Algebra 2

Applications • Equations • Graphs

Chapter 5 Resource Book

The Resource Book contains the wide variety of blackline masters available for Chapter 5. The blacklines are organized by lesson. Included are support materials for the teacher as well as practice, activities, applications, and assessment resources.

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Descriptions of Resources

This Chapter Resource Book is organized by lessons within the chapter in order to make your planning easier. The following materials are provided:

Tips for New Teachers These teaching notes provide both new and experienced teachers with useful teaching tips for each lesson, including tips about common errors and inclusion.

Parent Guide for Student Success This guide helps parents contribute to student success by providing an overview of the chapter along with questions and activities for parents and students to work on together.

Prerequisite Skills Review Worked-out examples are provided to review the prerequisite skills highlighted on the Study Guide page at the beginning of the chapter. Additional practice is included with each worked-out example.

Strategies for Reading Mathematics The first page teaches reading strategies to be applied to the current chapter and to later chapters. The second page is a visual glossary of key vocabulary.

Lesson Plans and Lesson Plans for Block Scheduling This planning template helps teachers select the materials they will use to teach each lesson from among the variety of materials available for the lesson. The block-scheduling version provides additional information about pacing.

Warm-Up Exercises and Daily Homework Quiz The warm-ups cover prerequisite skills that help prepare students for a given lesson. The quiz assesses students on the content of the previous lesson. (Transparencies also available)

Activity Support Masters These blackline masters make it easier for students to record their work on selected activities in the Student Edition.

Alternative Lesson Openers An engaging alternative for starting each lesson is provided from among these four types: ***Application, Activity, Graphing Calculator,*** or ***Visual Approach.*** (Color transparencies also available)

Graphing Calculator Activities with Keystrokes Keystrokes for four models of calculators are provided for each Technology Activity in the Student Edition, along with alternative Graphing Calculator Activities to begin selected lessons.

Practice A, B, and C These exercises offer additional practice for the material in each lesson, including application problems. There are three levels of practice for each lesson: A (basic), B (average), and C (advanced).

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Tips for New Teachers

For use with Chapter 5

Lesson 5.1

COMMON ERROR Some students do not know what to do when the x-coordinate of the vertex of a quadratic function is not an integer—for instance, for $y = 2x^2 + 5x - 7$ the vertex is at $\left(-\frac{5}{4}, -\frac{81}{8}\right)$. Remind students that the parabola is symmetric with respect to the vertical line with equation

$$x = \frac{-b}{2a}.$$

To graph the function, students must plot two points on one side of the axis of symmetry and then use symmetry to plot two points on the other side. However, if the x-coordinates of the first two points were integers, the x-coordinates of the symmetric points *will not* be integers. You might need to complete an example to show students how to find the coordinates of those points. Another option is to plot four *non-symmetrical* points of the parabola, two on each side of the axis of symmetry.

TEACHING TIP Show students that if a quadratic function is written in standard form, then c is the y-intercept for the graph of the function. This is always true because if $x = 0$, then $y = c$.

TEACHING TIP Students are already familiar with the equation and graph for an absolute value function of the form $y = a|x - h| + k$. Use this fact to introduce the vertex form of a quadratic equation, $y = a(x - h)^2 + k$. Compare these two functions and their graphs. Remind your students to be careful with the signs when they write the coordinates of the vertex of the graph.

Lesson 5.2

TEACHING TIP An alternative method to factor trinomials of the form $ax^2 + bx + c$ is the so-called *British method.* This method starts by finding the product $a \cdot c$—for $3x^2 - 17x + 10$ this product is 30. Then, find two factors of that product whose sum is b—in our example these factors are -2 and -15. Now rewrite the original trinomial using these factors—

$$3x^2 - 17x + 10 = 3x^2 - 2x - 15x + 10.$$

The final step is to use *grouping* to factor the resulting expression. In the example $x \cdot (3x - 2) - 5 \cdot (3x - 2)$, so $(x - 5) \cdot (3x - 2)$. This method also works when $a = 1$.

