CHAPTER 14

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Graph sine, cosine, and tangent functions. **(14.1)**	Graph the height of a boat moving over waves. **(p. 836)**
Graph translations and reflections of sine, cosine, and tangent graphs. **(14.2)**	Graph the height of a person rappelling down a cliff. **(p. 843)**
Use trigonometric identities to simplify expressions. **(14.3)**	Simplify the parametric equations that describe a carousel's motion. **(p. 854)**
Verify identities that involve trigonometric expressions. **(14.3)**	Show that two equations modeling the shadow of a sundial are equivalent. **(p. 853)**
Solve trigonometric equations. **(14.4, 14.6, 14.7)**	Solve an equation that models the position of the sun at sunrise. **(p. 860)**
Write sine and cosine models for graphs and data. **(14.5)**	Write models for temperatures inside and outside an igloo. **(p. 866)**
Use sum and difference formulas. **(14.6)**	Relate the length of an image to the length of an actual object when taking aerial photographs. **(p. 873)**
Use double- and half-angle formulas. **(14.7)**	Find the angle at which you should kick a football to make it travel a certain distance. **(p. 878)**
Use trigonometric functions to solve real-life problems. **(14.1–14.7)**	Model real-life patterns, such as the vibrations of a tuning fork. **(p. 833)**

How does Chapter 14 fit into the BIGGER PICTURE of algebra?

In Chapter 14 you continued your study of trigonometry, focusing more on algebra connections than geometry connections. You graphed trigonometric functions and studied characteristics of the graphs, just as you have done with other types of functions during this course.

In this chapter you saw how some algebraic skills are used in trigonometry, such as in solving a trigonometric equation in quadratic form. If you go on to study higher-level algebra, you will see how trigonometry is used in algebra, such as in finding the complex *n*th roots of a real number.

STUDY STRATEGY

How did you use multiple methods?

Here is an example of two methods used for Example 2 on page 849 following the **Study Strategy** on page 830.

Multiple Methods

(1) $\sec\theta\tan^2\theta + \sec\theta = \sec\theta(\sec^2\theta - 1) + \sec\theta$

$= \sec^3\theta - \sec\theta + \sec\theta$

$= \sec^3\theta$

(2) $\sec\theta\tan^2\theta + \sec\theta = \sec\theta(\tan^2\theta + 1)$

$= \sec\theta(\sec^2\theta)$

$= \sec^3\theta$

CHAPTER 14

Chapter Review

VOCABULARY

- periodic function, p. 831
- cycle, p. 831
- period, p. 831
- amplitude, p. 831
- frequency, p. 833
- trigonometric identities, p. 848

14.1 Graphing Sine, Cosine, and Tangent Functions

Examples on pp. 831–834

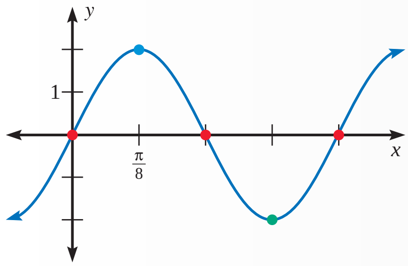
EXAMPLES You can graph a trigonometric function by identifying the characteristics and key points of the graph.

$y = 2 \sin 4x$

Amplitude $= |2| = 2$ Period $= \frac{2\pi}{|4|} = \frac{\pi}{2}$

Intercepts: $(0, 0)$; $\left(\frac{\pi}{4}, 0\right)$; $\left(\frac{\pi}{2}, 0\right)$

Maximum: $\left(\frac{\pi}{8}, 2\right)$ **Minimum:** $\left(\frac{3\pi}{8}, -2\right)$

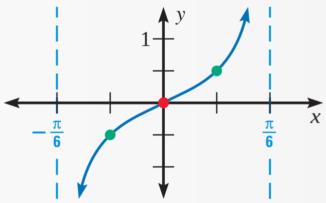


$y = \frac{1}{2} \tan 3x$

Period $= \frac{\pi}{|3|} = \frac{\pi}{3}$ **Intercept:** $(0, 0)$

Asymptotes: $x = \frac{\pi}{6}, x = -\frac{\pi}{6}$

Halfway points: $\left(\frac{\pi}{12}, \frac{1}{2}\right)$; $\left(-\frac{\pi}{12}, -\frac{1}{2}\right)$



Draw one cycle of the function's graph.

