LESSON 3.5

TEACHER'S NAME ______________________ CLASS ______ ROOM ______ DATE ______

Lesson Plan

1-day lesson (See *Pacing the Chapter,* TE pages 136C–136D) **For use with pages 170–176**

GOALS

1. **Graph linear equations in three variables and evaluate linear functions in two variables.**
2. **Use functions of two variables to model real-life situations.**

State/Local Objectives ____

____

✓ Check the items you wish to use for this lesson.

STARTING OPTIONS

____ Homework Check: TE page 166: Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 170 and 168, CRB page 69, or Transparencies

TEACHING OPTIONS

____ Lesson Opener (Activity): CRB page 70 or Transparencies
____ Graphing Calculator Activity with Keystrokes: CRB page 71
____ Examples: 1–4: SE pages 170–172
____ Extra Examples: TE pages 171–172 or Transparencies; Internet
____ Technology Activity: SE page 176
____ Closure Question: TE page 172
____ Guided Practice Exercises: SE page 173

APPLY/HOMEWORK

Homework Assignment

____ Basic 18–36 even, 39–45 odd, 49, 50, 53, 57–67 odd
____ Average 18–36 even, 39–53 odd, 57–67 odd
____ Advanced 18–36 even, 39–51 odd, 52–56, 57–67 odd

Reteaching the Lesson

____ Practice Masters: CRB pages 72–74 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 75–76 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson

____ Applications (Interdisciplinary): CRB page 78
____ Challenge: SE page 175; CRB page 79 or Internet

ASSESSMENT OPTIONS

____ Checkpoint Exercises: TE pages 171–172 or Transparencies
____ Daily Homework Quiz (3.5): TE page 175, CRB page 82, or Transparencies
____ Standardized Test Practice: SE page 175; TE page 175; STP Workbook; Transparencies

Notes ____

____

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LESSON 3.5

Lesson 3.5

TEACHER'S NAME ______________ CLASS ______ ROOM ______ DATE ______

Lesson Plan for Block Scheduling

Half-day lesson (See *Pacing the Chapter,* TE pages 136C–136D) **For use with pages 170–176**

GOALS

1. **Graph linear equations in three variables and evaluate linear functions in two variables.**
2. **Use functions of two variables to model real-life situations.**

State/Local Objectives ______________

CHAPTER PACING GUIDE	
Day	**Lesson**
1	3.1 (all)
2	3.2 (all)
3	3.3 (all); 3.4(all)
4	**3.5 (all)**; 3.6 (all)
5	Review/Assess Ch. 3

✓ **Check the items you wish to use for this lesson.**

STARTING OPTIONS

____ Homework Check: TE page 166: Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 170 and 168, CRB page 69, or Transparencies

TEACHING OPTIONS

____ Lesson Opener (Activity): CRB page 70 or Transparencies
____ Graphing Calculator Activity with Keystrokes: CRB page 71
____ Examples: 1–4: SE pages 170–172
____ Extra Examples: TE pages 171–172 or Transparencies; Internet
____ Technology Activity: SE page 176
____ Closure Question: TE page 172
____ Guided Practice Exercises: SE page 173

APPLY/HOMEWORK

Homework Assignment (See also the assignment for Lesson 3.6.)

____ Block Schedule: 18–36 even, 39–53 odd, 57–67 odd

Reteaching the Lesson

____ Practice Masters: CRB pages 72–74 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 75–76 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson

____ Applications (Interdisciplinary): CRB page 78
____ Challenge: SE page 175; CRB page 79 or Internet

ASSESSMENT OPTIONS

____ Checkpoint Exercises: TE pages 171–172 or Transparencies
____ Daily Homework Quiz (3.5): TE page 175, CRB page 82, or Transparencies
____ Standardized Test Practice: SE page 175; TE page 175; STP Workbook; Transparencies

Notes ______________

LESSON 3.5

NAME ______________________ DATE __________

Available as a transparency

WARM-UP EXERCISES

For use before Lesson 3.5, pages 170–176

Find the x- and y-intercepts of the graph of each equation.

1. $2x + 4y = 20$

2. $x - 3y = -15$

Use the linear equation to write y as a function of x.

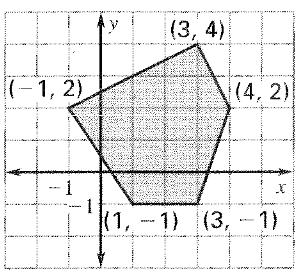
3. $2x - 6y = 24$

4. $3x + 5y = 12$

DAILY HOMEWORK QUIZ

For use after Lesson 3.4, pages 163–169

1. Find the minimum and maximum values of $C = 2x + 3y$ for the given feasible region.



2. Find the minimum and maximum values of $C = 7x + 5y$ subject to given constraints.

$x \geq 1 \qquad y \geq 2 \qquad x + 3y \leq 13 \qquad x + y \leq 7$

3. A cook plans to make burgers that contain ground meat and mushrooms. He will buy from 4 to 5 lb of meat at \$4.50/lb and from 0.5 to 0.8 lb of mushrooms at \$2.70/lb. He wants to use at most 0.15 times as much chopped mushrooms as ground meat. What are the greatest and least costs possible for the burgers?

LESSON **3.5**

NAME ______________________ DATE ________

Available as a transparency

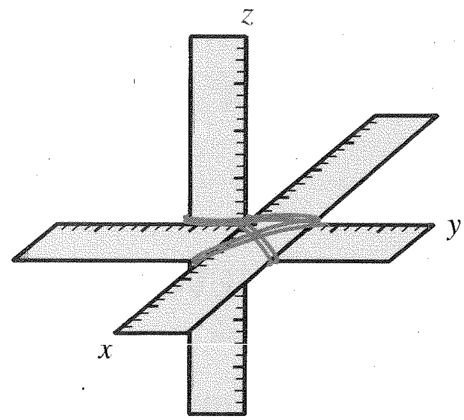
Activity Lesson Opener

For use with pages 170–175

SET UP: Work in a group.

YOU WILL NEED: • rulers • rubber band • tape

Use three rulers and a rubber band to form a 3-dimensional coordinate system as shown. In an *ordered triple,* such as (2, −1, 3), the numbers represent *x, y,* and *z* respectively.



1. Point to each of the following points.

a. $(1, 0, -1)$

b. $(-1, -1, -1)$

c. $(1, -1, 0)$

2. Complete the table of ordered triples for $x + y + z = 4$.

x	y	z
	0	0
0		0
0	0	
1		0

3. Cut a piece of paper into a triangle that is $5\frac{11}{16}$ inches on a side. Tape a corner of the triangle to the ruler at the first point on the table. Attach the other two corners to a ruler at the second and third points in the table. The triangle now represents the plane that is the graph of $x + y + z = 4$.

4. Plot the fourth point in the table by marking its location on the triangle.

5. Find at least 3 other points on the triangle and give their coordinates.

LESSON 3.5

NAME ______________________ DATE __________

Graphing Calculator Activity Keystrokes

For use with page 176

TI-92

Set Graph mode to the 3D setting.

MODE ▶ 5 ENTER

♦ [Y=] 5 − (1 ÷ 2) X − (5 ÷ 6) Y ENTER

♦ [WINDOW]

Choose the following.

eyeθ° = 20; eyeθ° = 70; xmin = 0; xmax = 20; xgrid = 14; ymin = 0; ymax = 20; ygrid = 14; zmin = 0; zmax = 20; zscl = 1.

F1 9

Choose the following.

Coordinates: RECT; Axes: BOX; Labels: ON; Style: HIDDEN SURFACE.
ENTER

♦ [GRAPH]

Evaluate $x = 3$ and $y = 2$ to see $z \approx 1.833$.

F5 1

3 ENTER 2 ENTER

LESSON **3.5**

NAME ______________________ DATE __________

Practice A

For use with pages 170–175

Plot the ordered triple in a three-dimensional coordinate system.

1. (0, 1, 2)
2. (1, 3, 1)
3. (2, 0, 1)
4. (2, 2, 4)
5. (1, 1, 0)
6. (2, 5, 0)
7. (−1, 2, 4)
8. (3, −2, 5)
9. (−2, 3, −1)

Find the *x*-intercept, *y*-intercept, and *z*-intercept of the graph of the linear equation.

10. $x + y + z = 4$
11. $2x + y + 3z = 6$
12. $3x + 4y + 6z = 12$
13. $2x - 5y + z = 20$
14. $7x + 2y - 21z = 14$
15. $12x + y - 3z = 9$

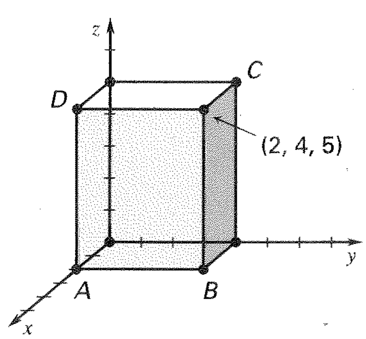
Evaluate the function for the given values.

