CHAPTER 13

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Evaluate trigonometric functions.	
• of acute angles **(13.1)**	Find the altitude of a kite. **(p. 771)**
• of any angle **(13.3)**	Find the horizontal distance traveled by a golf ball. **(p. 787)**
Find the sides and angles of a triangle.	
• solve right triangles **(13.1)**	Find the length of a zip-line at a ropes course. **(p. 774)**
• use the law of sines **(13.5)**	Find the distance between two buildings. **(p. 805)**
• use the law of cosines **(13.6)**	Find the angle at which two trapeze artists meet. **(p. 811)**
Measure angles using degree measure and radian measure. **(13.2)**	Find the angle generated by a figure skater performing a jump. **(p. 781)**
Find arc lengths and areas of sectors. **(13.2)**	Find the area irrigated by a rotating sprinkler. **(p. 781)**
Evaluate inverse trigonometric functions. **(13.4)**	Find the angle at which to set the arm of a crane. **(p. 794)**
Find the area of a triangle.	
• using two sides and the included angle **(13.5)**	Find the amount of paint needed for the side of a house. **(p. 806)**
• using Heron's formula **(13.6)**	Find the area of the Dinosaur Diamond. **(p. 812)**
Use parametric equations to model linear or projectile motion. **(13.7)**	Model the path of a leaping dolphin. **(p. 818)**
Use trigonometric and inverse trigonometric functions to solve real-life problems. **(13.1, 13.3–13.7)**	Find distances for a marching band on a football field. **(p. 787)**

How does Chapter 13 fit into the BIGGER PICTURE of algebra?

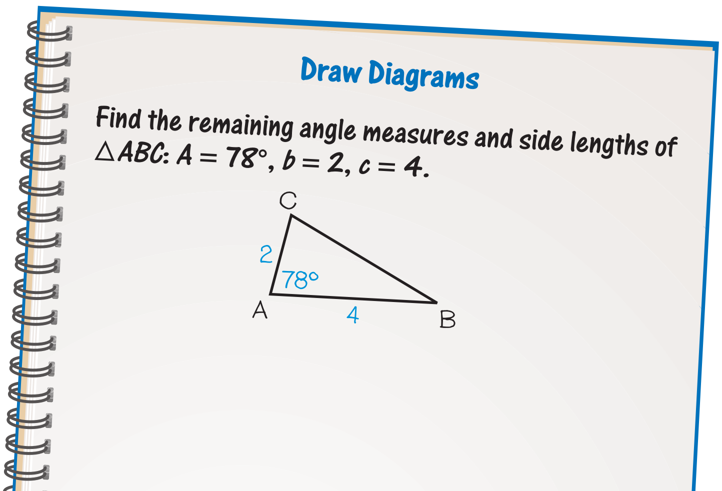
Trigonometry is closely tied to both algebra and geometry. In this chapter you studied trigonometric functions of *angles*, defined by ratios of side lengths of right triangles.

In the next chapter you will study trigonometric functions of *real numbers*, used to model periodic behavior. You will see even more connections between trigonometry and algebra as you graph trigonometric functions in a coordinate plane.

STUDY STRATEGY

How did you draw diagrams?

Here is an example of a diagram drawn for Exercise 22 on page 810, following the **Study Strategy** on page 768.



CHAPTER 13

Chapter Review

VOCABULARY

- sine, p. 769
- cosine, p. 769
- tangent, p. 769
- cosecant, p. 769
- secant, p. 769
- cotangent, p. 769
- solving a right triangle, p. 770
- angle of elevation, p. 771
- angle of depression, p. 771
- initial side of an angle, p. 776
- terminal side of an angle, p. 776
- standard position, p. 776
- coterminal angles, p. 777
- radian, p. 777
- sector, p. 779
- central angle, p. 779
- quadrantal angle, p. 785
- reference angle, p. 785
- inverse sine, p. 792
- inverse cosine, p. 792
- inverse tangent, p. 792
- law of sines, p. 799
- law of cosines, p. 807
- parametric equations, p. 813
- parameter, p. 813

