LESSON 5.3

TEACHER'S NAME ____________ CLASS ______ ROOM ______ DATE ______

Lesson Plan for Block Scheduling

Half-day lesson (See *Pacing the Chapter,* TE pages 246C–246D) **For use with pages 264–271**

GOALS
1. **Solve quadratic equations by finding square roots.**
2. **Use quadratic equations to solve real-life problems.**

State/Local Objectives ____________

CHAPTER PACING GUIDE	
Day	**Lesson**
1	5.1 (all)
2	5.2 (all)
3	**5.3 (all)**; 5.4 (begin)
4	5.4 (end); 5.5 (begin)
5	5.5 (end); 5.6 (all)
6	5.7 (all); 5.8 (all)
7	Review/Assess Ch. 5

✓ Check the items you wish to use for this lesson.

STARTING OPTIONS
____ Homework Check: TE page 260; Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 264 and 263, CRB page 38, or Transparencies

TEACHING OPTIONS
____ Motivating the Lesson: TE page 265
____ Lesson Opener (Application): CRB page 39 or Transparencies
____ Graphing Calculator Activity with Keystrokes: CRB page 40
____ Examples 1–4: SE pages 264–266
____ Extra Examples: TE pages 265–266 or Transparencies; Internet
____ Technology Activity: SE page 271
____ Closure Question: TE page 266
____ Guided Practice Exercises: SE page 267

APPLY/HOMEWORK
Homework Assignment (See also the assignment for Lesson 5.4.)
____ Block Schedule: 20–68 even, 69–73 odd, 74, 77–91 odd; Quiz 1: 1–11

Reteaching the Lesson
____ Practice Masters: CRB pages 41–43 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 44–45 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson
____ Applications (Real-Life): CRB page 47
____ Math & History: SE page 270; CRB page 48; Internet
____ Challenge: SE page 269; CRB page 49 or Internet

ASSESSMENT OPTIONS
____ Checkpoint Exercises: TE pages 265–266 or Transparencies
____ Daily Homework Quiz (5.3): TE page 269, CRB page 53, or Transparencies
____ Standardized Test Practice: SE page 269; TE page 269; STP Workbook; Transparencies
____ Quiz (5.1–5.3): SE page 270; CRB page 50

Notes ____________

Lesson 5.3

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NAME ______________________ DATE __________

Available as a transparency

Warm-Up Exercises

For use before Lesson 5.3, pages 264–271

Solve the equation.

1. $5x - 3 = 17$

2. $0 = -12 + 3t$

Find the value of y when $x = 0$, 1, and 2.

3. $y = -16x^2 + 24$

4. $y = -18x^2 + 321$

Daily Homework Quiz

For use after Lesson 5.2, pages 256–263

Factor the quadratic expression.

1. $x^2 - 14x - 15$

2. $5x^2 + 4x - 12$

3. $36x^2 - 49$

4. $25x^2 - 10x + 1$

Solve.

5. $16x^2 + 24x + 9 = 0$

6. $14x^2 + 11x + 3 = 2x^2 - 3x + 3$

7. Find the zeros of $y = 8x^2 - 18x$.

Lesson 5.3

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Available as a transparency

Application Lesson Opener

For use with pages 264–270

A number r is called a square root of s if $r^2 = s$. The notation $\sqrt{s}$ represents the positive square root of s.

According to the Vickers scale, the hardness H of a mineral is determined by the formula $Hd^2 = 1.89$, where d is the depth (in millimeters) of an indentation formed by hitting the mineral with a pyramid-shaped diamond.

If you know H, you can use a square root to solve the equation for d. Here is how to find the depth of indentation for a mineral whose hardness is 131:

$Hd^2 = 1.89$	Write the formula.
$131d^2 = 1.89$	Substitute 131 for H.
$d^2 = \frac{1.89}{131}$	Divide each side by 131.
$d = \pm\sqrt{\frac{1.89}{131}}$	Take square roots of each side.
$d \approx 0.12$	Use a calculator to find the *positive* square root.

The depth of an indentation is about 0.12 millimeter.

Use a calculator to find the depth of an indentation for each mineral. Round to the nearest hundredth of a millimeter.

1. Copper: $H = 140$

2. Galena: $H = 80$

3. Platinum: $H = 125$

4. Gold: $H = 50$

5. Hematite: $H = 755$

6. Graphite: $H = 12$

Lesson 5.3

LESSON 5.3

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Graphing Calculator Activity Keystrokes

For use with page 271

TI-82

Enter the function.

Y= 2 (X,T,θ − 3) x^2 − 5

Set the viewing window and graph.

WINDOW ENTER (-) 4 ENTER 8 ENTER 1 ENTER (-) 6 ENTER 4 ENTER 1 ENTER GRAPH

Find zero near $x \approx 1.4$:

2nd [CALC] 2

Use cursor keys, ◀ and ▶, to move the trace cursor to select the lower bound at $x \approx 1.2$. Press ENTER. Move the trace cursor to select the upper bound at $x \approx 1.6$. Press ENTER. Move the trace cursor to select the guess at $x \approx 1.4$. Press ENTER.

Find zero near $x \approx 4.6$

2nd [CALC] 2

Select the lower bound at $x \approx 4.4$. Press ENTER.

Select the upper bound at $x \approx 4.8$. Press ENTER.

Select the guess at $x \approx 4.6$. Press ENTER.

TI-83

Enter the function.

Y= 2 (X,T,θ,n − 3) x^2 − 5

Set the viewing window and graph.

WINDOW (-) 4 ENTER 8 ENTER 1 ENTER (-) 6 ENTER 4 ENTER 1 ENTER GRAPH

Find zeros near $x \approx 1.4$

2nd [CALC] 2

Use the keypad to select the lower bound at $x \approx 1.2$. 1.2 ENTER

Use the keypad to select the upper bound at $x \approx 1.6$. 1.6 ENTER

Use the keypad to select the guess at $x \approx 1.4$. 1.4 ENTER

Find zeros near $x \approx 4.6$:

2nd [CALC] 2

Select the lower bound $x \approx 4.4$. 4.4 ENTER

Select the upper bound at $x \approx 4.8$. 4.8 ENTER

Select the guess at $x \approx 4.6$. 4.6 ENTER

SHARP EL-9600c

Enter the function.

