9.6 Solving Rational Equations

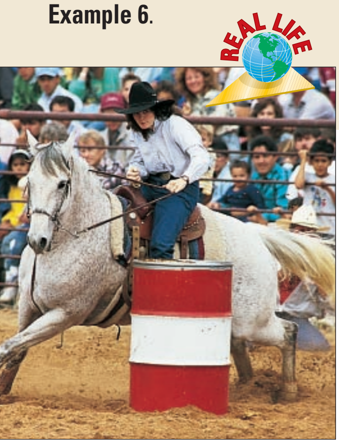
What *you should learn*

GOAL 1 Solve rational equations.

GOAL 2 Use rational equations to solve **real-life** problems, such as finding how to dilute an acid solution in **Example 5**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the year in which a certain amount of rodeo prize money was earned in **Example 6**.



GOAL 1 SOLVING A RATIONAL EQUATION

To solve a rational equation, multiply each term on both sides of the equation by the LCD of the terms. Simplify and solve the resulting polynomial equation.

EXAMPLE 1 *An Equation with One Solution*

Solve: $\frac{4}{x} + \frac{5}{2} = -\frac{11}{x}$

SOLUTION

The least common denominator is $2x$.

$$\frac{4}{x} + \frac{5}{2} = -\frac{11}{x}$$ Write original equation.

$$2x\left(\frac{4}{x} + \frac{5}{2}\right) = 2x\left(-\frac{11}{x}\right)$$ Multiply each side by $2x$.

$$8 + 5x = -22$$ Simplify.

$$5x = -30$$ Subtract 8 from each side.

$$x = -6$$ Divide each side by 5.

▶ The solution is -6. Check this in the original equation.

EXAMPLE 2 *An Equation with an Extraneous Solution*

Solve: $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

SOLUTION

The least common denominator is $x - 2$.

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$(x-2) \cdot \frac{5x}{x-2} = (x-2) \cdot 7 + (x-2) \cdot \frac{10}{x-2}$$

$$5x = 7(x-2) + 10$$

$$5x = 7x - 4$$

$$x = 2$$

▶ The solution appears to be 2. After checking it in the original equation, however, you can conclude that 2 is an extraneous solution because it leads to division by zero. So, the original equation has no solution.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 An Equation with Two Solutions

Solve: $\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$

SOLUTION

Write each denominator in factored form. The LCD is $(x+1)(x-1)$.

$$\frac{4x+1}{x+1} = \frac{12}{(x+1)(x-1)} + 3$$

$$(x+1)(x-1) \cdot \frac{4x+1}{x+1} = (x+1)(x-1) \cdot \frac{12}{(x+1)(x-1)} + (x+1)(x-1) \cdot 3$$

$$(x-1)(4x+1) = 12 + 3(x+1)(x-1)$$

$$4x^2 - 3x - 1 = 12 + 3x^2 - 3$$

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$$x + 2 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -2 \quad \text{or} \quad x = 5$$

▶ The solutions are −2 and 5. Check these in the original equation.

You can use **cross multiplying** to solve a simple rational equation for which each side of the equation is a single rational expression.

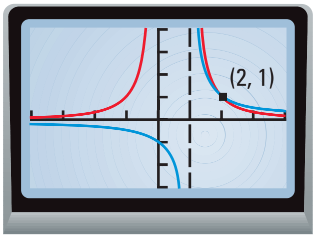
EXAMPLE 4 Solving an Equation by Cross Multiplying

Solve: $\frac{2}{x^2-x} = \frac{1}{x-1}$

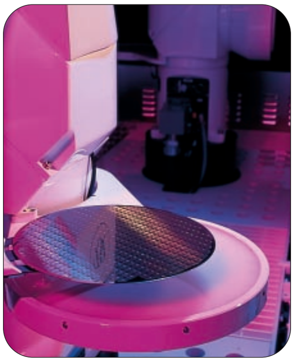
SOLUTION

$\frac{2}{x^2-x} = \frac{1}{x-1}$	**Write original equation.**
$2(x-1) = 1(x^2-x)$	**Cross multiply.**
$2x - 2 = x^2 - x$	**Simplify.**
$0 = x^2 - 3x + 2$	**Write in standard form.**
$0 = (x-2)(x-1)$	**Factor.**
$x = 2$ or $x = 1$	**Zero product property**

▶ The solutions appear to be 2 and 1. After checking them in the original equation, however, you see that the only solution is 2. The apparent solution $x = 1$ is extraneous. A graphic check shows that the graphs of the left and right sides of the equation, $y = \frac{2}{x^2-x}$ and $y = \frac{1}{x-1}$, intersect only at $x = 2$. At $x = 1$, the graphs have a common vertical asymptote.



