CHAPTER 9 Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Write and use variation models.	
• inverse variation **(9.1)**	Find the speed of a whirlpool's current. **(p. 535)**
• joint variation **(9.1)**	Find the heat loss through a window. **(p. 539)**
Graph rational functions.	
• simple rational functions **(9.2)**	Describe the frequency of an approaching ambulance siren. **(p. 545)**
• general rational functions **(9.3)**	Find the energy expenditure of a parakeet. **(p. 551)**
Perform operations with rational expressions.	
• multiply and divide **(9.4)**	Compare the velocities of two skydivers. **(p. 557)**
• add and subtract **(9.5)**	Write a model for the number of male college graduates in the United States. **(p. 566)**
Simplify complex fractions. **(9.5)**	Write a simplified model for the focal length of a camera lens. **(p. 564)**
Solve rational equations. **(9.6)**	Find the amount of water to add when diluting an acid solution. **(p. 570)**
Use rational models to solve real-life problems. **(9.1–9.6)**	Find the year in which a certain amount of rodeo prize money was earned. **(p. 570)**

How does Chapter 9 fit into the BIGGER PICTURE of algebra?

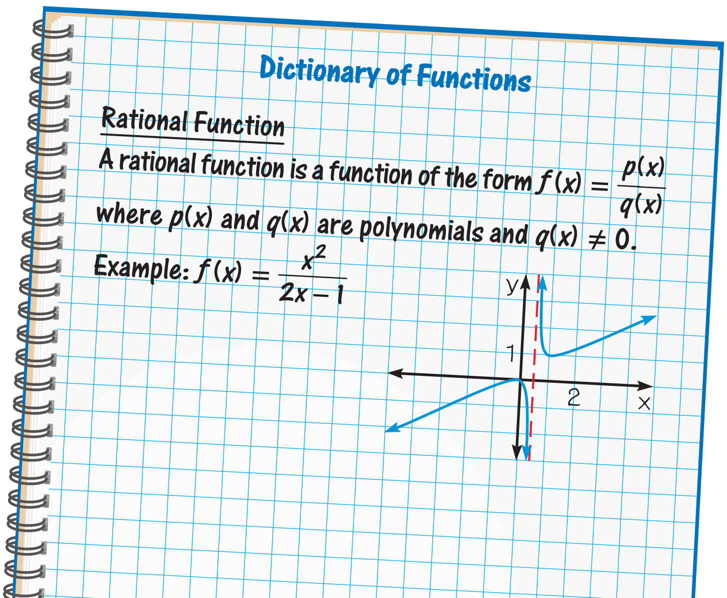
In Chapter 9 you studied rational functions. A rational function is the ratio of two polynomial functions, which you studied in Chapter 2 (linear functions), Chapter 5 (quadratic functions), and Chapter 6 (polynomial functions).

A hyperbola is the graph of one important type of rational function. In the next chapter you will learn more about hyperbolas, parabolas, circles, and ellipses, which together are called the conic sections.

STUDY STRATEGY

How did you make and use a dictionary of functions?

Here is an example of one entry in a dictionary of functions, following the **Study Strategy** on page 532.



CHAPTER 9

Chapter Review

VOCABULARY

- inverse variation, p. 534
- constant of variation, p. 534
- joint variation, p. 536
- rational function, p. 540
- hyperbola, p. 540
- branches of a hyperbola, p. 540
- simplified form of a rational expression, p. 554
- complex fraction, p. 564
- cross multiplying, p. 569

9.1 INVERSE AND JOINT VARIATION

Examples on pp. 534–536

EXAMPLES You can write an inverse or joint variation equation using a general equation for the variation and given values of the variables.

Inverse variation: $x = 5, y = 4$

$y = \frac{k}{x}$	*y* varies inversely with *x*.
$4 = \frac{k}{5}$	Substitute for *x* and *y*.
$20 = k$	Solve for *k*.

The inverse variation equation is $y = \frac{20}{x}$.