Y= 2 (X/θ/T/n − 3) x^2 − 5

Set the viewing window and graph.

WINDOW (-) 4 ENTER 8 ENTER 1 ENTER (-) 6 ENTER 4 ENTER 1 ENTER GRAPH

Find zeros of the function.

2ndF [Calc] 5

2ndF [Calc] 5

CASIO CFX-9850GA PLUS

From the main menu, choose GRAPH.

Enter the function.

2 (X,θ,T − 3) x^2 − 5 EXE

Set the viewing window and graph.

SHIFT F3 (-) 4 EXE 8 EXE 1 EXE (-) 6 EXE 4 EXE 1 EXE EXIT F6

Find zeros of the function.

SHIFT F5 F1

Use the cursor keys, ◀ or ▶, to move the cursor to the next zero (if any).

Lesson 5.3

LESSON 5.3

NAME ______________________________ DATE __________

Practice A

For use with pages 264–270

Simplify the expression.

1. $\sqrt{32}$
2. $\sqrt{12}$
3. $\sqrt{45}$
4. $\sqrt{125}$
5. $2\sqrt{18} \cdot \sqrt{2}$
6. $\sqrt{54} \cdot 2\sqrt{6}$
7. $\sqrt{\frac{49}{4}}$
8. $\sqrt{\frac{100}{9}}$
9. $\sqrt{\frac{1}{121}}$
10. $\sqrt{\frac{12}{25}}$
11. $\sqrt{\frac{72}{5}}$
12. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{4}{3}}$

Solve the equation.

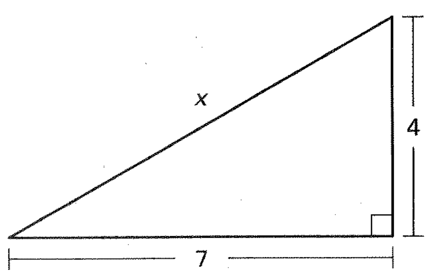
13. $x^2 = 9$
14. $x^2 = 144$
15. $x^2 = 128$
16. $x^2 - 36 = 0$
17. $x^2 - 1 = 0$
18. $x^2 - 8 = 0$
19. $2x^2 = 2$
20. $-4x^2 = -36$
21. $\frac{1}{2}x^2 = 32$
22. $x^2 - 3 = 1$
23. $x^2 + 2 = 7$
24. $16 - x^2 = -9$
25. $3x^2 - 1 = 5$
26. $\frac{1}{3}x^2 + 5 = 32$
27. $2x^2 - 11 = x^2 + 5$

Find the time it takes an object to hit the ground when it is dropped from a height of s feet. Use the falling-object model $h = -16t^2 + s$.

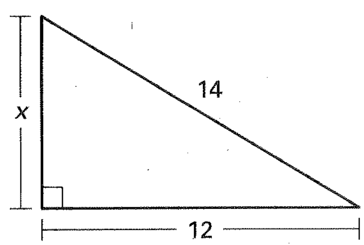
28. $s = 80$
29. $s = 160$
30. $s = 320$

Use the Pythagorean theorem to find x. Round your answer to the nearest hundredth.

31.



32.



33. ***Cost of a New Car*** From 1970 to 1990, the average cost of a new car, C (in dollars), can be approximated by the model $C = 30.5t^2 + 4192$, where t is the number of years since 1970. During which year was the average cost of a new car $12,000?

Lesson 5.3

LESSON 5.3

NAME ______________________________ DATE __________

Practice B

For use with pages 264–270

Simplify the expression.

1. $\sqrt{147}$
2. $\sqrt{60}$
3. $\sqrt{63}$
4. $4\sqrt{18} \cdot 2\sqrt{48}$
5. $\sqrt{8} \cdot \sqrt{18} \cdot 5\sqrt{4}$
6. $\sqrt{10} \cdot \sqrt{15}$
7. $\sqrt{\frac{225}{289}}$
8. $\sqrt{\frac{7}{3}} \cdot \sqrt{\frac{14}{3}}$
9. $\sqrt{15} \cdot \sqrt{\frac{35}{12}}$

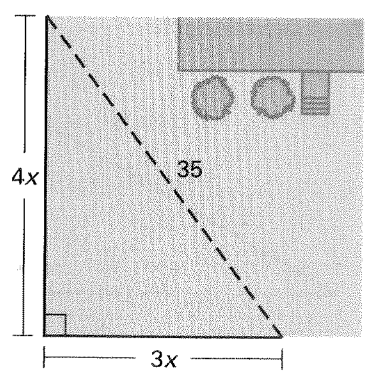
Solve the equation.

10. $x^2 = 324$
11. $x^2 - 81 = 0$
12. $5x^2 - 180 = 0$
13. $3x^2 - 100 = 332$
14. $\frac{2}{3}x^2 - 8 = 16$
15. $\frac{1}{2}x^2 - 5 = 5$
16. $x^2 + 1 = 3x^2 - 13$
17. $2(x^2 + 4) = 10$
18. $3(x^2 - 1) = 9$
19. $2(x + 3)^2 = 8$
20. $3(x - 2)^2 + 4 = 52$
21. $(3x + 1)^2 - 36 = 0$
22. $(2x - 3)^2 = 25$
23. $\frac{1}{2}(x - 4)^2 = 8$
24. $\frac{1}{4}(x + 1)^2 - 16 = 0$

25. ***Falling Object*** Use the falling-object model $h = -16t^2 + s$ where t is measured in seconds and h is measured in feet to find the time required for an object to reach the ground from a height of $s = 100$ feet and $s = 200$ feet. Does an object that is dropped from twice as high take twice as long to reach the ground? Explain your answer.

