



## **The Mathematics of Music**

# **OBJECTIVE** Explore the relationship between music and trigonometric functions.

**Materials:** plastic or glass bottle, container of water, CBL, CBL microphone, TI-82 or TI-83 graphing calculator with cable to link to the CBL

Sound is a variation in pressure transmitted through air, water, or other matter. Sound travels as a wave. The sound of a pure note can be represented using a sine wave (or a cosine wave; recall that a cosine wave is just a sine wave shifted horizontally). More complicated sounds can be modeled by the sum of several sine waves.

The pitch of a sound wave is determined by the wave's frequency. The greater the frequency, the higher the pitch.

Note	middle C	D	Е	F	G	А	В	С
Frequency (cycles/second)	262	294	330	349	392	440	494	523

### INVESTIGATION



1. Fill a 12–20 ounce plastic or glass bottle almost to the top with water. Blow across the top and listen to the note produced. Pour a small amount of water out and repeat. Continue to remove water and blow notes until the bottle is empty. What happens to the frequency of the notes as the water level decreases? How can you tell?

.....

- 2. Fill the bottle partway with water and blow across the top to create a note with constant pitch and volume. If you have trouble producing a steady stream of air, use a straw to blow across the bottle top. Use the CBL and the CBL microphone to collect the sound data and store it in the graphing calculator. Use the graphing calculator to graph the pressure of the sound as a function of time. The graph should resemble a sine wave.
- **3.** Use the graph of the sound data to calculate the frequency of the note—the number of complete cycles in one second.
- **4**. Write a sine function to describe the note.
- **5.** Choose a note from the table. Try producing the note as follows: Adjust the water level in the bottle, blow across the top, and use the CBL and graphing calculator to find the frequency of the resulting note. Repeat this process until the frequency of the note you produce is approximately equal to the frequency of the note you chose from the table.
- 6. Write a sine function to describe the note you chose from the table.

#### **PRESENT YOUR RESULTS**

Write a report to present your results.

- Explain how you used the CBL.
- Explain how you found the frequency of a note from the note's sine wave.
- Explain how you wrote the sine functions in Exercises 4 and 6.
- Include a sketch of the water level in your bottle for both notes you produced in Exercises 2 and 5.
- Include a graph of the sine wave for each of the two notes.
- Consider including a recording of the notes you produced. You might repeat the experiment to produce several different notes.
- Describe how you used your knowledge of trigonometric functions in this project.

#### EXTENSION



Choose a note to play and have a classmate also choose a note. Find sine functions y = f(x) and y = g(x) that model the two notes (as you did in Exercises 4 and 6 of the investigation). Then play the notes simultaneously and use the CBL and graphing calculator to graph the resulting sound wave. Compare this graph with the graph of y = f(x) + g(x). What do you notice?

### **MUSIC CONNECTION**

Bring in musical instruments and play a pure note on each. Use the CBL and graphing calculator to find the sine function that corresponds to each note. Observe what happens when you change the volume of the notes. Also, compare the sine waves for different instruments playing the same note.



The Mathematics

of Music

aurie Hernandez

Paul Green

Algebra 2 Mrs. Cheung