16. $f(x, y) = 3x + 2y, f(1, 5)$
17. $f(x, y) = x + 6y, f(0, 3)$
18. $f(x, y) = 3x - 2y, f(2, 3)$
19. $f(x, y) = x + y, f(-1, 4)$
20. $f(x, y) = 5x - y, f(3, -2)$
21. $f(x, y) = 7x - 2y, f(3, -3)$
22. $f(x, y) = -3x + 4y, f(1, 1)$
23. $f(x, y) = -4x - 3y, f(5, 2)$
24. $f(x, y) = 8x + 3y, f(2, -6)$

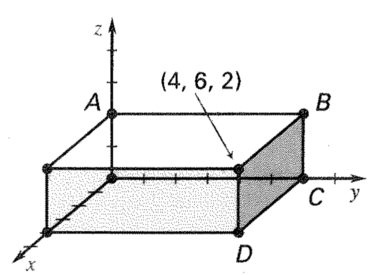
Write the linear equation as a function of *x* and *y*.

25. $2x + 3y + z = 12$
26. $3x - 2y + z = 1$
27. $x - 3y - z = 8$
28. $5x + 2y - z = 4$
29. $7x - 8y + z = 9$
30. $6x + y - z = 0$

31. ***Geometry*** Write the coordinates of the vertices *A*, *B*, *C*, and *D* of the rectangular prism shown, given that one vertex is the point (2, 4, 5).



32. ***Geometry*** Write the coordinates of the vertices *A*, *B*, *C*, and *D* of the rectangular prism shown, given that one vertex is the point (4, 6, 2).



33. ***Music Club*** A music club requires an initial purchase of \$50 worth of merchandise. After this initial fee, compact discs may be purchased for \$18 and audio cassettes may be purchased for \$12. Write an equation for the amount that you will spend as a function of the number of compact discs and audio cassettes that you buy. Make a table to show the different cost for several different numbers of compact discs and audio cassettes.

LESSON 3.5

NAME ______________________ DATE __________

Practice B

For use with pages 170–175

Plot the ordered triple in a three-dimensional coordinate system.

1. $(1, 3, 1)$
2. $(3, 5, 2)$
3. $(0, 0, -5)$
4. $(-4, 1, -3)$
5. $(0, -3, 2)$
6. $(-1, -3, -2)$

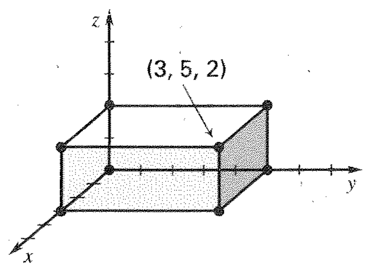
Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

7. $x + y + z = 8$
8. $x + 2y + z = 4$
9. $2x + y - 3z = 12$
10. $5x + 2y + 6z = 30$
11. $2x + y - z = 3$
12. $3x + 2y + z = 3$
13. $5x + y + 2z = 5$
14. $6x - y - z = 1$
15. $4x - 3y + 24z = 12$
16. $3x - 2y + 4z = -12$
17. $2x + 3y - 5z = -6$
18. $7x - 2y - 6z = 28$

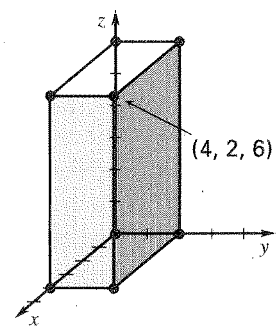
Write the linear equation as a function of *x* and *y*. Then evaluate the function for the given values.

19. $4x + y + z = 5, f(1, 5)$
20. $3x + 2y + z = 3, f(0, 2)$
21. $5x + 3y - z = 7, f(2, 6)$
22. $3x - y - z = 2, f(-2, 3)$
23. $2x + y + 2z = 6, f(4, 2)$
24. $3x + 2y - 4z = 24, f(-1, -3)$
25. $2x + y + 5z = 15, f(6, 3)$
26. $3x - 2y + 8z = 16, f(0, 2)$
27. $5x + 4y - 20z = 10, f(1, 5)$

28. ***Geometry*** Use the given point (3, 5, 2) to find the volume of the rectangular prism.



29. ***Geometry*** Use the given point (4, 2, 6) to find the volume of the rectangular prism.



30. ***Yearbook Advertisements*** The yearbook club's bank account has \$200 remaining from last year's advertising campaign. You are now trying to sell advertisements to local businesses for this year's yearbook. A quarter page ad costs \$35. A half page ad costs \$60. Write an equation for the total amount of money you may spend as a function of the number of quarter and half page ads that you sell. Evaluate the model if you sell 20 quarter page ads and 10 half page ads.

31. ***Baseball Game*** You and a group of your friends go to a professional baseball game. Your ticket costs \$12. Bottled water costs \$2 and hotdogs cost \$3.50. Write an equation for the cost of going to the game as a function of the number of bottled waters and hotdogs you purchase. Evaluate the model if you buy 3 bottled waters and 2 hotdogs.

LESSON 3.5

NAME ______________________________ DATE __________

Practice C

For use with pages 170–175

Plot the ordered triple in a three-dimensional coordinate system.

1. $(2, 1, 4)$
2. $(-1, 3, -2)$
3. $(-2, -4, -1)$
4. $\left(\frac{1}{2}, 3, \frac{3}{2}\right)$
5. $\left(-1, \frac{2}{3}, -3\right)$
6. $\left(-\frac{1}{2}, -2, \frac{5}{3}\right)$

Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

7. $5x + 4y - 2z = 20$
8. $7x - 2y + 14z = 14$
9. $2x - y + 3z = 3$
10. $4x + 3y - 5z = 2$
11. $-3x + y + 8z = 2$
12. $5x - 3y + 2z = 4$
13. $2x - y - 4z = -3$
14. $6x + 3y - 2z = -3$
15. $3x - 5y + 2z = -4$

Write the linear equation as a function of *x* and *y*. Then evaluate the function for the given values.

16. $6x - 2y + 3z = 12, f(0, 3)$
17. $2x - y + 4z = 1, f(1, 2)$
18. $3x + 2y - 4z = -3, f(-1, 0)$
19. $x + 2y + 6z = 9, f(-3, 4)$
20. $-5x - 3y + z = 2, f\left(\frac{1}{2}, 2\right)$
21. $3x - 7y + 2z = -14, f\left(-1, \frac{2}{3}\right)$

Write an equation of the plane having the given *x*-, *y*-, and *z*-intercepts. Explain the method you used.

22. *x*-intercept: 2
    y-intercept: 3
    z-intercept: 2
23. *x*-intercept: -3
    y-intercept: 2
    z-intercept: 5
24. *x*-intercept: $\frac{1}{2}$
    y-intercept: 1
    z-intercept: $-\frac{5}{2}$

25. ***Place-kicker*** In football a place kicker is responsible for kicking field goals worth 3 points and extra points after touchdowns worth 1 point. Write a model for the total number of points that a place-kicker can score in a game. In Superbowl XXXII, J. Elim kicked 4 extra points and 1 field goal for the Denver Broncos. Use the model to determine the total number of points scored by Elim.

26. ***Photography Studio*** A photography studio charges a \$14.95 sitting fee. A sheet of pictures can consist of one 8 x 10, two 5 x 7's, four 3 x 5's, or twenty-four wallets. The studio charges \$6.95 for a sheet of pictures. Holiday cards with your photo may be purchased. Twenty holiday cards cost \$24.95 plus \$10 for each addition 10-card order. Write a model for the total cost (not including tax) of buying pictures if you intend to purchase at least 20 holiday cards. Evaluate the model if you buy 40 holiday cards and two sheets of pictures.

27. ***N.B.A. Lifetime Leader*** Kareem Abdul-Jabbar is the N.B.A. lifetime leader in points scored with 38,387. Today, a player can score a three-point shot worth 3 points, a field goal worth 2 points, or a free throw worth 1 point. Write a model for the types of points needed to match Abdul-Jabbar's record. How many three-point shots are needed in a career to match the record if 12,000 field goals and 2000 free throws are scored?

LESSON 3.5

NAME ______________________ DATE ________

Reteaching with Practice

For use with pages 170–175

GOAL **Graph linear equations in three variables, evaluate linear functions in two variables, and use functions of two variables to model real-life situations**

VOCABULARY

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$, where a, b, and c are not all zero.

In a **three-dimensional coordinate system,** the xy-plane is in a horizontal position, with the **z-axis** as a vertical line through the origin.

An **ordered triple** is given by (x, y, z), and represents a point in space.

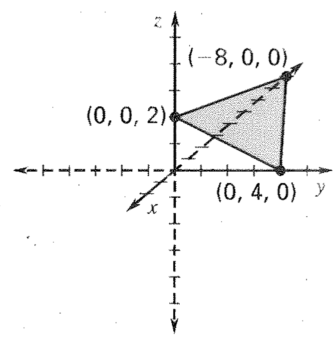
EXAMPLE 1 *Graphing a Linear Equation in Three Variables*

Sketch the graph of $-2x + 4y + 8z = 16$.