26. ***Truck Registrations*** From 1990 to 1993, the number of truck registrations (in millions) in the United States can be approximated by the model $R = 0.29t^2 + 45$ where t is the number of years since 1990. During which year were approximately 46.16 million trucks registered?

Short Cut Suppose your house is on a large corner lot. The children in the neighborhood cut across your lawn, as shown in the figure at the right. The distance across the lawn is 35 feet.



27. Use the Pythagorean theorem to find x.
28. Find the distance the children would have to travel if they did not cut across your lawn.
29. How many feet do the children save by taking the "short cut?"

LESSON 5.3

NAME ____________________ DATE __________

Practice C

For use with pages 264–270

Simplify the expression.

1. $\sqrt{294}$
2. $\sqrt{252}$
3. $3\sqrt{12} \cdot 5\sqrt{27}$
4. $\sqrt{21} \cdot \sqrt{24}$
5. $\sqrt{\frac{160}{162}}$
6. $\sqrt{35} \cdot \sqrt{21} \cdot \sqrt{7}$
7. $\sqrt{\frac{12}{27}} \cdot \sqrt{\frac{1}{4}}$
8. $\sqrt{\frac{20}{54}} \cdot \sqrt{\frac{8}{45}}$
9. $\sqrt{\frac{32}{15}} \cdot \sqrt{\frac{12}{5}} \cdot \sqrt{\frac{75}{72}}$

Solve the equation.

10. $x^2 - 289 = 0$
11. $x^2 - 13 = 0$
12. $\frac{1}{5}x^2 - 5 = 0$
13. $2x^2 - 10 = 0$
14. $\frac{1}{3}x^2 + 4 = 8$
15. $-\frac{2}{5}x^2 - 3 = -7$
16. $2x^2 + 7 = x^2 + 12$
17. $3(x^2 - 4) = 2x^2 - 1$
18. $2(x - 1)^2 = 8$
19. $3(x + 4)^2 = 9$
20. $5(x + 1)^2 = \frac{1}{4}$
21. $-\frac{1}{4}(x + 3)^2 + 5 = 2$
22. $\frac{3}{2}(x - 2)^2 - 4 = 2$
23. $2(x + 5)^2 + \frac{2}{3} = \frac{3}{4}$
24. $3\left(x - \frac{2}{3}\right)^2 + 4 = 13$
25. $\frac{(x + 3)^2}{4} - \frac{1}{2} = 6$
26. $\frac{(x - 1)^2}{3} + \frac{1}{6} = \frac{5}{6}$
27. $\frac{2(x - 2)^2}{5} - \frac{1}{10} = \frac{3}{5}$

Find the values of *a* for which the equation has two real-number solutions.

28. $x^2 = a$
29. $x^2 + 2 = a$
30. $3x^2 - 1 = a$
31. $x^2 + a = 2$
32. $a - x^2 = 4$
33. $2x^2 + a = 5$

34. ***Compound Interest*** The formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ gives the amount of money in an account, A after t years if the annual interest rate is r (in decimal form), n is the number of times interest is compounded per year, and P is the original principle. What interest rate is required to earn \$1 in two months if the principle is \$100 and interest is compounded monthly?

LESSON 5.3

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 264–270

GOAL **Solve quadratic equations by finding square roots and use quadratic equations to solve real-life problems**

VOCABULARY

If $b^2 = a$, then b is a **square root** of a. A positive number a has two square roots, $\sqrt{a}$ and $-\sqrt{a}$. The symbol $\sqrt{\ }$ is a **radical sign,** a is the **radicand,** and $\sqrt{a}$ is a **radical.**

Rationalizing the denominator is the process of eliminating square roots in the denominator of a fraction.

EXAMPLE 1 *Using Properties of Square Roots*

Simplify the expression.

a. $\sqrt{99} = \sqrt{9} \cdot \sqrt{11} = 3\sqrt{11}$

b. $\sqrt{6} \cdot \sqrt{8} = \sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

c. $\sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5}$

d. $\sqrt{\frac{36}{5}} = \frac{\sqrt{36}}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

Exercises for Example 1

Simplify the expression.

1. $\sqrt{60}$

2. $\sqrt{2} \cdot \sqrt{18}$

3. $\sqrt{\frac{81}{121}}$

EXAMPLE 2 *Solving a Quadratic Equation*

Solve $\frac{x^2}{6} - 4 = 10$.

SOLUTION

$\frac{x^2}{6} - 4 = 10$	Write original equation.
$\frac{x^2}{6} = 14$	Add 4 to each side.
$x^2 = 84$	Multiply both sides by 6.
$x = \pm\sqrt{84}$	Take square roots of both sides.
$x = \pm 2\sqrt{21}$	Simplify.

The solutions are $2\sqrt{21}$ and $-2\sqrt{21}$.

Exercises for Example 2

Solve the equation.

4. $4x^2 - 5 = -1$

5. $12 - 2y^2 = 4$

6. $\frac{p^2}{4} - 3 = 33$

Lesson 5.3

LESSON 5.3 CONTINUED

NAME ______________________ DATE ______

Reteaching with Practice

For use with pages 264–270

EXAMPLE 3 *Solving a Quadratic Equation*

Solve $5(x-7)^2 = 135$.

$5(x-7)^2 = 135$	Write original equation.
$(x-7)^2 = 27$	Divide both sides by 5.
$x - 7 = \pm\sqrt{27}$	Take the square roots of both sides.
$x - 7 = \pm 3\sqrt{3}$	Simplify.
$x = 7 \pm 3\sqrt{3}$	Add 7 to both sides.

The solutions are $7 + 3\sqrt{3}$ and $7 - 3\sqrt{3}$.

Exercises for Example 3

Solve the equation.