Joint variation: $x = 3, y = 8, z = 30$

$z = kxy$	*z* varies jointly with *x* and *y*.
$30 = k(3)(8)$	Substitute for *x*, *y*, and *z*.
$30 = 24k$	Multiply.
$k = \frac{30}{24} = \frac{5}{4}$	Solve for *k*.

The joint variation equation is $z = \frac{5}{4}xy$.

The variables *x* and *y* vary inversely. Use the given values to write an equation relating *x* and *y*. Then find *y* when *x* = 2.

1. $x = 1, y = 5$ **2.** $x = 15, y = \frac{2}{3}$ **3.** $x = \frac{1}{4}, y = 8$ **4.** $x = -2, y = 2$

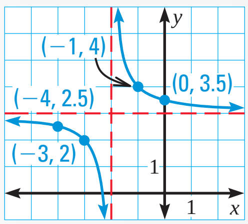
The variable *z* varies jointly with *x* and *y*. Use the given values to write an equation relating *x*, *y*, and *z*. Then find *z* when *x* = 5 and *y* = −6.

5. $x = 1, y = 12, z = 4$ **6.** $x = 6, y = 8, z = -6$ **7.** $x = \frac{3}{4}, y = 4, z = 9$

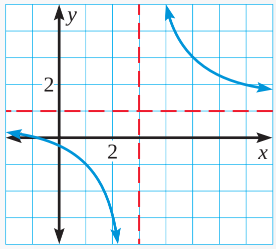
9.2 GRAPHING SIMPLE RATIONAL FUNCTIONS

Examples on pp. 540–542

EXAMPLE 1 To graph $y = \frac{1}{x+2} + 3$, note that the asymptotes are $x = -2$ and $y = 3$. Plot two points to the left of the vertical asymptote, such as $(-3, 2)$ and $(-4, 2.5)$, and two points to the right, such as $(-1, 4)$ and $(0, 3.5)$. Use the asymptotes and plotted points to draw the branches of the hyperbola. The domain is all real numbers except −2, and the range is all real numbers except 3.



EXAMPLE 2 To graph $y = \frac{x+1}{x-3}$, note that when the denominator equals zero, $x = 3$. So the vertical asymptote is $x = 3$. The horizontal asymptote, which occurs at the ratio of the x-coefficients, is $y = 1$. Plot some points to the left and right of the vertical asymptote. Use the asymptotes and plotted points to draw the branches of the hyperbola. The domain is all real numbers except 3, and the range is all real numbers except 1.



Graph the function. State the domain and range.

8. $y = \frac{3}{x-5}$ **9.** $y = \frac{1}{x+4} + 2$ **10.** $y = \frac{-6x}{x+2}$ **11.** $y = \frac{2x+5}{x-1}$

9.3 GRAPHING GENERAL RATIONAL FUNCTIONS

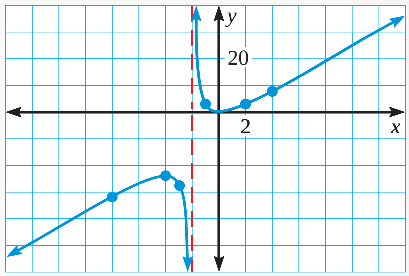
Examples on pp. 547–549

EXAMPLE To graph $y = \frac{3x^2}{x+2}$, note that the numerator has 0 as its only real zero, so the graph has one x-intercept at (0, 0). The only zero of the denominator is -2, so the only vertical asymptote is $x = -2$. The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote.

Plot some points to the left of $x = -2$.

Plot some points to the right of $x = -2$.

x	y
-8	-32
-4	-24
-3	-27
-1	3
2	3
4	8



Graph the function.

