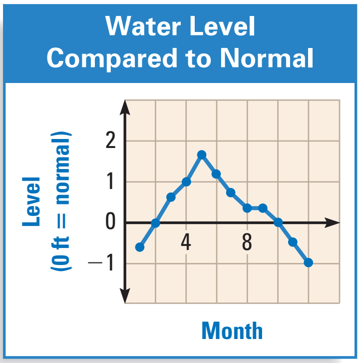


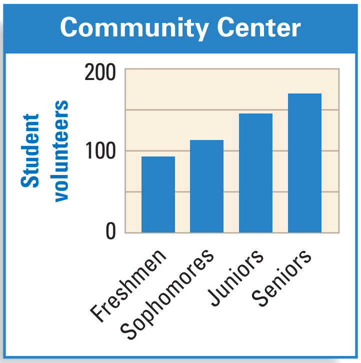
EXAMPLE According to the graph, how many juniors took AP English in 1999?

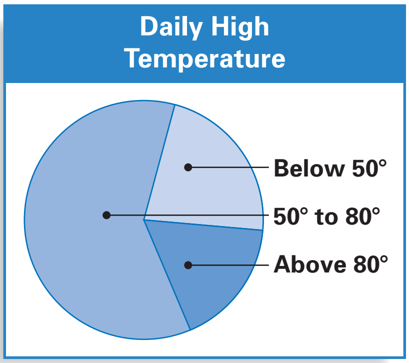
SOLUTION Find 1999 on the time axis, move up to the graph, and then move over to the student axis. Eleven juniors took AP English in 1999.

PRACTICE

Use the graphs to answer the questions.







1. What was the water level in June?
2. In what month was water level lowest?
3. How many sophomores volunteer at the community center?
4. How many more juniors than sophomores volunteer at the community center?
5. About what percent of the year is the average daily temperature above 80°?
6. Does the temperature graph support the statement that during most of the year the average temperature is above 50°?

7. Make a bar graph to represent choices of college majors.

College majors	English	History	Mathematics	Biology	Economics
Number of students	65	37	40	70	45

8. Make a circle graph to represent the favorite sports of students in a class.

Favorite sports	Basketball	Baseball	Hockey	Soccer	Swimming
Number of students	16	10	4	7	5

9. Make a line graph to represent the number of visitors to a city zoo.

Month	January	February	March	April	May
Number of visitors	450	400	410	470	500

Algebra

OPPOSITES

Two numbers that have the same absolute value but opposite signs are **opposites**. Multiplication by -1 changes a number to its opposite.

EXAMPLE Find the opposite of the number.

a. 13 **b.** -4

SOLUTION **a.** $13(-1) = -13$ **b.** $(-4)(-1) = 4$

To find the opposite of an expression, use the distributive property to multiply each term in the expression by -1.

EXAMPLE Simplify the expression $-(3 - 2x)$.

SOLUTION
$$-(3 - 2x) = -1(3 - 2x) = (-1)(3) + (-1)(-2x)$$
$$= -3 + 2x$$

PRACTICE

Find the opposite of the number.

1. 3 **2.** -27 **3.** 150 **4.** -13

5. 4.3 **6.** -9.28 **7.** $\frac{3}{5}$ **8.** $-\frac{1}{2}$

Simplify the expression.

9. $-(2a + b)$ **10.** $-(y - x)$ **11.** $-(a - b - c)$ **12.** $-(2y - 3x)$

13. $-(2 + x)$ **14.** $-(-3 - 11x)$ **15.** $-(4x - 1)$ **16.** $-(-x + 13)$

Simplify the expression.

17. $-(x^2 + 2x - 4)$ **18.** $-(x - (-2y))$ **19.** $-x - (3y + x)$ **20.** $-(-(x - y + 3))$

21. $-(3x - y + 11z)$ **22.** $x - (7y - 4x)$ **23.** $-9(-4x + 6y)$ **24.** $5x - (-(x - y))$

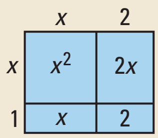
25. $-(2x + 7y) + 3x$ **26.** $4x - (-x + y)$ **27.** $a - (3b + 2a) + 6b$ **28.** $-2(-x + 3y - 6)$

MULTIPLYING BINOMIALS

A **binomial** is an expression that has two terms. You may find using a geometric model helpful when multiplying two binomials.

EXAMPLE Simplify $(x + 1)(x + 2)$.