7. $(y+3)^2 = 9$ **8.** $(w-1)^2 = 196$ **9.** $-2(x-3)^2 = -12$

10. $(r-8)^2 = 50$ **11.** $5(x-3)^2 = 50$ **12.** $\frac{1}{3}(z+3)^2 = 5$

EXAMPLE 4 *Modeling a Falling Object's Height with a Quadratic Function*

A person is trapped in a building 120 feet above the ground and wants to land on a rescue team's air cushon. How long before the person reaches safety?

SOLUTION

Use the falling object model $h = -16t^2 + h_0$, where h is the height (in feet) of the object after t seconds and h_0 is the object's initial height.

$0 = -16t^2 + 120$	Substitute 120 for h_0 and 0 for h.
$-120 = -16t^2$	Subtract 120 from each side.
$\frac{120}{16} = t^2$	Divide each side by -16.
$\sqrt{\frac{120}{16}} = t$	Take positive square root.
$2.7 \approx t$	Use a calculator.

The person will reach safety in about 2.7 seconds.

Exercises for Example 4

13. A coyote is standing on a cliff 254 feet above a roadrunner. If the coyote drops a boulder from the cliff, how much time does the roadrunner have to move out of its way?

14. An apple falls from a branch on a tree 30 feet above a man sleeping underneath. When will the apple strike the man?

Lesson 5.3

LESSON 5.3

NAME ______________________________ DATE ____________

Quick Catch-Up for Absent Students

For use with pages 264–271

The items checked below were covered in class on (date missed) ____________

Lesson 5.3: Solving Quadratic Equations by Finding Square Roots

____ **Goal 1:** Solve quadratic equations by finding sqare roots. (pp. 264–265)

Material Covered:

____ Activity: Investigating Properties of Square Roots

____ Example 1: Using Properties of Square Roots

____ Example 2: Solving a Quadratic Equation

____ Example 3: Solving a Quadratic Equation

Vocabulary:

square root, p. 264
radical sign, p. 264
radicand, p. 264
radical, p. 264
rationalizing the denominator, p. 265

____ **Goal 2:** Use a quadratic equation to solve real-life problems. (p. 266)

Material Covered:

____ Example 4: Modeling a Falling Object's Height with a Quadratic Function

Activity 5.3: Solving Quadratic Equations (p. 271)

____ **Goal:** Solve quadratic equations having real-number solutions using a graphing calculator.

____ Student Help: Keystroke Help

____ Other (specify) ______________________________

____

Homework and Additional Learning Support

____ Textbook (specify) pp. 267–270

____ Internet: Extra Examples at www.mcdougallittell.com

____ *Reteaching with Practice* worksheet (specify exercises) ____________

____ *Personal Student Tutor* for Lesson 5.3

Lesson 5.3

LESSON 5.3

NAME ______________________________ DATE __________

Real-Life Application: When Will I Ever Use This?

For use with pages 264–270

Public Golf Courses

There are two main types of golf courses, public and private. A public golf course is open to anyone. Public courses are usually very affordable but can be crowded at times. Public courses normally receive all their income from greens fees and occasional tournaments. There are 11,657 public golf courses in the United States.

Private golf courses are not open to the public. A yearly or monthly membership fee is charged per person or per family. This fee covers the greens fees charged by public courses. Private courses usually have a restaurant, tennis courts, and a swimming pool, which bring in additional income. There are 4708 private golf courses in the United States.

In Exercises 1–4, use the following information.

From 1992 to 1996, the annual income for all the public golf courses in the United States can be approximated by the model $I = 59t^2 + 2254$, where I is the annual income in millions of dollars and t is the year, with $t = 0$ corresponding to 1990.

1. In which year did the annual income increase to more than \$3,000,000?
2. In which year did the annual income increase to more than \$4,000,000?
3. Predict the year in which the annual income will increase to more than \$5,500,000.
4. In which year was the annual income \$2,254,000?

Lesson 5.3

NAME ______________________ DATE __________

Math and History Application

For use with page 270

HISTORY There is an active observatory that uses a liquid mirror telescope at the University of British Columbia. One of the problems the designers faced was controlling vibrations that might cause ripples on the surface of the mercury. Part of the solution was to place the mirror on an enormous concrete slab about 3 feet thick, which was bolted directly to bedrock.

The largest single-reflector antenna in the world, at Arecibo in Puerto Rico, also uses metal surfaces, but it is a very different kind of telescope. The Arecibo telescope collects radio signals rather than light waves. The "mirror" is actually a grid of about 40,000 curved aluminum panels laid out inside a natural depression in the middle of limestone hills. Instead of being parabolic like the mercury reflector, the dish at Arecibo is a spherical cap, that is, the shape that you would get by slicing the top off of a sphere. The radius of this sphere would be about 870 feet. Since a sphere doesn't have the same focusing property as a parabolic surface, the telescope uses two additional reflectors that are located in a movable carriage house suspended hundreds of feet above the primary reflector.

MATH The problems below explore some issues in the design of parabolic reflectors.

To collect the image reflected in their spinning mercury mirror, the astronomers need to place a collector a certain distance above the vertex of the parabolic surface. The distance d from the vertex to the collector is given by the formula $d = 4/(\pi^2 f^2)$, where f is the spinning frequency in revolutions per second and d is measured in feet. (In Section 10.5 you'll learn where this formula comes from. It gives the location of the *focus* of the parabolic mirror.)

1. For the spinning frequency of 0.5 revolution per second used in the Math and History feature on page 270, what is the distance d for the collector?

2. A more typical spinning frequency would be one revolution every 10 seconds. For this frequency, what is the distance d for the collector? (Remember to convert the frequency to revolutions per second.)

3. Suppose that astronomers want to place the collector 25 feet above the vertex of a liquid mercury mirror. What spinning frequency should they use? (*Hint*: Use the formula for d. You'll end up solving a quadratic equation.)