FOCUS ON APPLICATIONS



REAL LIFE **CHEMICAL ENGRAVING** Acid mixtures are used to engrave, or *etch*, electronic circuits on silicon wafers. A wafer like the one shown above can be cut into as many as 1000 computer chips.

GOAL 2 USING RATIONAL EQUATIONS IN REAL LIFE

EXAMPLE 5 *Writing and Using a Rational Model*

CHEMISTRY You have 0.2 liter of an acid solution whose acid concentration is 16 moles per liter. You want to dilute the solution with water so that its acid concentration is only 12 moles per liter. How much water should you add to the solution?

SOLUTION

VERBAL MODEL

$$\text{Concentration of new solution} = \frac{\text{Moles of acid in original solution}}{\text{Volume of original solution} + \text{Volume of water added}}$$

LABELS

Concentration of new solution $= \mathbf{12}$ (moles per liter)

Moles of acid in original solution $= \mathbf{16(0.2)}$ (moles)

Volume of original solution $= \mathbf{0.2}$ (liters)

Volume of water added $= \boldsymbol{x}$ (liters)

ALGEBRAIC MODEL

$$12 = \frac{16(0.2)}{0.2 + x}$$ Write equation.

$$12(0.2 + x) = 16(0.2)$$ Multiply each side by $0.2 + x$.

$$2.4 + 12x = 3.2$$ Simplify.

$$12x = 0.8$$ Subtract 2.4 from each side.

$$x \approx 0.067$$ Divide each side by 12.

▶ You should add about 0.067 liter, or 67 milliliters, of water.

EXAMPLE 6 *Using a Rational Model*

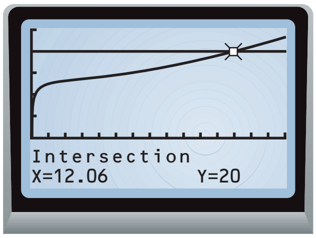
RODEOS From 1980 through 1997, the total prize money P (in millions of dollars) at Professional Rodeo Cowboys Association events can be modeled by

$$P = \frac{380t + 5}{-t^2 + 31t + 1}$$

where t represents the number of years since 1980. During which year was the total prize money about \$20 million? ▶ Source: Professional Rodeo Cowboys Association

SOLUTION

Use a graphing calculator to graph the equation $y = \frac{380x + 5}{-x^2 + 31x + 1}$. Then graph the line $y = 20$. Use the *Intersect* feature to find the value of x that gives a y-value of 20. As shown at the right, this value is $x \approx 12$. So, the total prize money was about \$20 million 12 years after 1980, in 1992.



STUDENT HELP

Study Tip
Example 6 can also be solved by setting the expression for P equal to 20 and solving the resulting equation algebraically.

GUIDED PRACTICE

Vocabulary Check ✓

1. Give an example of a rational equation that can be solved using cross multiplication.

Concept Check ✓

2. A student solved the equation $\frac{2}{x-3} = \frac{x}{x-3}$ and got the solutions 2 and 3. Which, if either, of these is extraneous? Explain how you know.

3. Describe two methods that can be used to solve a rational equation. Which method can always be used? Why?

4. Solve the equation $\frac{1}{x} = \frac{2}{x^2}$. Check the apparent solutions graphically. Explain how a graph can help you identify actual and extraneous solutions.

Skill Check ✓

Solve the equation using any method. Check each solution.

5. $\frac{7}{x} + \frac{3}{4} = \frac{5}{x}$

6. $\frac{x-2}{6} = \frac{x-2}{x-1}$

7. $3x + \frac{x}{3} = 5$

8. $\frac{x}{x-3} = 2 - \frac{2}{x-3}$

9. $\frac{5}{x-3} = \frac{2x}{x^2-9}$

10. $\frac{5x}{x-1} + 5 = \frac{15}{x-1}$

11. $\frac{2x}{x+3} = \frac{3x}{x-3}$

12. $\frac{2x}{x-4} = \frac{8}{x-4} + 3$

13. $\frac{2x}{2x+4} = \frac{3x}{x+2}$

14. **BASKETBALL STATISTICS** So far in the basketball season you have made 12 free throws out of the 20 free throws you have attempted, for a free-throw shooting percentage of 60%. How many consecutive free-throw shots would you have to make to raise your free-throw shooting percentage to 80%?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on p. 953.