12. $y = \frac{3x^2+1}{x^2-1}$ **13.** $y = \frac{x^3}{10}$ **14.** $y = \frac{x}{x^2-4}$ **15.** $y = \frac{3x^2-4x+1}{x^2-2x-3}$

9.4 MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Examples on pp. 554–557

EXAMPLE Dividing rational expressions is like dividing numerical fractions.

$$\frac{x^2-9}{5(x+2)} \div \frac{x-3}{5(x^2-4)} = \frac{x^2-9}{5(x+2)} \cdot \frac{5(x^2-4)}{x-3}$$ **Multiply by reciprocal.**

$$= \frac{(x+3)\cancel{(x-3)}\cancel{(5)}\cancel{(x+2)}(x-2)}{\cancel{5}\cancel{(x+2)}\cancel{(x-3)}}$$ **Factor and divide out common factors.**

$$= (x+3)(x-2)$$ **Simplified form**

Perform the indicated operation(s). Simplify the result.

16. $\frac{x^2-3x}{4x^2-8x} \cdot (4x^2-16)$ **17.** $5x \div \frac{1}{x-6} \cdot \frac{x^2-9}{x}$ **18.** $\frac{x^2-2x-3}{x+1} \div \frac{x^2+x-12}{x^2} - 1$

9.5 Addition, Subtraction, and Complex Fractions

Examples on pp. 562–564

EXAMPLES You can use the LCD to add or subtract rational expressions.

$$\frac{3}{x-3}-\frac{5}{x+2}=\frac{3(x+2)}{(x-3)(x+2)}-\frac{5(x-3)}{(x-3)(x+2)}$$ **Rewrite each expression using the LCD.**

$$=\frac{3(x+2)-5(x-3)}{(x-3)(x+2)}$$ **Subtract.**

$$=\frac{3x+6-5x+15}{(x-3)(x+2)}$$ **Multiply.**

$$=\frac{-2x+21}{(x-3)(x+2)}$$ **Simplified form**

To simplify a complex fraction, divide the numerator by the denominator.

$$\frac{\frac{2}{x}+4}{\frac{2x+1}{5x^2}}=\frac{\frac{2+4x}{x}}{\frac{2x+1}{5x^2}}=\frac{2+4x}{x}\cdot\frac{5x^2}{2x+1}=\frac{2(1+2x)(5x^2)}{x(2x+1)}=10x$$

Perform the indicated operation(s) and simplify.

19. $\frac{5}{x^2(x-2)}+\frac{x}{x-2}$ **20.** $\frac{x+5}{x-5}-\frac{3}{x+5}$ **21.** $\frac{x-2}{5x(x-1)}+\frac{1}{x-1}-\frac{3x+2}{x^2+4x-5}$

Simplify the complex fraction.

22. $\frac{\frac{x+3}{6}}{1+\frac{x}{3}}$ **23.** $\frac{\frac{x}{2}-4}{9+\frac{2}{x}}$ **24.** $\frac{\frac{1}{x+1}+\frac{1}{x-1}}{\frac{x}{x+1}}$ **25.** $\frac{\frac{4}{5-x}}{\frac{2}{5-x}+\frac{1}{3x-15}}$

9.6 Solving Rational Equations

Examples on pp. 568–570

EXAMPLES You can solve rational equations by multiplying each side of the equation by the LCD of the terms. If each side of the equation is a single rational expression, you can use cross multiplying. Check for extraneous solutions.

$$\frac{4}{x}+\frac{3}{2x}=11$$

$$(2x)\frac{4}{x}+(2x)\frac{3}{2x}=(2x)11$$ **Multiply each side by 2x.**

$$8+3=22x$$

$$x=\frac{1}{2}$$

$$\frac{2}{3x+6}=\frac{x+2}{x^2-10}$$

$$2(x^2-10)=(x+2)(3x+6)$$ **Cross multiply.**

$$2x^2-20=3x^2+12x+12$$

$$0=x^2+12x+32$$

$$0=(x+8)(x+4)$$

$$x=-8 \text{ or } x=-4$$

Solve the equation using any method. Check each solution.

26. $\frac{x}{x-1}=\frac{2x+10}{x+11}$ **27.** $\frac{x+3}{x}-1=\frac{1}{x-1}$ **28.** $\frac{2}{x-2}-\frac{2x}{3}=\frac{x-3}{3}$

29. $\frac{3x+2}{x+1}=2-\frac{2x+3}{x+1}$ **30.** $\frac{2}{x-6}=\frac{-5}{x+1}$ **31.** $1+\frac{3}{x-3}=\frac{4}{x^2-9}$

CHAPTER 9

Chapter Test

The variables *x* and *y* vary inversely. Use the given values to write an equation relating *x* and *y*. Then find *y* when *x* = 3.