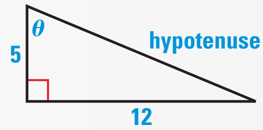
13.1 RIGHT TRIANGLE TRIGONOMETRY

Examples on pp. 769–771

EXAMPLE You can evaluate the six trigonometric functions of θ for the triangle shown. First find the hypotenuse length: $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13}$ $\quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{13}$ $\quad \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5}$

$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{12}$ $\quad \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{5}$ $\quad \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{12}$



Evaluate the six trigonometric functions of θ.

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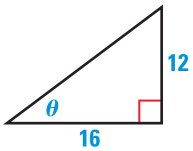

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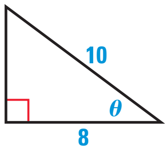

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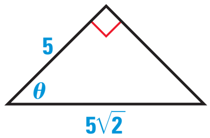

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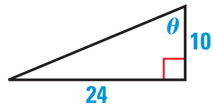


13.2 GENERAL ANGLES AND RADIAN MEASURE

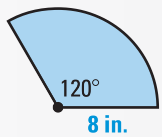
Examples on pp. 776–779

EXAMPLES You can measure angles using degree measure or radian measure.

$20° = 20°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{9}$ radians $\qquad \frac{7\pi}{6}$ radians $= \left(\frac{7\pi}{6} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right) = 210°$

Arc length of the sector at the right: $s = r\theta = 8\left(\frac{2\pi}{3}\right) = \frac{16\pi}{3}$ inches

Area of the sector at the right: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8^2)\left(\frac{2\pi}{3}\right) = \frac{64\pi}{3}$ square inches



Rewrite each degree measure in radians and each radian measure in degrees.

5. $30°$ **6.** $225°$ **7.** $-15°$ **8.** $\frac{3\pi}{4}$ **9.** $\frac{5\pi}{3}$ **10.** $\frac{\pi}{3}$

Find the arc length and area of a sector with the given radius r and central angle θ.

11. $r = 5$ ft, $\theta = \frac{\pi}{2}$ **12.** $r = 12$ in., $\theta = 25°$ **13.** $r = 16$ cm, $\theta = 210°$

13.3 Trigonometric Functions of Any Angle

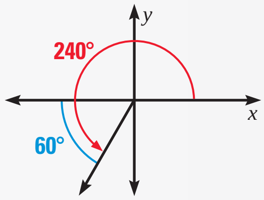
Examples on pp. 784–787

EXAMPLE You can evaluate the six trigonometric functions of $\theta = 240°$ using a reference angle: $\theta' = \theta - 180° = 240° - 180° = 60°$.

$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$ $\csc 240° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

$\cos 240° = -\cos 60° = -\frac{1}{2}$ $\sec 240° = -\sec 60° = -2$

$\tan 240° = +\tan 60° = \sqrt{3}$ $\cot 240° = +\cot 60° = \frac{\sqrt{3}}{3}$



Evaluate the function without using a calculator.

14. $\tan \frac{11\pi}{4}$ **15.** $\cos \frac{11\pi}{6}$ **16.** $\sec 225°$ **17.** $\sin 390°$ **18.** $\csc(-120°)$

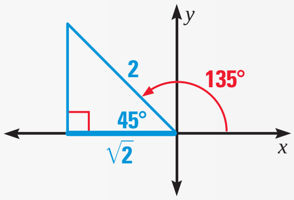
13.4 Inverse Trigonometric Functions

Examples on pp. 792–794

EXAMPLE You can find an angle within a certain range that corresponds to a given value of a trigonometric function.

To find $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, find θ so that $\cos\theta = -\frac{\sqrt{2}}{2}$ and $0° \le \theta \le 180°$.

So, $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135°$ $\left(\text{or } \frac{3\pi}{4} \text{ radians}\right)$.



Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

19. $\sin^{-1}\frac{\sqrt{2}}{2}$ **20.** $\tan^{-1}\frac{\sqrt{3}}{3}$ **21.** $\cos^{-1} 0$ **22.** $\tan^{-1}(-1)$ **23.** $\cos^{-1}\left(-\frac{1}{2}\right)$

13.5 The Law of Sines

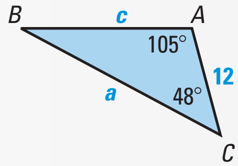
Examples on pp. 799–802

EXAMPLE You can solve the triangle shown using the law of sines.

The measure of the third angle is: $B = 180° - 105° - 48° = 27°$.

$$\frac{a}{\sin 105°} = \frac{12}{\sin 27°} \qquad \frac{c}{\sin 48°} = \frac{12}{\sin 27°}$$

$$a = \frac{12 \sin 105°}{\sin 27°} \approx 25.5 \qquad c = \frac{12 \sin 48°}{\sin 27°} \approx 19.6$$



Area of this triangle $= \frac{1}{2}bc \sin A = \frac{1}{2}(12)(19.6) \sin 105° \approx 114$ square units

13.5 continued

Solve △*ABC*. (*Hint:* Some of the "triangles" may have no solution and some may have two.)

24. $A = 45°, B = 60°, c = 44$ **25.** $B = 18°, b = 12, a = 19$ **26.** $C = 140°, c = 40, b = 20$

Find the area of the triangle with the given side lengths and included angle.

27. $C = 35°, b = 10, a = 22$ **28.** $A = 110°, b = 8, c = 7$ **29.** $B = 25°, a = 15, c = 31$

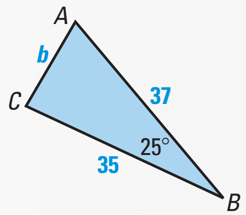
13.6 THE LAW OF COSINES

Examples on pp. 807–809

EXAMPLE You can solve the triangle below using the law of cosines.

Law of cosines: $b^2 = 35^2 + 37^2 - 2(35)(37)\cos 25° \approx 247$

$b \approx 15.7$



Law of sines: $\frac{\sin A}{35} \approx \frac{\sin 25°}{15.7}$, $\sin A \approx \frac{35 \sin 25°}{15.7}$, $A \approx 70.4°$

$C \approx 180° - 25° - 70.4° = 84.6°$

You can use Heron's formula to find the area of this triangle:

$s \approx \frac{1}{2}(35 + 15.7 + 37) \approx 44$, so area $\approx \sqrt{44(44 - 35)(44 - 15.7)(44 - 37)} \approx 280$ square units

Solve △*ABC*.

30. $a = 25, b = 18, c = 28$ **31.** $a = 6, b = 11, c = 14$ **32.** $B = 30°, a = 80, c = 70$

Find the area of △*ABC* having the given side lengths.

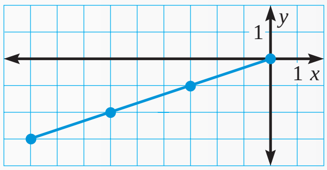
33. $a = 11, b = 2, c = 12$ **34.** $a = 4, b = 24, c = 26$ **35.** $a = 15, b = 8, c = 21$

13.7 PARAMETRIC EQUATIONS AND PROJECTILE MOTION

Examples on pp. 813–815

EXAMPLE You can graph the parametric equations $x = -3t$ and $y = -t$ for $0 \le t \le 3$. Make a table of values, plot the points (x, y), and connect the points.

t	0	1	2	3
x	0	−3	−6	−9
y	0	−1	−2	−3



To write an *xy*-equation for these parametric equations, solve the first equation for *t*: $t = -\frac{1}{3}x$. Substitute into the second equation: $y = \frac{1}{3}x$. The domain is $-9 \le x \le 0$.

Graph the parametric equations.