Lesson 5.3

COMMON ERROR Many students forget to take the negative root when they solve quadratic equations by finding square roots. Some of these students know that these equations usually have two solutions, but they mistakenly believe that such solutions are always opposite. For the equation $(x + 1)^2 = 16$ they might correctly find that $x = 3$ and then say the solutions are $x = \pm 3$. Remind your students to always use the symbol $\pm\sqrt{}$ to solve these problems to emphasize the need to take both the positive and the negative root. Show your students that the answers do not have to be opposite of each other and ask them to always check their answers.

Lesson 5.4

COMMON ERROR Students often forget to subtract the imaginary parts of two complex numbers when they find their difference. Remind them that they must subtract both their real parts and their imaginary parts.

TEACHING TIP Remind your students that they can use the *difference of squares pattern* to find the product of two complex conjugates. Better yet, use that pattern to show students that

$$(a + bi) \cdot (a - bi) = a^2 + b^2$$

Be careful to emphasize that this pattern can only be used when two complex conjugates are multiplied. For any other multiplication remind students they must use FOIL to find the answer.

Lesson 5.5

INCLUSION If you have students with learning disabilities, complete the activity on page 281 to provide them with a concrete model for *completing the square.* The algebra tiles will also help students to understand and memorize the steps followed in this process.

COMMON ERROR Students struggle finding the vertex form of a quadratic equation written in

Tips for New Teachers

For use with Chapter 5

standard form. Some of them might forget to factor a from the terms ax^2 and bx before proceeding to complete the square. Others might factor a, but forget to multiply the constant they add inside the parentheses by that same number a before adding that constant to the other side of the equation. Go over several of these problems with your students and emphasize the need to write all steps and to keep them organized.

Lesson 5.6

COMMON ERROR Students make all sorts of mistakes evaluating the quadratic formula. One error is to divide *only* the square root by $2a$, because students think that the quadratic formula is

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

or because they incorrectly use their calculators to evaluate the correct formula. You might want to spend some time solving easy quadratic equations already in standard form before moving on to more complicated ones, to make sure that students are able to evaluate the quadratic formula properly.

TEACHING TIP Take some time to review the five different methods covered in this chapter to solve quadratic equations—graphing, finding square roots, factoring, completing the square, and the quadratic formula. Ask your students when it is appropriate to use each of them and to make a list of pros and cons for each method.

TEACHING TIP Show students how to use the discriminant *when it is greater than zero* to decide whether the solutions of the corresponding quadratic equation will be rational or irrational numbers. Namely, if the discriminant is a perfect square, then the solutions will be rational numbers. Otherwise, they will be irrational numbers.

Lesson 5.7

TEACHING TIP Start this lesson by reviewing how to graph *linear* inequalities in two variables. Then ask students how graphing *quadratic* inequalities will be different. Follow this same method to discuss how to solve *systems* of quadratic inequalities.

TEACHING TIP Complete an example of a quadratic inequality in one variable with only one *critical x-value* and one with no *critical x-values*—for instance, $y \leq x^2 - x - 12$ and $y \leq x^2 + x + 2$. Solve them using the graphing method and discuss with your students whether they would be able to use the algebraic method. Talk about what kinds of solutions students can expect for these inequalities.

Lesson 5.8

TEACHING TIP Show your students that they can always write a system of three linear equations to find the standard form of a quadratic function given *any* three points on its graph. However, learning how to use the *vertex* form and the *intercept* form can save them a lot of time, because these forms have only one unknown, a. You could spend some time reviewing the three methods students can use to solve a linear system with three equations: elimination, Cramer's rule, and matrices.

Outside Resources

BOOKS/PERIODICALS

Schultz, Harris S. "Using a Digital Camera to Verify Quadratic Behavior." *Mathematics Teacher* (April 1999); pp. 292–293.

SOFTWARE

Harvey, Wayne, Judah Schwartz, and Michael Yerushalmy. *Visualizing Algebra: The Function Analyzer.* Pleasantville, NY; Sunburst Communications.