SOLUTION

Begin by finding the points where the graph intersects the axes. Let $y = 0$ and $z = 0$, and solve for x to get $x = -8$. The x-intercept is -8, so plot the point $(-8, 0, 0)$. In a similar way, you can find that the y-intercept is 4 and the z-intercept is 2.

After plotting $(-8, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 2)$, connect these points with lines to form the triangular region of the plane that lies in the second octant.



Exercises for Example 1

Sketch the graph of the equation. Label the three intercepts.

1. $x + y - z = 5$ **2.** $x - 5y + 2z = 10$ **3.** $-2x + 4y - 8z = 8$

EXAMPLE 2 *Evaluating a Function of Two Variables*

a. Write the linear equation $10x - 3y + 12z = -60$ as a function of x and y.

b. Evaluate the function for $f(3, -2)$.

SOLUTION

a. $10x - 3y + 12z = -60$ Write original equation.

$12z = -60 - 10x + 3y$ Isolate z-term.

$z = \frac{1}{12}(-60 - 10x + 3y)$ Solve for z.

$f(x, y) = \frac{1}{12}(-60 - 10x + 3y)$ Replace z with $f(x, y)$.

b. $f(3, -2) = \frac{1}{12}[-60 - 10(3) + 3(-2)]$ Substitute 3 for x and -2 for y.

$= \frac{1}{12}(-96)$ Simplify.

$= -8$ Multiply.

The graph of $10x - 3y + 12z = -60$ contains the point $(3, -2, -8)$.

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 170–175

Exercises for Example 2

Write the linear equation as a function of *x* and *y*. Then evaluate the function for the given values.

4. $12x - 2y + 4z = 12,\ f(0, 2)$
5. $-3x + 5y - z = 14,\ f(-1, 2)$
6. $4x + 3y + 2z = -12,\ f(-2, -4)$
7. $-6x - 2y - 2z = 18,\ f(3, 5)$

EXAMPLE 3 Modeling a Real-Life Situation

You own a specialty coffee shop which grinds gourmet coffee beans. The coffee grinder costs \$79. Colombian coffee costs \$10 per pound and the Kenyan costs \$8 per pound. Write a model for the total cost as a function of the number of pounds of the two types of coffee. Evaluate the model for 6 lb of Colombian and 3 lb of Kenyan.

SOLUTION

Verbal Model

[Total cost] = [Colombian cost] · [Amount of Colombian] + [Kenyan cost] · [Amount of Kenyan] + [Grinder cost]

Algebraic Model

$c = 10x + 8y + 79$

$c = 10(\mathbf{6}) + 8(\mathbf{3}) + 79$ Substitute 6 for *x* and 3 for *y*.

$= 163$ Simplify.

Exercises for Example 3

8. You are planting an apple orchard and decide to plant two types of trees: McIntosh and Red Delicious. The McIntosh trees cost \$20.85 each and the Red Delicious cost \$21.25 each. To plant the trees you need to buy a shovel which costs \$30 and peat moss which costs \$50. Write a model for the total amount you will spend as a function of the number of each type of tree. Evaluate the model for 25 McIntosh trees and 18 Red Delicious trees.

9. You are planning a cookout for the neighborhood and decide to serve hot dogs and hamburgers. The hot dogs cost \$2.79 per pound and the hamburger costs \$1.99 per pound. The condiments are \$12. Write a model for the total amount you will spend as a function of the number of pounds of hot dogs and hamburger. Evaluate the model for 5 lb of hot dogs and 2.5 lb of hamburger.

LESSON 3.5

NAME ______________________________ DATE __________

Quick Catch-Up for Absent Students

For use with pages 170–176

The items checked below were covered in class on (date missed) __________

Lesson 3.5: Graphing Linear Equations in Three Variables

____ **Goal 1**: Graph linear equations in three variables and evaluate linear functions in two variables. (pp. 170–171)

Material Covered:

____ Example 1: Plotting Points in Three Dimensions

____ Example 2: Graphing a Linear Equation in Three Variables

____ Student Help: Study Tip

____ Example 3: Evaluating a Function of Two Variables

Vocabulary:

three-dimensional coordinate system, p. 170

z-axis, p. 170

ordered triple, p. 170

octants, p. 170

linear equation in three variables, p. 171

function of two variables, p. 171

____ **Goal 2**: Use functions of two variables to model real-life situations. (p. 172)

Material Covered:

____ Example 4: Modeling a Real-Life Situation

Activity 3.5: Graphing Linear Equations in Three Variables (p. 176)

____ **Goal**: Graph a linear equation in three variables using a graphing calculator.

____ Student Help: Keystroke Help

____ Other (specify) ______________________________

____

Homework and Additional Learning Support

____ Textbook (specify) pp. 173–175

____

____ Internet: Extra Examples at www.mcdougallittell.com

____ *Reteaching with Practice* worksheet (specify exercises) ______________

____ *Personal Student Tutor* for Lesson 3.5

NAME ______________________________ DATE __________

Interdisciplinary Applications

For use with pages 170–175

Farming

BUSINESS Farmers analyze their fields in terms of the terrain, type of soil, and the predicted weather. They make a determination of what to plant based on past performance of their soil in combination with the weather patterns. *The Farmer's Almanac* gives the farmers some statistics to review to help them make a final decision of how much of each crop to plant. In consulting a seed catalogue, farmers determine how much seed they must purchase to plant each acre.

In Exercises 1 and 2, use the following information.

Farmer Brown needs to determine how to plant his fields. He has two crops to plant, corn and barley. The labor cost is the same no matter how he divides his acreage for planting, but the price of seed is different. He pays someone $1000 to do the planting no matter how much of each seed is planted.

The price of seed for corn is $10 per acre and the price of barley is $9 per acre.

1. Write a model for the total amount that will be spent to plant his fields if he plans on planting 100 acres.
2. Evaluate the model for several different combinations of barley and corn where the total number of acres planted is 100. Organize the results in a table.

In Exercises 3 and 4, use the following information.

Farmer Brown has land suitable for oats and wheat. The price of oats is $11.50 per acre and the price of wheat is $15.00 per acre. The farmer pays $1000 for planting.

3. Write a model for the amount that will be spent to plant his field in oats and wheat if he continues to plant the same 100 acres.
4. Evaluate the model for several different combinations of wheat and rye where the total number of acres planted is 100. Organize the results in a table.

NAME ______________________________ DATE __________

Challenge: Skills and Applications

For use with pages 170–175

1. An equation of a plane in space of the form $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is sometimes said to be in *point-normal form* (which is somewhat analogous to point-slope form for a line in a plane). The numbers a, b, and c are called *direction numbers* for the plane.

a. Explain the "point" part of this name.

b. The line through the origin and the point (a, b, c) is perpendicular to this plane. (Such a line is said to be "normal" to the plane.) A set of parametric equations for this line is given by

$$x = at \qquad y = bt \qquad z = ct.$$

Find the value of t at which this line intersects the plane, in terms of a, b, c, x_1, y_1, and z_1.

2. An equation of the form $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$ is said to be in *intercept form.*

a. Explain this name.

b. Write the given equation in the form $ax + by + cz = k$, where a, b, c, and k are given in terms of p, q, and r.

c. Give a set of direction numbers a, b, and c (see Exercise 1), in terms of p, q, and r, such that the line through the origin and (a, b, c) is a line perpendicular to the plane with the above equation, as in part (b) of Exercise 1.

3. The numbers

$$y_1z_2 - y_2z_1,\ x_2z_1 - x_1z_2,\ x_1y_2 - x_2y_1$$

are direction numbers (see Exercise 1) for the plane that passes through the origin (0, 0, 0), the point (x_1, y_1, z_1), and the point (x_2, y_2, z_2).

a. Give an equation of the plane in point-normal form (see Exercise 1) in terms of x_1, y_1, z_1, x_2, y_2, and z_2, using the fact that the plane passes through the origin.

b. Show that (x_1, y_1, z_1) is on the plane.

LESSON 3.6

TEACHER'S NAME ______________________ CLASS ______ ROOM ______ DATE ______

Lesson Plan

1-day lesson (See *Pacing the Chapter,* TE pages 136C–136D) For use with pages 177–184

GOALS
1. **Solve systems of linear equations in three variables.**
2. **Use linear systems in three variables to model real-life situations.**

State/Local Objectives ____

____

✓ Check the items you wish to use for this lesson.