4. If your mirror has a radius of 4 feet and is spinning at one revolution every 5 seconds, how far below the origin is the lowest point of the mirror? You'll need to use the equation for the parabolic cross-section given on page 270. The answer will show you that the typical shape of a liquid mirror is much flatter than indicated in the schematic picture on page 270.

LESSON 5.4

TEACHER'S NAME ______________________ CLASS ______ ROOM ______ DATE ______

Lesson Plan

2-day lesson (See *Pacing the Chapter,* TE pages 246C–246D) **For use with pages 272–280**

GOALS
1. **Solve quadratic equations with complex solutions and perform operations with complex numbers.**
2. **Apply complex numbers to fractal geometry.**

State/Local Objectives ____

____

✓ Check the items you wish to use for this lesson.

STARTING OPTIONS

____ Homework Check: TE page 267; Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 272 and 269, CRB page 53, or Transparencies

TEACHING OPTIONS

____ Motivating the Lesson: TE page 273
____ Lesson Opener (Activity): CRB page 54 or Transparencies
____ Examples: Day 1: 1–4, SE pages 272–274; Day 2: 5–7, SE pages 274–276
____ Extra Examples: Day 1: TE pages 273–274 or Transp.; Day 2: TE pages 274–276 or Transp.
____ Closure Question: TE page 276
____ Guided Practice: SE page 277 Day 1: Exs. 1–2, 4–10, 15; Day 2: Exs. 3, 11–14, 16

APPLY/HOMEWORK

Homework Assignment

____ Basic Day 1: 18–24 even, 30–42 even, 48–72 even, 80–85; Day 2: 26–28 even, 43–46, 86, 92, 97–99, 101–113 odd
____ Average Day 1: 18–72 even, 80–86; Day 2: 74, 76, 87–94, 97–99, 102–114 even
____ Advanced Day 1: 18–72 even, 80–86; Day 2: 74, 76, 87–99, 100–114 even

Reteaching the Lesson

____ Practice Masters: CRB pages 55–57 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 58–59 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson

____ Applications (Interdisciplinary): CRB page 61
____ Challenge: SE page 280; CRB page 62 or Internet

ASSESSMENT OPTIONS

____ Checkpoint Exercises: Day 1: TE pages 273–274 or Transp.; Day 2: TE pages 274–276 or Transp.
____ Daily Homework Quiz (5.4): TE page 280, CRB page 65, or Transparencies
____ Standardized Test Practice: SE page 280; TE page 280; STP Workbook; Transparencies

Notes ____

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LESSON 5.4

TEACHER'S NAME ______________________ CLASS ______ ROOM ______ DATE ______

Lesson Plan for Block Scheduling

1-day lesson (See *Pacing the Chapter,* TE pages 246C–246D) For use with pages 272–280

GOALS

1. **Solve quadratic equations with complex solutions and perform operations with complex numbers.**
2. **Apply complex numbers to fractal geometry.**

State/Local Objectives ______________________

CHAPTER PACING GUIDE	
Day	**Lesson**
1	5.1 (all)
2	5.2 (all)
3	5.3 (all); **5.4 (begin)**
4	**5.4 (end)**; 5.5 (begin)
5	5.5 (end); 5.6 (all)
6	5.7 (all); 5.8 (all)
7	Review/Assess Ch. 5

✓ **Check the items you wish to use for this lesson.**

STARTING OPTIONS

____ Homework Check: TE page 267; Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 272 and 269, CRB page 53, or Transparencies

TEACHING OPTIONS

____ Motivating the Lesson: TE page 273
____ Lesson Opener (Activity): CRB page 54 or Transparencies
____ Examples: Day 3: 1–4, SE pages 272–274; Day 4: 5–7, SE pages 274–276
____ Extra Examples: Day 3: TE pages 273–274 or Transparencies; Day 4: TE pages 274–276 or Transparencies
____ Closure Question: TE page 276
____ Guided Practice: SE page 277 Day 3: Exs. 1–2, 4–10, 15; Day 4: Exs. 3, 11–14, 16

APPLY/HOMEWORK

Homework Assignment (See also the assignments for Lessons 5.3 and 5.5.)

____ Block Schedule: Day 3: 18–72 even, 80–86; Day 4: 74, 76, 87–94, 97–99, 102–114 even

Reteaching the Lesson

____ Practice Masters: CRB pages 55–57 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 58–59 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson

____ Applications (Interdisciplinary): CRB page 61
____ Challenge: SE page 280; CRB page 62 or Internet

ASSESSMENT OPTIONS

____ Checkpoint Exercises: Day 3: TE pages 273–274 or Transparencies; Day 4: TE pages 274–276 or Transparencies
____ Daily Homework Quiz (5.4): TE page 280, CRB page 65, or Transparencies
____ Standardized Test Practice: SE page 280; TE page 280; STP Workbook; Transparencies

Notes ______________________

NAME ______________________ DATE ________

Available as a transparency

Warm-Up Exercises

For use before Lesson 5.4, pages 272–280

Simplify.

1. $\sqrt{200}$
2. $\sqrt{75}$
3. $\sqrt{20}$
4. $\sqrt{98}$

Daily Homework Quiz

For use after Lesson 5.3, pages 264–271

Simplify the expression.

1. $\sqrt{63}$
2. $\sqrt{\frac{3}{25}}$
3. $\sqrt{8} \cdot \sqrt{10}$
4. $\sqrt{\frac{5}{3}}$
5. Solve $3x^2 + 2 = 62$.
6. Solve $\frac{1}{2}(x + 3)^2 = 5$.

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Available as a transparency

Activity Lesson Opener

For use with pages 272–280

SET UP: Work in pairs.

YOU WILL NEED: colored pencils

The *imaginary unit* is defined as $i = \sqrt{-1}$. It can be used to create *complex numbers* in the form $a + bi$.

Although i is *not* a variable, it behaves just like a variable when you are adding and subtracting. Note that $i^2 = -1$.