1. $x = -4, y = 9$ **2.** $x = \frac{1}{2}, y = 5$ **3.** $x = 12, y = \frac{2}{3}$ **4.** $x = 6, y = -1$

The variable *z* varies jointly with *x* and *y*. Use the given values to write an equation relating *x*, *y*, and *z*. Then find *z* when *x* = −2 and *y* = 4.

5. $x = 5, y = 4, z = 2$ **6.** $x = -3, y = 2, z = 18$ **7.** $x = \frac{1}{3}, y = \frac{3}{4}, z = \frac{5}{2}$

Graph the function.

8. $y = \frac{-1}{x+1} - 2$ **9.** $y = \frac{4}{x-2}$ **10.** $y = \frac{x}{2x+5}$ **11.** $y = \frac{4x-3}{x-4}$

12. $y = \frac{6}{x^2+4}$ **13.** $y = \frac{-3x^2}{2x-1}$ **14.** $y = \frac{x^2-2}{x^2-9}$ **15.** $y = \frac{x^2-2x+15}{x+1}$

Perform the indicated operation. Simplify the result.

16. $\frac{x^2-4}{x+3} \cdot \frac{x^2+4x+3}{2x-4}$ **17.** $\frac{4x-8}{x^2-3x+2} \div \frac{3x-6}{x-1}$ **18.** $\frac{x+4}{x^2-25} \cdot (x^2+3x-10)$

19. $\frac{5}{6x} + \frac{7}{18x}$ **20.** $\frac{x-1}{x-2} - \frac{x-4}{x+1}$ **21.** $\frac{3x}{x^2-10x+21} + \frac{5}{x-3}$

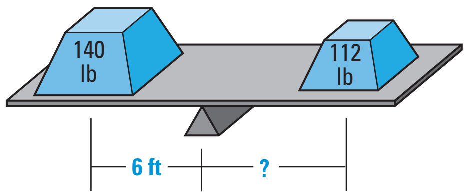
Simplify the complex fraction.

22. $\dfrac{1+\frac{3}{x}}{2-\frac{5}{x^2}}$ **23.** $\dfrac{\frac{4+x}{10}}{\frac{x^2-16}{8}}$ **24.** $\dfrac{\frac{2}{x-1}+5}{\frac{x}{3}}$ **25.** $\dfrac{36}{\frac{1}{x}+\frac{7}{2x}}$

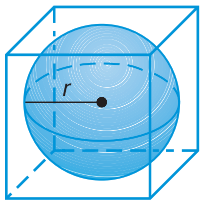
Solve the equation using any method. Check each solution.

26. $\frac{9}{x} + \frac{11}{5} = \frac{31}{x}$ **27.** $\frac{-15}{x} = \frac{x+16}{4}$ **28.** $\frac{8}{x+3} = \frac{5}{x-3}$ **29.** $\frac{4x}{x+3} = \frac{37}{x^2-9} - 3$

30. **SCIENCE CONNECTION** A lever pivots on a support called a *fulcrum*. For a balanced lever, the distance *d* (in feet) an object is from the fulcrum varies inversely with the object's weight *w* (in pounds). An object weighing 140 pounds is placed 6 feet from a fulcrum. How far from the fulcrum must a 112 pound object be placed to balance the lever?



31. **GEOMETRY CONNECTION** A sphere with radius *r* is inscribed in a cube as shown. Find the ratio of the volume of the cube to the volume of the sphere. Write your answer in simplified form.



32. **STARTING A BUSINESS** You start a small bee-keeping business, spending \$500 for equipment and bees. You figure it will cost \$1.25 per pound to collect, clean, bottle, and label the honey. How many pounds of honey must you produce before your average cost per pound is \$1.79?