STARTING OPTIONS
____ Homework Check: TE page 173; Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 177 and 175, CRB page 82, or Transparencies

TEACHING OPTIONS
____ Motivating the Lesson: TE page 178
____ Lesson Opener (Visual Approach): CRB page 83 or Transparencies
____ Examples 1–4: SE pages 178–180
____ Extra Examples: TE pages 178–180 or Transparencies; Internet
____ Closure Question: TE page 180
____ Guided Practice Exercises: SE page 181

APPLY/HOMEWORK
Homework Assignment
____ Basic 12–27 multiples of 3, 34, 42–69 multiples of 3; Quiz 3: 1–14
____ Average 12–30 multiples of 3, 35–39, 42, 45–71 odd; Quiz 3: 1–14
____ Advanced 12–33 multiples of 3, 36–44, 45–71 odd; Quiz 3: 1–14

Reteaching the Lesson
____ Practice Masters: CRB pages 84–86 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 87–88 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson
____ Applications (Real-Life): CRB page 90
____ Challenge: SE page 183; CRB page 91 or Internet

ASSESSMENT OPTIONS
____ Checkpoint Exercises: TE pages 178–180 or Transparencies
____ Daily Homework Quiz (3.6): TE page 184, or Transparencies
____ Standardized Test Practice: SE page 183; TE page 184; STP Workbook; Transparencies
____ Quiz (3.5–3.6): SE page 184

Notes ____

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LESSON 3.6

TEACHER'S NAME ______________________ CLASS ______ ROOM ______ DATE ______

Lesson Plan for Block Scheduling

Half-day lesson (See *Pacing the Chapter,* TE pages 136C–136D) **For use with pages 177–184**

GOALS
1. **Solve systems of linear equations in three variables.**
2. **Use linear systems in three variables to model real-life situations.**

State/Local Objectives ______________________________________

CHAPTER PACING GUIDE	
Day	**Lesson**
1	3.1 (all)
2	3.2 (all)
3	3.3 (all); 3.4(all)
4	3.5 (all); **3.6 (all)**
5	Review/Assess Ch. 3

✓ **Check the items you wish to use for this lesson.**

STARTING OPTIONS

____ Homework Check: TE page 173; Answer Transparencies
____ Warm-Up or Daily Homework Quiz: TE pages 177 and 175, CRB page 82, or Transparencies

TEACHING OPTIONS

____ Motivating the Lesson: TE page 178
____ Lesson Opener (Visual Approach): CRB page 83 or Transparencies
____ Examples 1–4: SE pages 178–180
____ Extra Examples: TE pages 178–180 or Transparencies; Internet
____ Closure Question: TE page 180
____ Guided Practice Exercises: SE page 181

APPLY/HOMEWORK

Homework Assignment (See also the assignment for Lesson 3.5.)

____ Block Schedule: 12–30 multiples of 3, 35–39, 42, 45–71 odd; Quiz 3: 1–14

Reteaching the Lesson

____ Practice Masters: CRB pages 84–86 (Level A, Level B, Level C)
____ Reteaching with Practice: CRB pages 87–88 or Practice Workbook with Examples
____ Personal Student Tutor

Extending the Lesson

____ Applications (Real Life): CRB page 90
____ Challenge: SE page 183; CRB page 91 or Internet

ASSESSMENT OPTIONS

____ Checkpoint Exercises: TE pages 178–180 or Transparencies
____ Daily Homework Quiz (3.6): TE page 184, or Transparencies
____ Standardized Test Practice: SE page 183; TE page 184; STP Workbook; Transparencies
____ Quiz (3.5–3.6): SE page 184

Notes ______________________________________

LESSON 3.6

NAME ______________________ DATE __________

Available as a transparency

WARM-UP EXERCISES

For use before Lesson 3.6, pages 177–184

Solve the linear system.

1. $x - 5y = 10$
   $3x - 15y = 15$

2. $y = 2x + 3$
   $-4x + 2y = 6$

3. $x - y = -5$
   $x + 3y = 11$

Lesson 3.6

DAILY HOMEWORK QUIZ

For use after Lesson 3.5, pages 170–176

1. Plot $(3, 2, -4)$ in a three-dimensional coordinate system.

2. Sketch the graph of $-15x + 10y - 6z = -30$. Label the points where the graph crosses the x-, y-, and z- axes.

3. Write $8x - 5y + 2z = 9$ as a function of x and y. Then evaluate $f(-1, 6)$.

NAME ______________________ DATE ________

Practice B

For use with pages 177–184

Decide whether the given ordered triple is a solution of the system.

1. (0, 0, 3)
$3x + 4y + z = 3$
$-2x + 7y + 2z = 6$
$-10x + 12y - z = -3$

2. (−1, −2, 5)
$2x + y - 5z = -29$
$6x + 4y - z = -19$
$x + y + 2z = 7$

3. (0, 0, 0)
$x + y + z = 0$
$2x + 3y - z = 0$
$3x + y - 4z = 1$

4. (−1, −3, −2)
$x - 5y + 6z = 2$
$3x - y + 8z = -16$
$4x + 2y - 7y = 4$

5. (5, 7, 1)
$x + y + z = 13$
$2x - 7y + 5z = -34$
$3x + y + 4z = 25$

6. (−4, 8, −9)
$x + 2y - 3z = 39$
$2x + y - 7z = -63$
$3x + y + z = -13$

Use any algebraic method to solve the system.

7. $x - y + 2z = 4$
$x - 3z = 1$
$2y - z = -15$

8. $x + y - z = 6$
$2y - 3z = 4$
$-y + 2z = -1$

9. $x + 2y - z = 3$
$x - 3y + z = -1$
$-x + y - 3z = 5$

10. $x - 2y - z = 3$
$x + y + 2z = 9$
$2x + 3y + z = 0$

11. $2x + 3y + 2z = 1$
$x + 4y - z = 7$
$3x + y + 3z = -2$

12. $x - 2y + 3z = -7$
$4x + 5y + z = 4$
$-x + y - 2z = 5$

13. $8x + 2y - z = -25$
$3x - 3y + 5z = 10$
$-5x + 6y - 2z = 17$

14. $x + 5y - 2z = -16$
$-x - 7y + 3z = 23$
$3x - 10y - 5z = 5$

15. $3x + 2y - 8z = 4$
$6x + 4y - 16z = 8$
$12x + 8y - 32z = 16$

16. ***Pet Store Supplies*** A pet store receives a shipment of pet foods at the beginning of each month. Over a three month period, the store received 1770 pounds of dog food, 1165 pounds of cat food, and 365 pounds of bird seed. Write and solve a system of equations to find the number of pounds of pet food in each of the three shipments.

Pet food	*1st shipment*	*2nd shipment*	*3rd shipment*
Dog food	60%	50%	50%
Cat food	25%	35%	45%
Bird seed	15%	15%	5%

17. ***Movie Rental Store*** The table below shows the percent of comedies, drama, and action videos available at a video store. Write and solve a system of equations to find out how many comedies, dramas, and action movies are at the store. Assume that the store has a collection of 3405 general videos to be rented, 1070 children's videos to be rented, and 1225 videos for sale.

Store section	*Comedy*	*Drama*	*Action*
General rental	55%	65%	60%
Children's rental	25%	10%	20%
Videos for sale	20%	25%	20%

LESSON 3.6

NAME ______________________________ DATE __________

Practice C

For use with pages 177–184

Lesson 3.6

Solve the system using the linear combination method.

1. $2x + 3y + z = 10$
$2x - 3y - 3z = 22$
$4x - 2y + 3z = -2$

2. $2x - y + 4z = -4$
$3x + 2y + z = -2$
$5x - 2y + 3z = 0$

3. $5x - 3y + 2z = 3$
$2x + 4y - z = 7$
$x - 11y + 4z = 3$

Solve the system using the substitution method.

4. $3x - y + 2z = 3$
$2x - 3y + 5z = 4$
$-2x + y - z = -4$

5. $x - 3y + 5z = 3$
$4x + 5y - 2z = 7$
$3x + 2y + 4z = 9$

6. $2x + 3y - z = 2$
$x - 5y + 3z = 8$
$5x + y + z = 12$

Solve the system using any algebraic method.

7. $2x + 5y - 4z = -7$
$4x + 2y + 3z = 8$
$2x - 8y + 5z = 11$

8. $6x + 3y - 9z = 7$
$2x + 2y + 9z = -1$
$5x - y - 6z = 3$

9. $2x + 3y + 6z = \frac{7}{2}$
$3x + 4y + 7z = 4$
$5x - 2y - 4z = -\frac{11}{4}$

10. $6x - 6y + 2z = -5$
$12x + 3y + 4z = 0$
$4x + 9y + 2z = 6$

11. $3x + 3y - 4z = 3$
$x + 2y - 8z = -1$
$6x - 9y - 4z = 12$

12. $x + 2y - z = 4$
$3x - y + 4z = -2$
$6x + 5y + z = 10$

Solve the system of equations.