Each expression in parts (a)–(j) is equal to one of the following complex numbers. Determine which number, and color the corresponding region the appropriate color.

$-7 + 24i$	$9 - 7i$	$15 + 3i$	$22 + 32i$
Red	Blue	Green	Black or dark gray

a. $(5 - 2i) + (4 - 5i)$

b. $(1 + 15i) - (8 - 9i)$

c. $(6 + 5j) + (9 - 2i)$

d. $-i(7 + 9i)$

e. $(3 + 4i)^2$

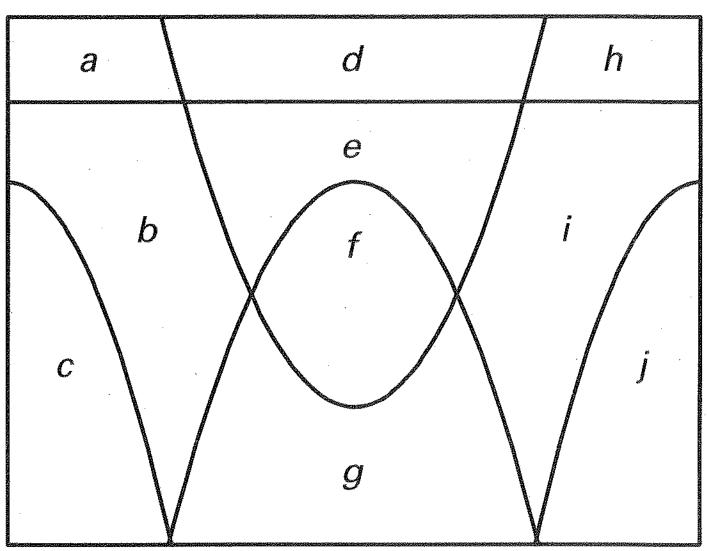
f. $2(11 + 16i)$

g. $(5 - i)(3 + 7i)$

h. $(3 + i)(2 - 3i)$

i. $-(4 - 3i)^2$

j. $(3i)(1 - 5i)$



LESSON 5.4

NAME ______________________________ DATE ____________

Practice A

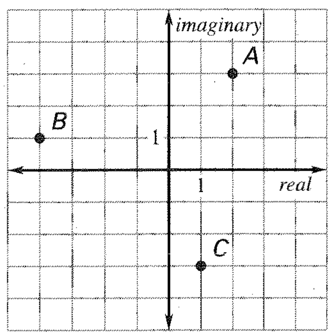
For use with pages 272–280

Solve the equation.

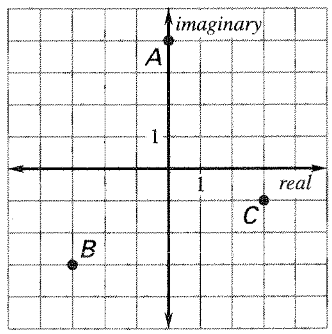
1. $x^2 = -16$
2. $x^2 = -81$
3. $x^2 + 144 = 0$
4. $x^2 + 5 = 4$
5. $x^2 - 1 = 3$
6. $x^2 - 7 = 4x^2 + 5$

Identify the complex numbers plotted in the complex plane.

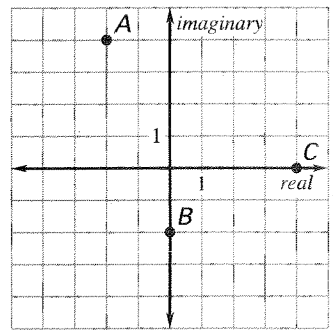
7.



8.



9.



Write the expression as a complex number in standard form.

10. $(5 + 3i) + (2 + 4i)$
11. $(3 - 2i) + (1 + i)$
12. $(7 + 2i) - (3 + 3i)$
13. $(5 + i) - (3 - 8i)$
14. $i + (11 - 5i)$
15. $i - (6 + i) + (4 - 2i)$
16. $i(4 + i)$
17. $3i(-1 + 2i)$
18. $-4i(3 - 7i)$
19. $(1 + 3i)(1 - i)$
20. $(5 - i)(1 - 2i)$
21. $(2 + 3i)(3 + 4i)$
22. $\frac{3}{1 + i}$
23. $\frac{5}{2 - i}$
24. $\frac{3 - i}{2 + i}$

Find the absolute value of the complex number.

25. $1 + i$
26. $2 - i$
27. $6 + i$
28. $1 - 2i$
29. $3 + 4i$
30. $5 - 4i$

Plot the numbers in a complex plane.

31. $2i$
32. -3
33. $1 + 3i$
34. $4 - 3i$
35. $-1 + 2i$
36. $-2 - 4i$

Lesson 5.4

LESSON 5.4

NAME ______________________ DATE __________

Practice B

For use with pages 272–280

Solve the equation.

1. $x^2 = -64$
2. $x^2 + 1 = 0$
3. $x^2 + 5 = 14$
4. $x^2 = -12$
5. $x^2 + 48 = 0$
6. $x^2 + 3 = -24$
7. $2x^2 - 9 = 3x^2$
8. $x^2 - 16 = 5x^2$
9. $11x^2 + 1 = 2x^2$
10. $-2(x + 1)^2 = 72$
11. $4(x - 2)^2 = -1$
12. $3(x + 5)^2 + 147 = 0$

Plot the number in a complex plane.

13. $3i$
14. -2
15. $2 + 4i$
16. $3 - 4i$
17. $-2 + i$
18. $-4 - 3i$

Write the expression as a complex number in standard form.