ACTIVITIES/MANIPULATIVES

Craine, Timothy V. "A Graphical Approach to the Quadratic Formula." *Activities: Mathematics Teacher* (January 1996); pp. 34–38, 44–46.

Breuningsen, Chris, Bill Bower, Linda Antinone, and Elisa Breuningsen. "That's the Way the Ball Bounces." *Real-World Math with the CBL System.* Activity 9: pp. 49–54. Texas Instruments, 1995.

CHAPTER 5

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Parent Guide for Student Success

For use with Chapter 5

Chapter Overview One way that you can help your student succeed in Chapter 5 is by discussing the lesson goals in the chart below. When a lesson is completed, ask your student to interpret the lesson goals for you and to explain how the mathematics of the lesson relates to one of the key applications listed in the chart.

Lesson Title	*Lesson Goals*	*Key Applications*
5.1: Graphing Quadratic Functions	Graph quadratic functions and use quadratic functions to solve real-life problems.	• Civil Engineering • Automobiles • Physiology
5.2: Solving Quadratic Equations by Factoring	Factor to solve quadratic equations. Find zeros of quadratic functions.	• Business • Environment • Home Electronics
5.3: Solving Quadratic Equations by Finding Square Roots	Solve quadratic equations by finding square roots and use quadratic equations to solve real-life problems.	• History • Astronomy • Television
5.4: Complex Numbers	Solve quadratic equations with complex solutions and perform operations with complex numbers.	• Fractal Geometry • Mandelbrot Set • Electricity
5.5: Completing the Square	Solve equations by completing the square and write quadratic functions in vertex form.	• Traffic Engineering • Sports • Firefighting
5.6: The Quadratic Formula and the Discriminant	Solve equations using the quadratic formula. Use the formula to solve real-world problems.	• Entertainment • Aviation • Earth Science
5.7: Graphing and Solving Quadratic Inequalities	Graph quadratic inequalities in two variables. Solve quadratic inequalities in one variable.	• Driving • Theater • Forestry
5.8: Modeling with Quadratic Functions	Write functions given characteristics of their graphs. Use technology to find quadratic models for data.	• Fuel Economy • Botany • Baseball

Study Strategy

Troubleshooting is the study strategy featured in Chapter 5 (see page 248). Encourage your student to look back over each lesson and try to solve exercises that were initially too difficult. Give what help you can or suggest that your student ask a classmate or the teacher for help.

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Parent Guide for Student Success

For use with Chapter 5

Key Ideas **Your student can demonstrate understanding of key concepts by working through the following exercises with you.**

Lesson	Exercise
5.1	A company sells posters. The company's monthly revenue R can be modeled by the function $R = 4000 + 600x - 100x^2$, where x is the dollar increase in the selling price over \$4. What selling price maximizes revenue and what is the maximum revenue?
5.2	Write the quadratic function $y = 2x^2 + 6x - 20$ in intercept form and give the zeros of the function.
5.3	A 12 by 30 foot rectangular garden has a diagonal path from corner to corner with length c. According to the Pythagorean Theorem, $c^2 = 1044$. Find the length of the path to the nearest tenth of a foot.
5.4	Write the quotient $\frac{2 + 3i}{3 - i}$ as a complex number in standard form.
5.5	Write the quadratic function $y = x^2 - 6x + 8$ in vertex form. What is the vertex of the graph?
5.6	A 10 by 14 meter garden will have a sidewalk of width x along the two 14 meter sides and width $2x$ along the two 10 meter sides. Approximate the value of x that will make the area covered by the sidewalks and the garden total 174 square meters.
5.7	Solve $x^2 - 2x \leq 15$.
5.8	Write a quadratic function in standard form whose graph passes through the points (2, 1), (0, 3), and (−2, 9).

Home Involvement Activity

You Will Need: A tape measure

Directions: Measure the length and width of your living room to the nearest quarter of a foot. Suppose you want an area rug to fit in your living room so that the distance to the wall on either end of the rug is twice the distance to the wall on either side of the rug. You also want the area of the rug to be 80% of the area of your living room. What should be the dimensions of the rug?