13. $w + x + y + z = 1$
$2w + x - y - z = 4$
$w - x - 2y + 2z = 2$
$3w + 2x + y + z = 7$

14. $w + 2x - y + 3z = 3$
$w + x + 2y - 2z = 3$
$2w + 2x + 2y - z = 6$
$3w - x - y + 4z = 12$

Polynomial Curve Fitting **In Exercises 15–18, use the following information.** You can use a system of equations to find a polynomial of degree n whose graph passes through $(n + 1)$ points. Consider a polynomial of degree 2, $y = ax^2 + bx + c$. Suppose $(1, 3)$, $(-1, -3)$, and $(2, 12)$ lie on the graph. Using the point $(1, 3)$, the following equation can be derived:

$$y = ax^2 + bx + c$$
$$3 = a(1)^2 + b(1) + c$$
$$3 = a + b + c.$$

The equation $a + b + c = 3$ becomes the first equation in the system.

15. Write the equation in the system that corresponds to the point $(-1, -3)$.

16. Write the equation in the system that corresponds to the point $(2, 12)$.

17. Write a system of equations for the coefficients of a polynomial of degree 2 that passes through $(1, 3)$, $(-1, -3)$, and $(2, 12)$. Solve the system.

18. Write the polynomial.

LESSON 3.6

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 177–184

GOAL **Solve systems of linear equations in three variables and use linear systems in three variables to model real-life situations**

VOCABULARY

A **system of three linear equations** includes three equations in the same variables.

A **solution** of a linear system in three variables is an ordered triple (x, y, z) that satisfies all three equations. The linear combination method you learned in Lesson 3.2 can be extended to solve a system of linear equations in three variables.

The Linear Combination Method (3-Variable Systems)

Step 1: Use the linear combination method to rewrite the linear system in three variables as a linear system in *two* variables.

Step 2: Solve the new linear system for both of its variables.

Step 3: Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

Note: If you obtain a false equation, such as $0 = 1$, in any step, then the system has no solution. If you do not obtain a false solution, but obtain an identity, such as $0 = 0$, then the system has infinitely many solutions.

EXAMPLE 1 Using the Linear Combination Method

Solve the system.

$x + y + z = 2$ **Equation 1**

$-x + 3y + 2z = 8$ **Equation 2**

$4x + y = 4$ **Equation 3**

SOLUTION

Since Equation 3 does not have a z-term, eliminate the z from one of the other equations.

$-x + 3y + 2z = 8$ Add -2 times the first equation to the second.

$\underline{-2x - 2y - 2z = -4}$

$-3x + y = 4$ New Equation 2

Now solve the system of the new Equation 2 and Equation 3.

$-3x + y = 4$ New Equation 2

$\underline{-4x - y = -4}$ Add -1 times Equation 3 to Equation 2.

$-7x = 0$ Solve for x: $x = 0$.

Substitute 0 for x into Equation 3 and solve for y: $y = 4$.

Substitute $x = 0$ and $y = 4$ into either original Equation 1 or 2 and solve for z: $0 + 4 + z = 2$, so $z = -2$.
The solution is the ordered triple $(0, 4, -2)$.

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 177–184

Exercises for Example 1

Solve the system using any algebraic method.

1. $5x + 2y - z = -7$
$x - 2y + 2z = 0$
$3y + z = 17$

2. $x - 2y - 3z = -1$
$2x + y + z = 6$
$x + 3y - 2z = 13$

3. $x + y + z = 6$
$2x - y + z = 3$
$3x - z = 0$

EXAMPLE 2 Solving a System with Many Solutions

Solve the system.

$2x + y + z = 0$ **Equation 1**

$x - 2y - 2z = 0$ **Equation 2**

$x + y + z = 0$ **Equation 3**

SOLUTION

$x - 2y - 2z = 0$ Equation 2
$\underline{-x - y - z = 0}$ -1 times Equation 3
$-3y - 3z = 0$ New Equation 2

$2x + y + z = 0$ Equation 1
$\underline{-2x - 2y - 2z = 0}$ -2 times Equation 3
$-y - z = 0$ New Equation 1

$-3y - 3z = 0$
$\underline{3y + 3z = 0}$
$0 = 0$

Because $0 = 0$ is always a true equation, the system has infinitely many solutions. To describe the solution, express two of the variables in terms of the third. One way to do this is to express x and y in terms of z. Using the new Equation 2 you get $y = -z$ when you solve for y. Now substitute z for z and $-z$ for y in any of the original equations and solve for x. The solution is any ordered triple of the form $(0, -z, z)$.

Exercises for Example 2

Solve the system using any algebraic method and describe the solution.

4. $x + 3y + z = 0$
$x + y - z = 0$
$x - 2y - 4z = 0$

5. $3x - 2y + 4z = 1$
$x + y - 2z = 3$
$2x - 3y + 6z = 8$

6. $x - y + 2z = 4$
$x + z = 6$
$2x - 3y + 5z = 4$

7. $x - 2y + z = -6$
$2x - 3y = -7$
$-x + 3y - 3z = 11$

LESSON 3.6

NAME ______________________________ DATE __________

Quick Catch-Up for Absent Students

For use with pages 177–184

The items checked below were covered in class on (date missed) __________

Lesson 3.6: Solving Systems of Linear Equations in Three Variables

____ **Goal 1**: Solve systems of linear equations in three variables. (pp. 177–179)

Material Covered:

____ Example 1: Using the Linear Combination Method

____ Student Help: Look Back

____ Example 2: Solving a System with No Solution

____ Example 3: Solving a System with Many Solutions

Vocabulary:

system of three linear equations in three variables, p. 177

solution of a system of three linear equations, p. 177

____ **Goal 2**: Use linear systems in three variables to model real-life situations. (p. 180)

Material Covered:

____ Example 4: Writing and Solving a Linear System

____ Other (specify) ______________________________

____

Homework and Additional Learning Support

____ Textbook (specify) pp. 181–184

____

____ Internet: Extra Examples at www.mcdougallittell.com

____ *Reteaching with Practice* worksheet (specify exercises) __________

____ *Personal Student Tutor* for Lesson 3.6

Lesson 3.6

LESSON 3.6

NAME ______________________________ DATE __________

Real-Life Application: When Will I Ever Use This?

For use with pages 177–184

Basketball Statistics

Basketball coaches like to begin a game with the players who can score early in the game. You are the statistician for the girls' basketball team and the coach needs you to help him in determining the starting line-up for the upcoming game. You are analyzing the statistics of the results of the first quarter of the last game. Only 7 shots were made and 14 points were scored. How many three-point shots, baskets, and free throws were made if there were as many three-point shots as baskets and free throws combined?

1. Write an equation that shows the combination of type of shots made.
2. Write another equation that represents the value of each type of shot.
3. Write a third equation that shows the relationship of the comparison of three-point shots with free throws.
4. Solve the system of equations to determine how many of each type of shot was made.
5. Will these results help the coach in making the starting line up? Why or why not?

NAME ______________________ DATE ________

Challenge: Skills and Applications

For use with pages 177–184

1. Find a, b, and c so that the graph of the equation $y = ax^2 + bx + c$ passes through the points $(2, 1)$, $(-3, 31)$, and $(4, 17)$.

2. a. For the system of equations

$$3x - 2y + z = 8$$
$$2x + y - 4z = -4$$
$$5x - y - 3z = 4,$$

if (x_0, y_0, z_0) is a solution of the first two equations, it is also a solution of the third equation. Explain how you know this is true.

b. By eliminating z from the first two equations, write a relationship between x and y that must be true for any solution. Find a solution of the system. Explain why there must be infinitely many solutions.

3. If a linear system has the form

$$ax + by + cz = 0$$
$$dx + ey + fz = 0$$
$$gz + hy + kz = 0,$$

it is called *homogeneous.*

a. Explain why any homogeneous system has at least one solution.

b. Show that if (x_0, y_0, z_0) is a solution of a homogeneous system, then so is (rx_0, ry_0, rz_0) for any real number r. Use this fact to state a general fact about the nature of the solution set of a homogeneous system (i. e. the number of solutions the system has).

c. Show that if (x_1, y_1, z_1) and (x_2, y_2, z_2) are solutions of a homogeneous system, then so is $(x_1 + x_2, y_1 + y_2, z_1 + z_2)$.

d. Suppose System (*) is like the one above, but with nonzero numbers replacing the 0's on the right-hand side of each equation. Suppose also that (x_*, y_*, z_*) is a solution of System (*), and (x_0, y_0, z_0) is a solution of the homogeneous system. What can you say about the triple $(x_* + x_0, y_* + y_0, z_* + z_0)$?