SOLUTION Draw a rectangle and divide the width and length into two parts: x and 1 for the width, and x and 2 for the length. Divide the rectangle into four regions and find the area of each region. The product of $x + 1$ and $x + 2$ is the sum of the areas of the four regions.



$$\begin{aligned}(x + 1)(x + 2) &= x^2 + 2x + x + 2\\ &= x^2 + 3x + 2\end{aligned}$$

The **FOIL** method can also be used to multiply two binomials. **FOIL** stands for **F**irst, **O**uter, **I**nner, and **L**ast, which is the order in which you multiply terms.

$$\begin{aligned}(2x + 3)(x - 1) &= \overset{F}{2x(x)} + \overset{O}{2x(-1)} + \overset{I}{3(x)} + \overset{L}{3(-1)}\\ &= 2x^2 - 2x + 3x - 3\\ &= 2x^2 + x - 3\end{aligned}$$

EXAMPLE Simplify $(3x + 2)(x - 1)$.

SOLUTION

$$\begin{aligned}(3x + 2)(x - 1) &= 3x \cdot x + 3x \cdot (-1) + 2 \cdot x + 2 \cdot (-1) && \textbf{Multiply using FOIL.}\\ &= 3x^2 - 3x + 2x - 2 && \textbf{Simplify.}\\ &= 3x^2 - x - 2 && \textbf{Combine like terms}\end{aligned}$$

PRACTICE

Simplify.

1. $(x + 1)(x + 1)$ **2.** $(2 - 4x)(1 + 2x)$ **3.** $(4x + 1)(2 + x)$

4. $(3x + 2)(x - 1)$ **5.** $(1 - 2x)(x + 3)$ **6.** $(-2x + 1)(3x - 4)$

7. $(2x - 5)(2x + 5)$ **8.** $(6x + 3)(1 - x)$ **9.** $(5x + 3)(x - 2)$

10. $(a + 1)(b + 1)$ **11.** $(y + 3)(2y - 3)$ **12.** $(-x + 2)(-3x - 2)$

13. $(a + b)(c + d)$ **14.** $(x + 2)(x + 3)$ **15.** $(2x - 1)(-2x - 1)$

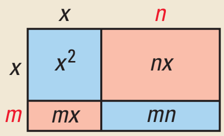
16. $(x + 2)(2x - y)$ **17.** $(x - y)(x + y)$ **18.** $(3y - a)(y + 3a)$

19. $(3x + 5)(x - 2)$ **20.** $(x + 0)(x - 1)$ **21.** $(-4x + 12)(3x + 8)$

FACTORING

To factor a polynomial of the form $x^2 + bx + c$, you may find a geometric model helpful.

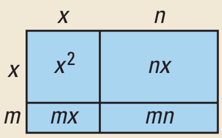
As when multiplying binomials, draw a rectangle and divide the width and length into two parts: x and m for the width, and x and n for the length. Divide the rectangle into four regions and find the area of each region. You can see that m and n must be factors of c and that the sum of m and n must be equal to b.



$$(x + m)(x + n) = x^2 + \underbrace{nx + mx}_{bx} + \underbrace{mn}_{c}$$
$$= x^2 + bx + c$$

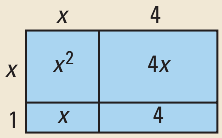
EXAMPLE Factor $x^2 + 5x + 4$.

SOLUTION



You want $mx + nx = 5x$ and $mn = 4$.

The integral factors of 4 are 1 and 4, −1 and −4, 2 and 2, and −2 and −2. Since $1x + 4x = 5x$, $m = 1$ and $n = 4$.



$x^2 + 5x + 4 = (x + 1)(x + 4)$

You can factor polynomials of the form $x^2 + bx + c$ without using a geometric model by listing the factors of c and finding the pair of factors that has a sum equal to b.

EXAMPLE Factor $x^2 - 2x - 15$.

SOLUTION You want $x^2 - 2x - 15 = (x + m)(x + n)$ where $mn = -15$ and $m + n = -2$.

Factors of −15	1, −15	−1, 15	3, −5	−3, 5
Sum of factors ($m + n$)	−14	14	−2	2

▶ The table shows that the values of m and n you want are $m = 3$ and $n = -5$. So, $x^2 - 2x - 15 = (x + 3)(x - 5)$.

PRACTICE

Factor.