19. $(3 + 2i) + (-5 + 8i)$
20. $(-2 - 4i) + (3 - 6i)$
21. $\left(\frac{1}{3} + \frac{1}{2}i\right) + \left(\frac{2}{3} - 2i\right)$
22. $(4 + 2i) - (-1 + 5i)$
23. $(5 - 8i) - (2 + 9i)$
24. $\left(\frac{1}{2} - \frac{2}{3}i\right) - \left(\frac{2}{3} - \frac{1}{4}i\right)$
25. $(5 - 4i)(3 + 6i)$
26. $(2 + 5i)^2$
27. $(4 + 8i)(4 - 8i)$
28. $\dfrac{6}{2 + 3i}$
29. $\dfrac{3 + i}{-2 + i}$
30. $\dfrac{2 - i}{\sqrt{3} - i}$
31. $2(2 + i) + (1 - i)^2$
32. $\dfrac{1}{3 - 5i} - (6 - 2i)$
33. $(1 - 5i)(2 + i) - i(3 - 4i)$

Find the absolute value of the complex number.

34. $-4 + 3i$
35. $\sqrt{2} - i$
36. $\sqrt{3} + \sqrt{2}i$

Write the complex number in standard form.

37. i
38. i^2
39. i^3
40. i^4
41. i^5
42. i^6
43. i^7
44. i^8

45. ***Pattern Recognition*** Using the information from Exercises 37–44, write a general statement about the standard form of i^n where n is a positive integer. Use this statement to write i^{231} in standard form.

Lesson 5.4

LESSON 5.4

NAME ______________________ DATE __________

Practice C

For use with pages 272–280

Solve the equation.

1. $3x^2 + 5 = x^2 - 2$
2. $4(x + 1)^2 + 12 = 0$
3. $6(x - 4)^2 + 3 = 0$
4. $2(x^2 + 7) = x^2 + 10$
5. $-3(x + 6)^2 - 7 = 0$
6. $-2(x - 1)^2 + 3 = 6$

Determine whether *a* and *b* are greater than zero, less than zero or equal to zero for the given complex number *a + bi*.

7. $a + bi$ lies in the first quadrant of the complex plane
8. $a + bi$ lies in the second quadrant of the complex plane
9. $a + bi$ lies in the third quadrant of the complex plane
10. $a + bi$ lies in the fourth quadrant of the complex plane
11. $a + bi$ lies on the positive real axis of the complex plane
12. $a + bi$ lies on the negative imaginary axis of the complex plane

Perform the given operation and find the absolute value of the complex number.

13. $(3 + 2i) + (-4 + i)$
14. $(5 - 2i) + (3 - 5i)$
15. $(6 + i) - (3 + 2i)$
16. $(3 - 2i) - (4 - 5i)$
17. $(2 + i) - (3 + 4i) + 3i$
18. $(2 + 6i)(1 - 2i)$
19. $(-2 - 3i)(1 + 2i)$
20. $\dfrac{2 + 3i}{4 + i}$
21. $\dfrac{1 - 2i}{3 - 5i}$

Determine whether the complex number *c* belongs to the Mandelbrot set. Use absolute value to justify your answer.

22. $c = -3$
23. $c = 2 + i$
24. $c = 2i$

Determine whether the complex number is *real, imaginary, pure imaginary,* or *neither.*

25. The sum of a complex number and its conjugate.
26. The difference of a complex number and its conjugate.
27. The product of a complex number and its conjugate.
28. The quotient of a complex number and its conjugate.

Lesson 5.4

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 272–280

GOAL **Solve quadratic equations with complex solutions and perform operations with complex numbers**

VOCABULARY

The **imaginary unit** i is defined as $i = \sqrt{-1}$.

A **complex number** written in **standard form** is a number $a + bi$, where a and b are real numbers.

If $b \neq 0$, then $a + bi$ is an **imaginary number.**

If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number.**

Sum of complex numbers:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference of complex numbers:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

In the **complex plane,** the horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis.**

The expressions $a + bi$ and $a - bi$ are called **complex conjugates.** The product of complex conjugates is always a real number.

EXAMPLE 1 *Solving a Quadratic Equation*

Solve $2x^2 - 12 = -40$

SOLUTION

$2x^2 - 12 = -40$	Write original equation.
$2x^2 = -28$	Add 12 to each side.
$x^2 = -14$	Divide each side by 2.
$x = \pm\sqrt{-14}$	Take square roots of each side.
$x = \pm i\sqrt{14}$	Write in terms of i.

Exercises for Example 1

Solve the equation.

1. $x^2 = -16$ **2.** $5y^2 = -40$ **3.** $r^2 - 4 = -8$

4. $-2t^2 - 8 = 46$ **5.** $3x^2 + 1 = -35$ **6.** $y^2 + 25 = 0$

EXAMPLE 2 *Adding and Subtracting Complex Numbers*

Write $(6 + 3i) - (-4 - 2i) - 7i$ as a complex number in standard form.

$(6 + 3i) - (-4 - 2i) - 7i = (6 + 4) + (3 + 2 - 7)i$	Definition of complex addition
$= 10 - 2i$	Simplify and use standard form.

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Reteaching with Practice

For use with pages 272–280

Exercises for Example 2

Write the expression as a complex number in standard form.

7. $(5 + 4i) + (7 + 2i)$ **8.** $(-6 + 3i) + (5 + i)$ **9.** $i - (5 - 6i)$

10. $(12 - 8i) - (6 - 6i)$ **11.** $(6 - 7i) + (-3 - i)$ **12.** $12 - (8 - 10i)$

EXAMPLE 3 Multiplying Complex Numbers

Write $(7 - 3i)(1 - 4i)$ as a complex number in standard form.

SOLUTION

$(7 - 3i)(1 - 4i) = 7 - 28i - 3i + 12i^2$ Use FOIL.

$= 7 - 31i + 12(-1)$ Simplify and use $i^2 = -1$.

$= -5 - 31i$ Standard form

Exercises for Example 3

Write as a complex number in standard form.

13. $-2i(5 + i)$ **14.** $4i(3 - 5i)$ **15.** $(2 + 3i)(2 - 3i)$

16. $(4 - i)(-2 + 6i)$ **17.** $(5 + 3i)^2$ **18.** $(8 + i)(2 + i)$

EXAMPLE 4 Dividing Complex Numbers

Write the quotient $\frac{7 - 4i}{2 + i}$ in standard form.