CHAPTER 3

NAME ______________________ DATE __________

Chapter Review Games and Activities

For use after Chapter 3

A Random Random Random Review

1. Members of the class can be divided into two teams as they enter the classroom. Even-numbered students are to sit on one side of the room and odd-numbered students on the other side.
2. Use a calculator or a random number table to generate random numbers to form groups of 4 or 5 within each team. (This may vary according to class size.)
3. Next, the teacher should generate a random number from one to six. That number will correspond to the section number in the Chapter 3 Review Exercises on pages 186–188. For example, if the number 4 was generated, section 3.4 would be chosen. An additional random number should next be generated from 13–16, since these numbers are the problem numbers in the section. This will then be the problem that all teams will work on together in their groups.
4. The teacher should determine an appropriate amount of time for groups to solve the problem.
5. After sufficient time has elapsed, the teacher should generate another random number to determine which person on the team is chosen to explain their answer to the problem. (For example, ask each team to determine a person whose birthday is closest to the day this activity is taking place, or whose phone number or Social Security number ends in a randomly chosen digit.) If that randomly-chosen person can answer and explain the problem correctly, his or her team receives one point. If not, the other team has the opportunity to select a random person to explain the answer. If the second team gives the correct answer, they receive the point, and it is also their turn to answer the next question since teams alternate going first in presenting the answers.
6. The next and succeeding problems are chosen according to the directions in step 3. It is suggested to end the activity approximately 10 minutes before the end of the period. The team with the most points is determined the winner.
7. The teacher should choose an appropriate "prize" for the winners. Examples of prizes may be that the homework before the exam would consist of randomly-generated problems from the Review Exercises that were not completed during this activity. The winning team would not be required to complete the entire assignment, or possibly they would each receive a bonus point on their upcoming exam.

Alternative Plan

Students can be assigned the entire Review section for homework the night before this activity, and their answers can be checked according to the directions above in number 3. In this manner, there would be time for more problems to be reviewed, but the random selection makes it fair.

CHAPTER 3

NAME ______________________ DATE __________

Chapter Test B

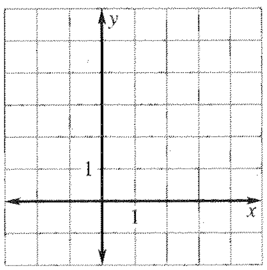
For use after Chapter 3

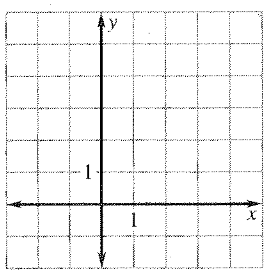
Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

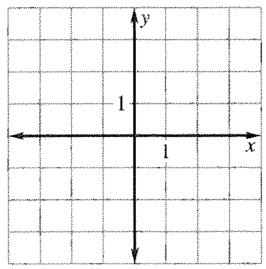
1. $y = 3x$
$y = -x + 4$

2. $y = x + 1$
$y = -x + 3$

3. $x + 2y = -2$
$-3x - 6y = 6$







Solve the system using any algebraic method.

4. $x + y = 2$
$y = 2x + 5$

5. $y - 2x = -5$
$y - x = -3$

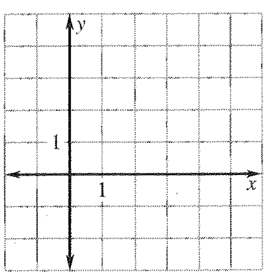
6. $2x - y = -8$
$2x + y = 4$

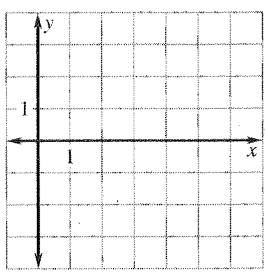
Graph the system of linear inequalities.

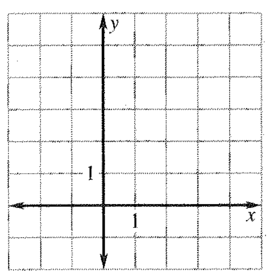
7. $x + 2y \geq 4$
$x - y \leq 3$

8. $y \leq 2$
$x > 3$

9. $x \geq 0$
$y \geq 0$
$2x + y \leq 4$







Find the minimum and maximum values of the objective function subject to the given constraints.

10. Objective Function: $C = 4x + 5y$

Constraints: $x \geq 0$
$y \geq 0$
$x + y \leq 6$

11. Objective Function: $C = 3x + 2y$

Constraints: $x \geq 0$
$y \geq 0$
$x + 3y \leq 15$
$4x + y \leq 16$

Answers

1. Use grid at left. ____________
2. Use grid at left. ____________
3. Use grid at left. ____________
4. ____________
5. ____________
6. ____________
7. Use grid at left.
8. Use grid at left.
9. Use grid at left.
10. ____________
11. ____________

Review and Assess

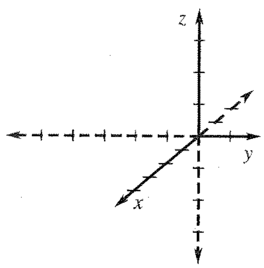
NAME ______________________________ DATE __________

Chapter Test B

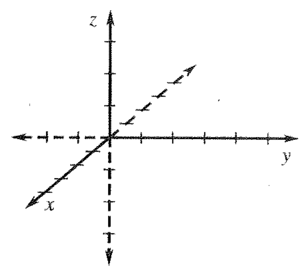
For use after Chapter 3

Plot the ordered triple in a three-dimensional coordinate system.

12. $(3, -4, 2)$

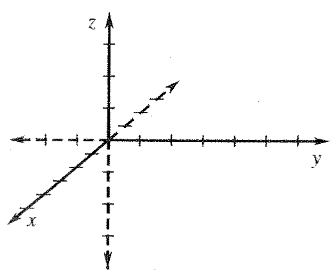


13. $(-4, 1, -2)$

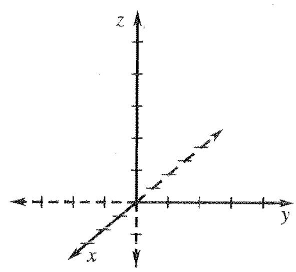


Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

14. $x + y + 2z = 10$



15. $4x + 2y + z = 4$



16. Write the linear equation $2x + 3y + z = 6$ as a function of x and y. Then evaluate the function when $x = 2$ and $y = 3$.

Solve the system using any algebraic method.

17. $x + 4y + z = 12$

$y - 3z = -7$

$z = 3$

18. $x + y + 2z = 5$

$x + 2y + z = 8$

$2x + 3y - z = 1$

19. ***Earning money*** You work at a grocery store. Your hourly wage is greater after 6:00 P.M. than it is during the day. One week you work 20 daytime hours and 20 evening hours and earn $280. Another week you work 30 day time hours and 12 evening hours and earn a total of $276. What is your daytime rate? What is your evening rate?

20. ***Telethon*** During a recent telethon, people pledged $25 or $50. Twice as many people pledged $25 as $50. Altogether, $18,000 was pledged. How many people pledged $25?

12. Use grid at left.

13. Use grid at left.

14. Use grid at left.

15. Use grid at left.

16. ______________

17. ______________

18. ______________

19. ______________

20. ______________

CHAPTER 3

NAME ______________________ DATE __________

Chapter Test C

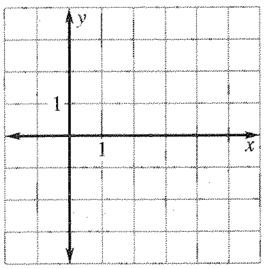
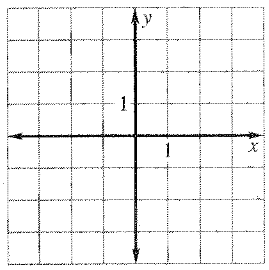
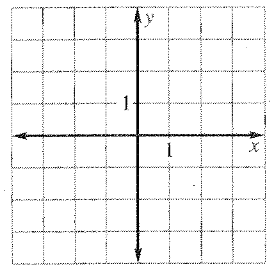
For use after Chapter 3

Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

1. $2x - y = 5$
$-x + y = -3$

2. $2x - 3y = -6$
$-3y + 2x = 3$

3. $3x + y = 1$
$2y = 2 - 6x$

Solve the system using any algebraic method.

4. $3x - 2y = 10$
$5x + 3y = -15$

5. $2x - 4y = -6$
$-x + 2y = 3$

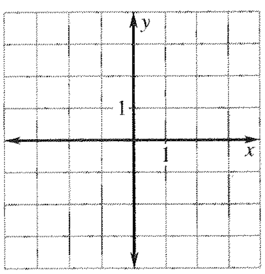
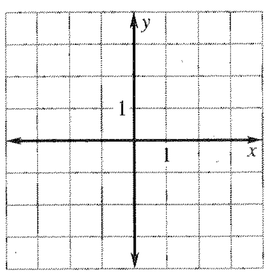
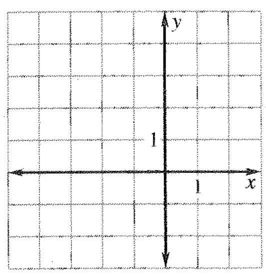
6. $3x - 5y + 10 = 0$
$-9x + 15y = -30$

Graph the system of linear inequalities.

7. $x \leq 0$
$y \geq 0$

8. $x + y > -1$
$3x - 2y > 4$

9. $-y \leq -2x - 3$
$x + 2 \leq 0$

Find the minimum and maximum values of the objective function subject to the given constraints.