1. $x^2 + 5x + 6$

2. $x^2 + 7x + 10$

3. $x^2 + 9x + 20$

4. $x^2 - 7x + 10$

5. $x^2 + 6x + 9$

6. $x^2 - 10x + 21$

7. $x^2 - 5x - 24$

8. $x^2 + 3x - 28$

9. $x^2 + 3x + 2$

10. $x^2 - 4x - 12$

11. $x^2 + x - 6$

12. $x^2 + 6x - 16$

13. $x^2 + 14x + 49$

14. $x^2 + 5x - 6$

15. $x^2 - 8x - 20$

16. $x^2 + 9x - 36$

17. $x^2 - 18x + 81$

18. $x^2 + 5x + 4$

19. $x^2 - 8x + 15$

20. $x^2 + 10x + 9$

21. $x^2 - 21x + 80$

22. $x^2 - 4x - 5$

23. $x^2 - 3x - 4$

24. $x^2 + 8x + 12$

25. $x^2 - 9x + 20$

26. $x^2 + 8x + 16$

27. $x^2 + 10x + 25$

28. $x^2 + x - 30$

29. $x^2 + 6x + 8$

30. $x^2 + 4x - 21$

LEAST COMMON DENOMINATOR

To add or subtract rational expressions with unlike denominators, first find the **least common denominator (LCD)** of the original rational expressions. To find the least common denominator of two rational expressions, follow these steps.

1. Factor each denominator. If a constant factor is negative, multiply the numerator and denominator of the expression by -1.
2. Find the *least common multiple* (*LCM*) of the constant factors in the factored denominators. (For help with LCM, see page 908.)
3. For each different variable factor that appears in any denominator, write the factor as many times as it appears in the denominator having the greatest number of that factor.
4. Write the LCD as the product of the results of Steps 2 and 3.

EXAMPLE Find the LCD of the pair of rational expressions.

a. $\frac{5}{6x^2}, \frac{x}{4x^2 - 12x}$

b. $\frac{x+1}{x^2 + 4x + 4}, \frac{2}{x^2 - 4}$

SOLUTION

a.
1. $6x^2 = 6 \cdot x \cdot x$
   $4x^2 - 12x = 4 \cdot x \cdot (x - 3)$
2. LCM of 6 and 4 is 12.
3. LCM of variable factors is $x \cdot x \cdot (x - 3)$.
4. LCD is $12x^2(x - 3)$.

▶ LCD of $\frac{5}{6x^2}$ and $\frac{x}{4x^2 - 12x}$ is $12x^2(x - 3)$.

b.
1. $x^2 + 4x + 4 = (x + 2) \cdot (x + 2)$
   $x^2 - 4 = (x + 2) \cdot (x - 2)$
2. LCM of 1 and 1 is 1.
3. LCM of variable factors is $(x + 2) \cdot (x + 2) \cdot (x - 2)$.
4. LCD is $(x + 2)^2(x - 2)$.

▶ LCD of $\frac{x+1}{x^2 + 4x + 4}$ and $\frac{2}{x^2 - 4}$ is $(x + 2)^2(x - 2)$.

PRACTICE

Find the least common denominator of the pair of rational expressions.

1. $\frac{1}{2x}, \frac{1}{2}$

2. $\frac{1}{3y}, \frac{3}{-4y}$

3. $\frac{2}{45k}, \frac{-1}{30k^2}$

4. $\frac{9}{z(z+1)}, \frac{15}{z^3}$

5. $\frac{4}{6x}, \frac{6}{2y}$

6. $\frac{5}{12a}, \frac{a}{9}$

7. $\frac{10}{3z}, \frac{1}{z^2}$

8. $\frac{-4}{9k}, \frac{1}{3k^2}$

9. $\frac{b}{b-1}, \frac{3}{(b+1)^2}$

10. $\frac{7}{12d}, \frac{d+4}{-12d^2}$

11. $\frac{-3n+4}{2n+4}, \frac{5}{n+2}$

12. $\frac{w}{w+9}, \frac{w-1}{w^2+18w+81}$

13. $\frac{x-11}{6+9x}, \frac{-x+11}{18x+12}$

14. $\frac{5g}{3g^3-21}, \frac{g^3+3}{2g^3-14}$

15. $\frac{h}{20-15h}, \frac{9h}{6h-8}$

16. $\frac{7-q^2}{q-8q^3}, \frac{5q^2}{24q^3-3q}$

17. $\frac{7e}{6-10e}, \frac{e+9}{12-20e}$

18. $\frac{4x}{5x^3-20x}, \frac{9-x}{4x-x^3}$

19. $\frac{12c+1}{c^3-4c^2}, \frac{c^3-3}{c^2-c^3}$

20. $\frac{5}{6-2p^5}, \frac{7}{-2p^5+6}$

21. $\frac{7}{36x^2}, \frac{11}{-9x}$

22. $\frac{16}{x-5}, \frac{6}{7x-35}$

23. $\frac{x+4}{3x^2+2x}, \frac{6}{3x+2}$

24. $\frac{6x+9}{2x^3+5x^2}, \frac{15}{-4x^2}$