1. $y = \sin \frac{1}{4}x$

2. $y = \frac{1}{2} \cos \pi x$

3. $y = \tan 2\pi x$

4. $y = 3 \tan \frac{2}{3}x$

14.2 Translations and Reflections of Trigonometric Graphs

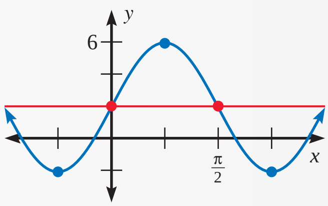
Examples on pp. 840–843

EXAMPLE To graph $y = 2 - 4 \cos 2\left(x + \frac{\pi}{4}\right)$, start with the graph of $y = 4 \cos 2x$. Translate the graph **left $\frac{\pi}{4}$ units** and **up 2 units**, and reflect it in the line $y = 2$.

Amplitude $= |-4| = 4$ Period $= \frac{2\pi}{|2|} = \pi$

On $y = 2$: $(0, 2)$; $\left(\frac{\pi}{2}, 2\right)$

Maximum: $\left(\frac{\pi}{4}, 6\right)$ **Minimums:** $\left(-\frac{\pi}{4}, -2\right)$; $\left(\frac{3\pi}{4}, -2\right)$



Graph the function.

5. $y = 5 \sin (2x + \pi)$ **6.** $y = -4 \cos (x - \pi)$ **7.** $y = 2 + \tan \left(\frac{1}{2}x + \pi\right)$

14.3 VERIFYING TRIGONOMETRIC IDENTITIES

Examples on pp. 848–851

EXAMPLE You can verify identities such as $\sin x + \cot x \cos x = \csc x$.

$\sin x + \cot x \cos x = \sin x + \left(\frac{\cos x}{\sin x}\right) \cos x$ — Reciprocal identity

$= \frac{\sin^2 x + \cos^2 x}{\sin x}$ — Write as one fraction.

$= \frac{1}{\sin x}$ — Pythagorean identity

$= \csc x$ — Reciprocal identity

Simplify the expression.

8. $\tan (-x) \cos (-x)$ **9.** $\csc^2 (-x) \cos^2 \left(\frac{\pi}{2} - x\right)$ **10.** $\sin^2 \left(\frac{\pi}{2} - x\right) - 2 \sin^2 x + 1$

Verify the identity.

11. $\sin^2 (-x) = \frac{\tan^2 x}{\tan^2 x + 1}$ **12.** $1 - \cos^2 x = \tan^2 (-x) \cos^2 x$

14.4 SOLVING TRIGONOMETRIC EQUATIONS

Examples on pp. 855–858

EXAMPLE You can find the general solution of a trigonometric equation or just the solution(s) in an interval.

$3 \tan^2 x - 1 = 0$ — Write original equation.

$3 \tan^2 x = 1$ — Add 1 to each side.

$\tan^2 x = \frac{1}{3}$ — Divide each side by 3.

$\tan x = \pm\frac{\sqrt{3}}{3}$ — Take square roots of each side.

There are two solutions in the interval $0 \le x < \pi$: $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$. The general solution of the equation is: $x = \frac{\pi}{6} + n\pi$ or $x = \frac{5\pi}{6} + n\pi$ where n is any integer.

Find the general solution of the equation.

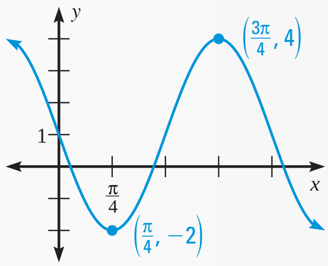
13. $2 \sin^2 x \tan x = \tan x$ **14.** $\sec^2 x - 2 = 0$ **15.** $\cos 2x + 2 \sin^2 x - \sin x = 0$

16. $\tan^2 3x = 3$ **17.** $2 \sin x - 1 = 0$ **18.** $\sin x (\sin x + 1) = 0$

14.5 MODELING WITH TRIGONOMETRIC FUNCTIONS

Examples on pp. 862–864

EXAMPLE You can write a model for the sinusoid at the right. Since the maximum and minimum values of the function do not occur at points equidistant from the x-axis, the curve has a vertical shift. To find the value of k, add the maximum and minimum values and divide by 2.



$$k = \frac{M + m}{2} = \frac{4 + (-2)}{2} = \frac{2}{2} = 1$$

The period is $\frac{2\pi}{b} = \pi$, so $b = 2$. Because the minimum occurs at $\frac{\pi}{4}$, the graph is a sine curve that involves a reflection but no horizontal shift. The amplitude is

$|a| = \frac{M - m}{2} = \frac{4 - (-2)}{2} = 3$. Since $a < 0$, $a = -3$. The model is $y = 1 - 3 \sin 2x$.

Write a trigonometric function for the sinusoid with maximum at *A* and minimum at *B*.