10. Objective function: $C = 4x + y$
Constraints: $x \geq 0$
$y \geq 0$
$x + y \leq 3$

11. Objective function: $C = 6x + 7y$
Constraints: $x \geq 0$
$y \geq 0$
$4x + 3y \geq 24$
$x + 3y \geq 15$

Answers

1. Use grid at left. ______
2. Use grid at left. ______
3. Use grid at left. ______
4. ______
5. ______
6. ______
7. Use grid at left.
8. Use grid at left.
9. Use grid at left.
10. ______
11. ______

Review and Assess

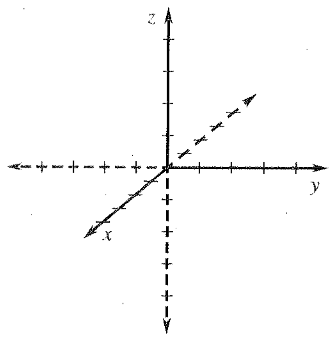
NAME ______________________ DATE __________

Chapter Test C

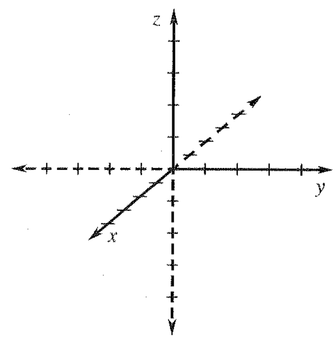
For use after Chapter 3

Plot the ordered triple in a three-dimensional coordinate system.

12. $(2, 1, 4)$

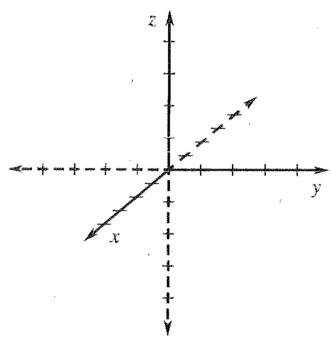


13. $(-3, 4, -4)$

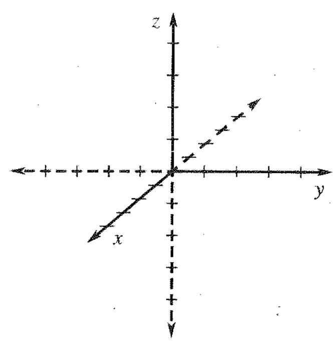


Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

14. $x + y - z = 4$



15. $-3x - 3y + 3z = 12$



16. Write the linear equation $2x + 3y + z = 12$ as a function of x and y. Then evaluate the function when $x = 4$ and $y = 1$.

Solve the system using any algebraic method.

17.
$$-3x + 4y = -6$$
$$5x - 3z = -22$$
$$3y + 2z = -1$$

18.
$$3x + 2y + 2z = -3$$
$$2x + 3y + 3z = -2$$
$$-3x - 5y + z = -9$$

19. ***Stamps*** Postcard stamps are 20¢ each, while letter stamps are 33¢ each. If you have 50 stamps worth \$12.60, how many of each type do you have?

20. ***Numbers*** The sum of the digits of a two-digit number is 9. If the digits are reversed, the new number is 27 more than the original number. Find the original number.

12. Use grid at left.

13. Use grid at left.

14. Use grid at left.

15. Use grid at left.

16. ____________

17. ____________

18. ____________

19. ____________

20. ____________

NAME ______________________ DATE __________

SAT/ACT Chapter Test

For use after Chapter 3

1. Solve the system using any algebraic method.

$x + y = 2$
$x - y = 0$

Ⓐ $(-1, -1)$ Ⓑ $(1, 1)$
Ⓒ $(0, 0)$ Ⓓ $(-1, 1)$

2. The ordered pair $(-6, 2)$ is a solution of which system?

Ⓐ $4x - 3y = 7$, $x + y = -4$

Ⓑ $2x - 3y = -18$, $x - y = 8$

Ⓒ $10x + 13y = -34$, $-15x - 17y = 56$

Ⓓ $x + y = 10$, $x - y = -8$

3. How many solutions does the following system have?

$x + y = 4$
$2x + 2y = 8$

Ⓐ no solution
Ⓑ infinitely many solutions
Ⓒ two solutions
Ⓓ one solution

4. How many solutions does the following system have?

$5x - 3y = -4$
$x + 2y = 7$

Ⓐ no solution
Ⓑ infinitely many solutions
Ⓒ two solutions
Ⓓ one solution

5. At which point does the graph of $3x + 5y - z = 15$ cross the y-axis?

Ⓐ $(0, 3, 0)$ Ⓑ $(5, 0, 0)$
Ⓒ $(0, 0, -15)$ Ⓓ $(0, -3, 0)$

6. What geometric figure does the graph of the system form?

$y \le -\frac{1}{2}x + 2$
$x \ge 0$
$y \ge 0$

Ⓐ square Ⓑ triangle
Ⓒ rectangle Ⓓ rhombus

In Questions 7–9, choose the statement below that is true about the given numbers.

Ⓐ The number in column A is greater.
Ⓑ The number in column B is greater.
Ⓒ The two numbers are equal.
Ⓓ The relationship cannot be determined from the given information.

7.

Column A	*Column B*
$x \ge 2$	$x \ge -2$

Ⓐ Ⓑ Ⓒ Ⓓ

8.

Column A	*Column B*
x-value of the solution of $x + y = 10$, $x - y = 2$	y-value of the solution of $x + y = 10$, $x - y = 2$

Ⓐ Ⓑ Ⓒ Ⓓ

9.

Column A	*Column B*
value of z in $(4, 5, 3)$	value of y in $(-4, 3, 2)$

Ⓐ Ⓑ Ⓒ Ⓓ

NAME ______________________________ DATE __________

Alternative Assessment and Math Journal

For use after Chapter 3

JOURNAL

1. Consider the system of equations. $\begin{aligned} 3x + 2y + z &= 10 \\ x + y - 2z &= 4 \\ -2x + y - 3z &= 12 \end{aligned}$

(a) Explain how to solve the system of equations. Be sure to include all steps. You do not need to solve the system, just explain the steps. (b) Why is it necessary to create a new system of two equations in two variables? (c) Explain a second method for solving the system of equations. (d) List the three possible outcomes for intersection when sketching the graph of three planes in a three-dimensional coordinate system. (e) Give examples of at least two of the possible outcomes from part d by creating systems with those characteristics.

MULTI-STEP PROBLEM

2. Use the constraints below to answer the following.

$$x \geq 0$$
$$y \geq 0$$
$$2x + y \leq 120$$
$$x + 3y \leq 150$$
$$2x + 2y \leq 140$$

a. Sketch the graph of the constraints and shade the appropriate area to find the feasible region.

b. What are the vertices of the feasible region?

c. Explain why $x \geq 0$ and $y \geq 0$ are constraints in most linear programming problems.

d. Write an objective function for the given constraints to satisfy the following conditions.

- (30, 40) is where the maximum occurs
- (50, 20) is where the maximum occurs
- (60, 0) is where the maximum occurs

e. For the objective function $Ax + By = 100$ and the given constraints, find A and B such that (0, 50) is the maximum.

3. ***Writing*** Write a linear programming problem that could be described by the constraints in Exercise 2.

Alternative Assessment Rubric

For use after Chapter 3

JOURNAL SOLUTION

1. a–e. Complete answers should address these points:

   a. *Sample answer:* Use the linear combination method to rewrite the linear system in three variables as a linear system in *two* variables. Solve the new linear system for both of its variables. Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

   b. • Explain that with two variables, one can graph to determine the solution, or can eliminate one variable to solve for the other (substitution method).

   c. • A second method would be to graph the equations in space.

   d. • Explain that three equations in three variables could either intersect in one point, intersect in a line, or not at all.

   e. • Examples should include two of the three cases: a system that intersects in one point, one that intersects in a line, or one that does not intersect at all.

MULTI-STEP PROBLEM SOLUTION

2. a. Check graph.

   b. (60, 0), (0, 50), (30, 40), (50, 20), (0, 0)

   c. *Sample answer:* In most linear programming problems, the variables cannot be negative; this ensures that the solutions are first quadrant values.

   d. *Sample answer:* (30, 40) $P = 8x + 10y$, (50, 20) $P = 5x + 3y$, (60, 0) $P = 5x + y$

   e. *Sample answer:* $A = -1, B = 2$

3. Answers will vary.

MULTI-STEP PROBLEM RUBRIC

4 Students complete all parts of the questions accurately. Explanations are clear and logical. Students can explain why different objective quantities yield different maximum values. Student examples illustrate an understanding of the concepts. Student linear programming problem accurately models constraints and objective quantities.

3 Students complete the questions and explanations. Explanations may be somewhat vague, but show some understanding of Objective Quantities. Student examples illustrate the different concepts. Student linear programming problem models constraints and objective quantities.