36. $x = 3t + 1$ and $y = 3t + 6$ for $0 \le t \le 5$ **37.** $x = 2t + 4$ and $y = -4t + 2$ for $2 \le t \le 5$

Write an *xy*-equation for the parametric equations. State the domain.

38. $x = 5t$ and $y = t + 7$ for $0 \le t \le 20$ **39.** $x = 2t - 3$ and $y = -4t + 5$ for $0 \le t \le 8$

CHAPTER 13

Chapter Test

Evaluate the six trigonometric functions of θ.

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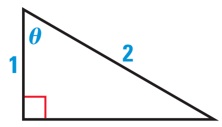

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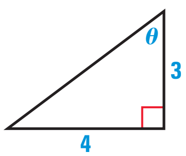

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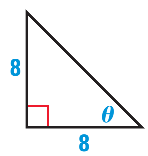

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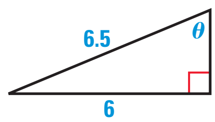


Rewrite each degree measure in radians and each radian measure in degrees.

5. $120°$ **6.** $360°$ **7.** $-60°$ **8.** $\frac{\pi}{9}$ **9.** 5π **10.** $-\frac{5\pi}{4}$

Find the arc length and area of a sector with the given radius r and central angle θ.

11. $r = 4$ ft, $\theta = 240°$ **12.** $r = 20$ cm, $\theta = 45°$ **13.** $r = 12$ in., $\theta = 150°$

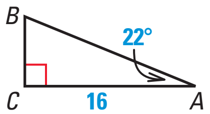
Evaluate the function without using a calculator.

14. $\cos 180°$ **15.** $\sec(-30°)$ **16.** $\cot 495°$ **17.** $\sin \frac{7\pi}{6}$ **18.** $\tan\left(-\frac{\pi}{4}\right)$ **19.** $\csc\left(-\frac{7\pi}{4}\right)$

Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

20. $\sin^{-1} 1$ **21.** $\tan^{-1}\sqrt{3}$ **22.** $\cos^{-1}\frac{\sqrt{3}}{2}$ **23.** $\tan^{-1} 0$ **24.** $\cos^{-1} 1$ **25.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Solve $\triangle ABC$.

26.


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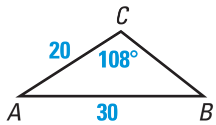

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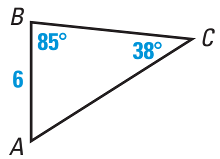

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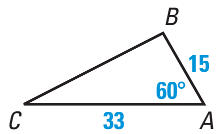
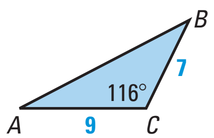


30. $A = 120°, a = 14, b = 10$ **31.** $B = 40°, a = 7, c = 10$ **32.** $C = 105°, a = 4, b = 3$

Find the area of $\triangle ABC$.

33.


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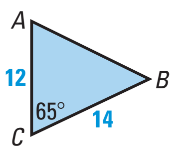

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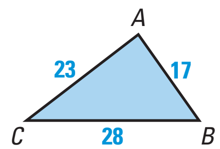

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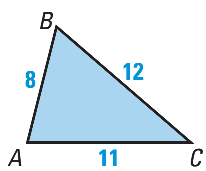

Graph the parametric equations. Then write an *xy*-equation and state the domain.

37. $x = 2t - 3$ and $y = -5t + 6$ for $1 \le t \le 4$ **38.** $x = t - 4$ and $y = -t + 6$ for $0 \le t \le 6$

39. **BOAT RIDE** A boat travels 50 miles due west before adjusting its course 25° north of west and traveling an additional 35 miles. How far is the boat from its point of departure?

40. **PROJECTILE MOTION** You throw a ball at an angle of 50°, from a height of 6 feet, and with an initial speed of 25 feet per second. Write a set of parametric equations for the path of the ball. How far from you does the ball land?