19. $A\left(\frac{\pi}{2}, 2\right), B\left(\frac{3\pi}{2}, -2\right)$ **20.** $A(0, 6), B(2\pi, 0)$ **21.** $A(0, 1), B\left(\frac{\pi}{2}, -1\right)$

14.6 USING SUM AND DIFFERENCE FORMULAS

Examples on pp. 869–871

EXAMPLE You can use formulas to evaluate trigonometric functions of the sum or difference of two angles.

$$\sin 105° = \sin (45° + 60°) = \sin 45° \cos 60° + \cos 45° \sin 60°$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

Find the exact value of the expression.

22. $\sin 150°$ **23.** $\cos 195°$ **24.** $\tan 15°$ **25.** $\tan \frac{7\pi}{12}$ **26.** $\cos \frac{13\pi}{12}$

14.7 USING DOUBLE- AND HALF-ANGLE FORMULAS

Examples on pp. 875–878

EXAMPLE You can use formulas to evaluate some trigonometric functions.

$$\tan \frac{\pi}{12} = \tan \left(\frac{1}{2} \cdot \frac{\pi}{6}\right) = \frac{1 - \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$$

Find the exact value of the expression.

27. $\tan 165°$ **28.** $\sin 67.5°$ **29.** $\cos \frac{5\pi}{8}$ **30.** $\cos \frac{\pi}{12}$ **31.** $\sin 6\pi$

CHAPTER 14

Chapter Test

Draw one cycle of the function's graph.

1. $y = 3\cos\frac{1}{4}x$ **2.** $y = 4\sin\frac{1}{2}\pi x$ **3.** $y = \frac{5}{2}\tan x$ **4.** $y = -2\tan 2x$

5. $y = -3 + 2\cos(x - \pi)$ **6.** $y = 1 - \cos x$ **7.** $y = 5 + \sin\frac{1}{2}x$ **8.** $y = 5 + 2\tan(x + \pi)$

Simplify the expression.

9. $\cos\left(x - \frac{\pi}{2}\right)$ **10.** $\frac{\cos 2x + \sin^2 x}{\cos^2 x}$ **11.** $\frac{\tan 2x}{2\tan x} - \frac{\sec^2 x}{1 - \tan^2 x}$ **12.** $\frac{4\sin x\cos x - 2\sin x\sec x}{2\tan x}$

Verify the identity.

13. $-2\cos^2 x\tan(-x) = \sin 2x$ **14.** $\tan\frac{x}{2} = \csc x - \cot x$ **15.** $\cos 3x = \cos^3 x - 3\sin^2 x\cos x$

Solve the equation in the interval $0 \le x < 2\pi$. Check your solutions.

16. $-6 + 10\cos x = -1$ **17.** $\tan^2 x - 2\tan x + 1 = 0$ **18.** $\tan(x + \pi) + 2\sin(x + \pi) = 0$

Find the general solution of the equation.

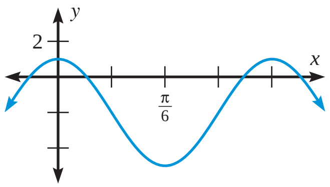
19. $4 - 3\sec^2 x = 0$ **20.** $\cos x - \sin x\sin 2x = 0$ **21.** $\cos x\csc^2 x + 3\cos x = 7\cos x$

Find the exact value of the expression.

22. $\sin 345°$ **23.** $\tan 112.5°$ **24.** $\cos 375°$ **25.** $\tan\frac{13\pi}{12}$ **26.** $\sin\frac{\pi}{8}$ **27.** $\cos\frac{41\pi}{12}$

Find the amplitude and period of the graph. Then write a trigonometric function for the graph.

28.



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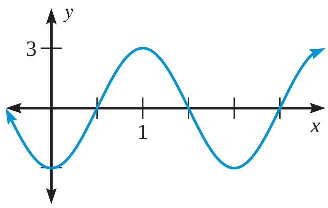

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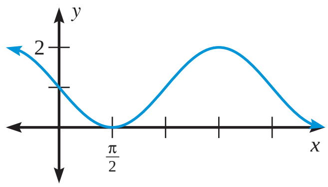

31. **TIDES** The depth of the ocean at a swim buoy reaches a maximum of 6 feet at 3 A.M. and a minimum of 2 feet at 9 A.M. Write a trigonometric function that models the water depth y (in feet) as a function of time t (in hours). Assume that $t = 0$ represents 12:00 A.M.

32. **TEMPERATURES** The average daily temperature T (in degrees Fahrenheit) in Baltimore, Maryland, is given in the table. The variable t is measured in months, with $t = 0$ representing January 1. Use a graphing calculator to write a trigonometric model for T as a function of t.

▶ Source: U.S. National Oceanic and Atmospheric Administration

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
T	75	79	87	94	98	101	104	105	100	92	87	77