SOLUTION

$\frac{7 - 4i}{2 + i} = \frac{7 - 4i}{2 + i} \cdot \frac{2 - i}{2 - i}$ Multiply by $2 - i$, the conjugate of $2 + i$.

$= \frac{14 - 7i - 8i + 4i^2}{4 - 2i + 2i - i^2}$ Use FOIL.

$= \frac{14 - 15i - 4}{4 + 1}$ Simplify and use $i^2 = -1$.

$= \frac{10 - 15i}{5}$ Simplify.

$= 2 - 3i$ Standard form

Exercises for Example 4

Write as a complex number in standard form.

19. $\frac{6}{1 - i}$ **20.** $\frac{5i}{2 + i}$ **21.** $\frac{2 - 3i}{5i}$ **22.** $\frac{1 - i}{1 + i}$

NAME ______________________________ DATE __________

Quick Catch-Up for Absent Students

For use with pages 272–280

The items checked below were covered in class on (date missed) __________

Lesson 5.4: Complex Numbers

____ **Goal 1:** Solve quadratic equations with complex solutions and perform operations with complex numbers. (pp. 272–274)

Material Covered:

____ Example 1: Solving a Quadratic Equation

____ Example 2: Plotting Complex Numbers

____ Example 3: Adding and Subtracting Complex Numbers

____ Example 4: Multiplying Complex Numbers

____ Example 5: Dividing Complex Numbers

Vocabulary:

imaginary unit i, p. 272
complex number, p. 272
standard form of a complex number, p. 272
imaginary number, p. 272
pure imaginary number, p. 272
complex plane, p. 273
complex conjugates, p. 274

____ **Goal 2:** Apply complex numbers to fractal geometry. (pp. 275–276)

Material Covered:

____ Example 6: Finding Absolute Values of Complex Numbers

____ Example 7: Determining if a Complex Number Is in the Mandelbrot Set

Vocabulary:

absolute value of a complex number, p. 275

____ Other (specify) ______________________________

____

Homework and Additional Learning Support

____ Textbook (specify) pp. 277–280

____

____ *Reteaching with Practice* worksheet (specify exercises) ______________

____ *Personal Student Tutor* for Lesson 5.4

NAME _______________ DATE _______

Interdisciplinary Application

For use with pages 272–280

Fractal Geometry

GEOMETRY Fractals are extremely irregular lines or surfaces formed of an infinite number of similarly irregular sections. To construct a fractal, a pattern needs to be replicated, or iterated over and over. Often these patterns can be represented by a function. To iterate a function, you input a starting value and solve the function. Then you input the solution into the same function and solve.

For example, to iterate the function $f(x) = 5x - 1$ for the starting value $x = i$, you begin by replacing i for x in the function. You should get an answer of $5i - 1$. This completes the first iteration. Now you use your answer from the first iteration $(5i - 1)$ to find the second iteration. Using the same function, replace x with $(5i - 1)$. The answer to the second iteration is $25i - 5$.

Usually the iteration is written as an ordered pair. Using the example above, the first iteration would be $(i, 5i - 1)$ and the second iteration would be $(5i - 1, 25i - 5)$. Repeating this process and plotting the points would create a fractal. A computer is typically used to accomplish this task.

In Exercises 1–3, find the first, second, and third iteration of the function for the given starting value. Write your answers as ordered pairs.

1. $f(x) = x + 7$ for $x = 3i$

2. $f(x) = x^2$ for $x = 1 + i$

3. $f(x) = x^2 + 3$ for $x = 2 - i$

In Exercise 4, find the first and second iteration. Write your answers as ordered pairs.

4. $f(x) = 2x^2 - 1$ for $x = i + 4$

LESSON 5.4

NAME ____________________ DATE __________

Challenge: Skills and Applications

For use with pages 272–280

1. Suppose $z = a + bi$ is a complex number, and $w = x + yi$ is another complex number such that $z + w$ is a real number and zw is also a real number. Show that w must be the conjugate of z. (*Hint:* A complex number is real if and only if its imaginary part is 0; use this fact to solve for x and y in terms of a and b.)

2. **a.** Explain why the following fact is true for all *real* numbers p and q: $|pq| = |p||q|$.

   b. Let $z = a + bi$ and $w = c + di$. Find $(|z||w|)^2 = |z|^2|w|^2$, in terms of a, b, c, and d.

   c. Find zw and $|zw|^2$ in terms of a, b, c, and d. How is $|zw|^2$ related to the number you found in part (b)? How does this prove the fact stated in part (a) is also true for complex numbers?

3. Let $z = a + bi$, and denote the conjugate of z, $a - bi$, by $\bar{z}$.

   a. Let $w = c + di$ be another complex number. Show that $\overline{z + w} = \bar{z} + \bar{w}$.

   b. For z and w as above, show that $\overline{zw} = \bar{z} \cdot \bar{w}$.

4. You may notice that the Mandelbrot set appears to have the real axis as a line of symmetry. In this problem, you will verify this conjecture.

   a. Let $z = a + bi$. Show that $|z| = |\bar{z}|$.

   b. By using repeated substitution, write out, in terms of c, the values of z_0, z_1, z_2, z_3, and z_4, the first 5 values that you need to check to find out whether c is in the Mandelbrot set. (You may leave parentheses in your answer.)

   c. Suppose z_0', z_1', z_2', z_3', and z_4' are the first 5 values that you need to check to find out whether *the conjugate of* c, $\bar{c}$, is in the Mandelbrot set. Explain why each z_j' is the conjugate of z_j. (*Hint:* Use the results of Problem 3 above.)

   d. Use the result of parts (a) and (c) to explain why the real axis is a line of symmetry for the Mandelbrot set. (*Hint:* The reflection of a point z in the real axis is the point $\bar{z}$.)

Lesson 5.4