Answers

5.1: \$7, \$4900 **5.2:** $y = 2(x - 2)(x + 5)$, 2, −5 **5.3:** 32.3 ft **5.4:** $\frac{3}{10} + \frac{11}{10}i$

5.5: $y = (x - 3)^2 - 1$; (3, −1) **5.6:** about 0.47 m **5.7:** $-3 \leq x \leq 5$ **5.8:** $y = 0.5x^2 - 2x + 3$

CHAPTER 5

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Prerequisite Skills Review

For use before Chapter 5

EXAMPLE 1

Solving Linear Equations

Solve the equation.

a. $3(x + 7) = 15$ **b.** $4x + 1 = 8 - 6x$

SOLUTION

a. $3(x + 7) = 15$ Write original equation.

$3x + 21 = 15$ Distributive property

$3x = -6$ Subtract 21 from each side.

$x = -2$ Divide each side by 3.

The solution is -2. Check this in the original equation.

b. $4x + 1 = 8 - 6x$ Write original equation.

$10x + 1 = 8$ Add $6x$ to each side.

$10x = 7$ Subtract 1 from each side.

$x = \frac{7}{10}$ Divide each side by 10.

The solution is $\frac{7}{10}$. Check this in the original equation.

Exercises for Example 1

Solve the equation.

1. $5x - 4 = 11$ **2.** $-2(x - 3) = -10$ **3.** $-6x - 5 = 9x + 8$

EXAMPLE 2

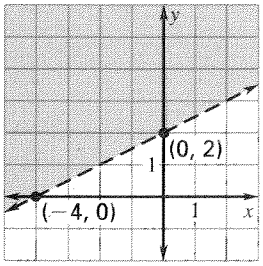
Graphing Linear Inequalities in Two Variables

Graph the inequality.

a. $-x + 2y > 4$ **b.** $4x - 3y \leq 6$

SOLUTION

a. *Graph* the boundary line $-x + 2y = 4$. Use a dashed line because $-x + 2y > 4$. *Test* the point $(0, 0)$. Because $(0, 0)$ is not a solution of the inequality, shade the half-plane above the line.



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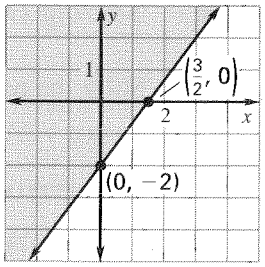
Prerequisite Skills Review

For use before Chapter 5

b. *Graph* the boundary line $4x - 3y = 6$. Use a solid line because $4x - 3y \leq 6$. *Test* the point $(0, 0)$. Because $(0, 0)$ is a solution of the inequality, shade the half-plane above the line.



Exercises for Example 2

Graph the inequality.

4. $3x + 2y > -4$

5. $x - 4y \leq -12$

6. $5x + y \geq 5$

7. $-3x - 6y < 18$

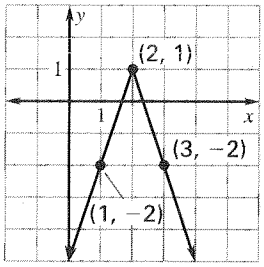
EXAMPLE 3

Graphing an Absolute Value Function

Graph $y = -3|x - 2| = 1$.

SOLUTION

To graph $y = -3|x - 2| + 1$, plot the vertex $(2, 1)$. Then plot another point on the graph, such as $(1, -2)$. Use symmetry to plot a third point, $(3, -2)$. Connect these three points with a V-shaped graph. Note that $a = -3 < 0$ and $|a| = 3$, so the graph opens down and is narrower than the graph of $y = -|x|$.



Exercises for Example 3

Graph the function and label the vertex.

8. $y = |x + 4|$

9. $y = 4|x + 3| - 2$

10. $y = -|x - 5| + 2$

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Strategies for Reading Mathematics

For use with Chapter 5

Strategy: Reading Properties

When a new property is introduced, you need to understand what it says and how it is applied. Consider the properties of square roots introduced on p. 264.