2 Students complete questions and explanations. Explanations are not clear. Examples are incomplete or inaccurate.

1 Students' work is very incomplete. Solutions or reasoning are incorrect. Examples are incomplete or inaccurate.

CHAPTER 3

NAME ______________________ DATE __________

Project: Puzzling Inequalities

For use with Chapter 3

OBJECTIVE **Create and solve puzzles using systems of inequalities.**

MATERIALS Graph paper, colored pencils

INVESTIGATION In this project, you will create a puzzle by making a drawing, and then providing clues to classmates so that they can recreate your drawing.

Background To create your clues, you will need to find a system of inequalities that defines a given polygon-shaped region in the plane. To find the inequality corresponding to one side of a polygon, first identify two points on the side (usually vertices of the polygon). Then find an equation of the line containing the two points. Finally, change the equals sign to a $\leq$ or $\geq$ sign, as appropriate.

1. Create a drawing by shading several polygons in the coordinate plane. You can use a different color for each polygon, if desired. The vertices of each polygon should be grid intersections, and each polygon should be a relatively simple shape that could represent the feasible region for a linear programming problem. Keep your drawing secret from the other students in your class.
2. Find a system of inequalities corresponding to each polygon you drew. For each system of inequalities, write down the correct color for the shading. You've designed a puzzle!
3. Check your work by solving your own puzzle. By graphing each system of inequalities in the correct color, you should be able to recreate your original drawing.
4. Now exchange puzzles with other students in your class. Solve another student's puzzle by graphing systems of inequalities.
5. See if your drawing matches the other student's solution.

PRESENT YOUR RESULTS In addition to giving your own puzzle and its solution, write an evaluation of the puzzle you solved. Give suggestions for improvement, if you have any. What general advice would you give to someone who is trying to create a puzzle?

Review and Assess

Project: Teacher's Notes

For use with Chapter 3

GOALS
- Write an inequality by finding the equation of a line given two points on the line.
- Write a system of inequalities to describe a polygon-shaped region.
- Solve systems of inequalities graphically.

MANAGING THE PROJECT If students seem confused about how to create a puzzle, you may wish to provide an example. The sample answer given in the back of the resource book may be used for this purpose.

RUBRIC **The following rubric can be used to assess student work.**

4 The student creates an attractive and accurate puzzle. The student draws an accurate solution for another student's puzzle. The student writes a meaningful critique of the other student's puzzle, and gives helpful general advice to anyone who wants to create a puzzle.

3 The student creates an attractive puzzle, but may make one or two mistakes in writing the inequalities to represent the puzzle. The student draws a reasonably accurate solution for another student's puzzle. Most of the student's critique and advice is on-target.

2 The student attempts to create a puzzle and to solve another student's puzzle. However, work may be incomplete or reflect misunderstandings. For example, the student may not know when to use a $\leq$ sign and when to use a $\geq$ sign. The critique and advice may indicate a limited grasp of certain ideas or may lack key supporting evidence.

1 Portions of the student's puzzle are missing or do not show an understanding of key ideas. The student is unable to solve another student's puzzle. The report does not give a reasonable critique or advice.

CHAPTER 3

NAME ______________________________ DATE ____________

Cumulative Review

For use after Chapters 1–3

Tell what property the statement illustrates. (1.1)

1. $3 \cdot 4 = 4 \cdot 3$

2. $4 \cdot \frac{1}{4} = 1$

3. $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$

Select and perform an operation to answer the question. (1.1)

4. What is the sum of 55 and -8?

5. What is the difference of -2 and -8?

6. What is the product of 9 and -5?

7. What is the quotient of -15 and $-\frac{3}{2}$?

Simplify the expression. (1.2)

8. $7x^2 + 5x - 9 + 3x^2 - 2x - 7$

9. $3(x - 8) + 5(2x - 6)$

10. $4(x^2 - x + 7) + 3(2x^2 + x)$

11. $8(4x + 2y) - 2(5x - 8y)$

Solve the equation. Check your solution. (1.3)

12. $5x + 7 = 22$

13. $3a + 5 = 7a + 21$

14. $2(x + 8) = -2(x - 12)$

15. $3(-2x + 8) = 4(x + 2) - 4$

16. $\frac{9}{2}x - 2 = 3x + 4$

17. $\frac{1}{2}x + \frac{5}{3} = \frac{2}{3}x - \frac{5}{6}$

Solve the equation for *y*. (1.4)

18. $x + xy = 8$

19. $6x - 4y = 12$

20. $-x = 3y + 18$

21. $6x + 5y + 30 = 0$

22. $-xy + 8 = x$

23. $x = 12 + xy$

Solve the inequality. Then graph the solution. (1.6–1.7)

24. $3(n - 4) < 9$

25. $4 - 4x > 5(3 + x)$

26. $\frac{1}{2}x + 8 \geq 12$

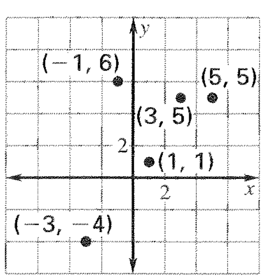
27. $3x + 7 \geq 10$

28. $4x - 2 < 6$ or $3x + 1 > 22$

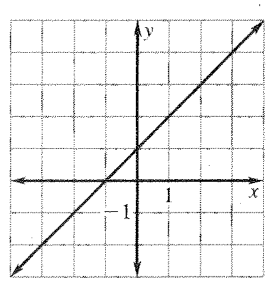
29. $-5 < 2x + 1 < 15$

Use the vertical line test to determine whether the relation is a function. (2.1)

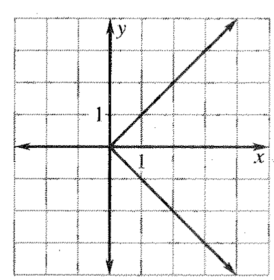
30.



31.



32.



Tell which line is steeper. (2.2)

33. Line 1: through $(-3, 5)$ and $(0, -1)$
Line 2: through $(1, 10)$ and $(6, -14)$

34. Line 1: through $(4, 5)$ and $(8, 5)$
Line 2: through $(6, 3)$ and $(8, -4)$

35. Line 1: through $(2, 3)$ and $(3, 6)$
Line 2: through $(0, 7)$ and $(2, 9)$

36. Line 1: through $(0, 0)$ and $(4, 2)$
Line 2: through $(-3, -2)$ and $(-4, -4)$

NAME ______________________ DATE ____________

Cumulative Review

For use after Chapters 1–3

Find the slope and y-intercept of the line. (2.3)

37. $y = 4x + 6$

38. $y = -\frac{2}{3}x - 5$

39. $y = 10$

40. $3x - 2y = 14$

41. $x + 8y = 16$

42. $9x + y = 0$

Write an equation of the line that passes through the given point and has the given slope. (2.4)

43. $(0, 7), m = 5$

44. $(-6, 4), m = 0$

45. $(5, 1), m = \frac{2}{3}$

46. $(4, -1), m = -\frac{2}{3}$

47. $(5, 0), m = -4$

48. $(-2, -1), m = -3$

Graph the inequality in a coordinate plane. (2.6)

49. $x \le -3$

50. $2y > -10$

51. $y \ge 3x + 2$

52. $y < -4 - 2x$

53. $3x + 4y > 12$

54. $\frac{2}{3}x + \frac{1}{2}y > 1$

Graph the absolute value function. Then identify the vertex, tell whether the graph opens up or down, and tell whether the graph is wider, narrower, or the same width as the graph of $y = |x|$. (2.8)

55. $f(x) = -|x - 7| + 1$

56. $f(x) = |x + 3| - 2$

57. $f(x) = -|x| + 2$

58. $f(x) = |x + 2|$

59. $f(x) = 2|x| + 2$

60. $f(x) = -\frac{1}{2}|x| + 4$

Graph the system of linear inequalities. (3.3)

61. $y \ge 5$
$x \le 2$

62. $x + y \ge 4$
$2x - y \le 3$

63. $5x + 3y \le 6$
$2x - 4y > 8$

64. $y > x - 5$
$y < 2x + 1$

65. $x - y \ge 5$
$3x + y \le -8$

66. $x > -6$
$x - y \ge 0$

Solve the system using either the linear combination method or the substition method. (3.6)

67. $x + 2y + z = 2$
$2x - 3y + 2z = -10$
$x + 3y + z = 4$

68. $x + y + z = 0$
$5x + 3y + z = 10$
$x + y = -z$

69. $x + y + z = 10$
$2x + y - 2z = -7$
$6x + 4y - 2z = 6$

70. ***Size of House*** In 1997, the chairman of an Oriental Holding Company was reported to have sold a property for \$98.8 million. At \$2,863 per square foot, it was the world's most expensive house. How big was the house to the nearest square foot? **(1.1)**

71. ***Surface Area*** Lake Superior, the largest of the Great Lakes, has a surface area of 20,600 square miles. This is 3300 square miles larger than five times the size of Lake Ontario, the smallest. What is the surface area of Lake Ontario? **(1.5)**