In the activity you may have discovered the following properties of square roots. You can use these properties to simplify expressions containing square roots.

Properties of Square Roots $(a > 0, b > 0)$

Product Property: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ **Quotient Property:** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

A square root expression is considered simplified if (1) no radicand has a perfect-square factor other than 1, and (2) there is no radical in a denominator.

EXAMPLE **Simplify the expression.**

a. $\sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$ **b.** $\sqrt{6} \cdot \sqrt{15} = \sqrt{90} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$

c. $\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}} = \frac{\sqrt{7}}{4}$ **d.** $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$

STUDY TIP

Reading Properties

Pay careful attention to the introduction of the property, its definition, and the examples showing how it is applied. These will all help you understand how to use a new property.

Questions

1. Rewrite parts (a) and (c) of the example above, using the properties of square roots to justify each step.
2. In what way does the expression given in part (d) need to be simplified?

   Simplify $\sqrt{\frac{13}{5}}$.
3. Is the expression $\sqrt{125}$ simplified? Why or why not? If not, use the properties of square roots to write it in simplest form.
4. Simplify each expression.

   a. $\sqrt{50}$ **b.** $\sqrt{10} \cdot \sqrt{22}$ **c.** $\sqrt{\frac{3}{49}}$ **d.** $\frac{1}{\sqrt{3}}$

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Strategies for Reading Mathematics

For use with Chapter 5

Visual Glossary

The Study Guide on page 248 lists the key vocabulary for Chapter 5 as well as review vocabulary from previous chapters. Use the page references on page 248 or the Glossary in the textbook to review key terms from prior chapters. Use the visual glossary below to help you understand some of the key vocabulary in Chapter 5. You may want to copy these diagrams into your notebook and refer to them as you complete the chapter.

GLOSSARY

quadratic function (p. 249)
A function of the form $y = ax^2 + bx + c$ where $a \neq 0$. Its $\cup$-shaped graph is called a parabola.

vertex of a parabola (p. 249)
The point on a parabola that lies on the axis of symmetry. This point is the lowest or highest point on a parabola with a vertical axis of symmetry.

axis of symmetry (p. 249)
The axis of symmetry is the vertical line through the vertex of the graph of a quadratic function.

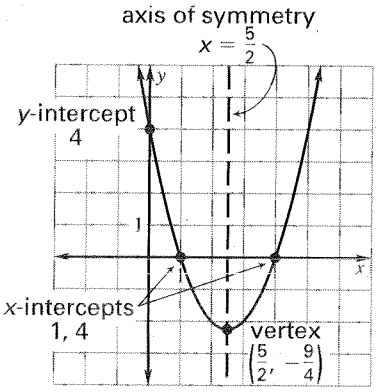
quadratic formula (p. 291)
A formula that gives the solutions of any quadratic equation. If a, b, and c are real numbers with $a \neq 0$, the solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

discriminant (p. 293)
The expression $b^2 - 4ac$ under the radical sign in the quadratic formula.

Graphing a Quadratic Function

The graph of a quadratic function is a smooth curve called a parabola. To graph a parabola, locate the axis of symmetry and the point where the parabola intersects it, plot a few additional points, and connect them with a smooth curve.



$y = x^2 - 5x + 4$

vertex at $x = -\frac{-5}{2} = \frac{5}{2}$

$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 4 = -\frac{9}{4}$

Using the Quadratic Formula

The discriminant, $D = b^2 - 4ac$, shows the number and type of roots of a quadratic function. If $D > 0$, there are two distinct real roots. If $D = 0$, there is one real root. If $D < 0$, there are two distinct complex roots. The quadratic formula gives the solutions of an equation of the form $ax^2 + bx + c = 0$.

The roots of $3x^2 - 6x + 2 = 0$ are:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{12}}{6}$$ ← Discriminant $12 > 0$. 2 real roots

$$= 1 \pm \frac{\sqrt{3}}{3}$$