CHECKING SOLUTIONS **Determine whether the given *x*-value is a solution of the equation.**

15. $\frac{2x-3}{x+3} = \frac{3x}{x+4}$; $x = -1$

16. $\frac{x}{2x+1} = \frac{5}{4-x}$; $x = -1$

17. $\frac{4x-3}{x-4} + 1 = \frac{x}{x-3}$; $x = 2$

18. $\frac{3x}{x-6} = 5 + \frac{18}{x-6}$; $x = 6$

19. $\frac{x}{x-3} = \frac{6}{x-3}$; $x = 6$

20. $\frac{2}{x(x+2)} + \frac{3}{x} = \frac{4}{x-2}$; $x = 2$

LEAST COMMON DENOMINATOR **Solve the equation by using the LCD. Check each solution.**

STUDENT HELP

HOMEWORK HELP
Examples 1–3: Exs. 15–32, 42–50
Example 4: Exs. 15, 16, 19, 33–50
Examples 5, 6: Exs. 54–59

21. $\frac{3}{2} + \frac{1}{x} = 2$

22. $\frac{3}{x} + x = 4$

23. $\frac{3}{2x} - \frac{9}{2} = 6x$

24. $\frac{8}{x+2} + \frac{8}{2} = 5$

25. $\frac{3x}{x+1} + \frac{6}{2x} = \frac{7}{x}$

26. $\frac{2}{3x} + \frac{2}{3} = \frac{8}{x+6}$

27. $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

28. $\frac{x-3}{x-4} + 4 = \frac{3x}{x}$

29. $\frac{7x+1}{2x+5} + 1 = \frac{10x-3}{3x}$

30. $\frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2}$

31. $\frac{4(x-1)}{x-1} = \frac{2x-2}{x+1}$

32. $\frac{2(x+7)}{x+4} - 2 = \frac{2x+20}{2x+8}$

CROSS MULTIPLYING **Solve the equation by cross multiplying. Check each solution.**

33. $\frac{3}{4x} = \frac{5}{x+2}$

34. $\frac{-3}{x+1} = \frac{4}{x-1}$

35. $\frac{x}{x^2-8} = \frac{2}{x}$

36. $\frac{x}{2x+7} = \frac{x-5}{x-1}$

37. $\frac{-2}{x-1} = \frac{x-8}{x+1}$

38. $\frac{2(x-2)}{x^2-10x+16} = \frac{2}{x+2}$

39. $\frac{8(x-1)}{x^2-4} = \frac{4}{x-2}$

40. $\frac{x^2-3}{x+2} = \frac{x-3}{2}$

41. $\frac{-1}{x-3} = \frac{x-4}{x^2-27}$

CHOOSING A METHOD **Solve the equation using any method. Check each solution.**

42. $\frac{x-2}{x+2} = \frac{3}{x}$

43. $\frac{3}{x+2} = \frac{6}{x-1}$

44. $\frac{3x}{x+1} = \frac{12}{x^2-1} + 2$

45. $\frac{3x+6}{x^2-4} = \frac{x+1}{x-2}$

46. $\frac{x-4}{x} = \frac{6}{x^2-3x}$

47. $\frac{2x}{4-x} = \frac{x^2}{x-4}$

48. $\frac{2x}{x-3} = \frac{3x}{x^2-9} + 2$

49. $\frac{x}{2x-6} = \frac{2}{x-4}$

50. $\frac{2}{x+1} + \frac{x}{x-1} = \frac{2}{x^2-1}$

LOGICAL REASONING **In Exercises 51–53, *a* is a nonzero real number. Tell whether the algebraic statement is *always true, sometimes true,* or *never true.* Explain your reasoning.**

51. For the equation $\frac{1}{x-a} = \frac{x}{x-a}$, $x = a$ is an extraneous solution.

52. The equation $\frac{3}{x-a} = \frac{x}{x-a}$ has exactly one solution.

53. The equation $\frac{1}{x-a} = \frac{2}{x+a} + \frac{2a}{x^2-a^2}$ has no solution.

54. **FOOTBALL STATISTICS** At the end of the 1998 season, the National Football League's all-time leading passer during regular season play was Dan Marino with 4763 completed passes out of 7989 attempts. In his debut 1998 season, Peyton Manning made 326 completed passes out of 575 attempts. How many consecutive completed passes would Peyton Manning have to make to equal Dan Marino's pass completion percentage?

INTERNET **DATA UPDATE** of National Football League data at www.mcdougallittell.com

55. **PHONE CARDS** A telephone company offers you an opportunity to sell prepaid, 30 minute long-distance phone cards. You will have to pay the company a one-time setup fee of \$200. Each phone card will cost you \$5.70. How many cards would you have to sell before your average total cost per card falls to \$8?

56. **RIVER CURRENT** It takes a paddle boat 53 minutes to travel 5 miles up a river and 5 miles back, going at a steady speed of 12 miles per hour (with respect to the water). Find the speed of the current.

57. **BIOLOGY CONNECTION** The number f of flies eaten by a praying mantis in 8 hours can be modeled by

$$f = \frac{26.6d}{d + 0.0017}$$

where d is the density of flies available (in flies per cubic centimeter). Approximate the density of flies (in flies per cubic *meter*) when a praying mantis eats 15 flies in 8 hours. (*Hint:* There are 1,000,000 cm^3 in 1 m^3.) ▶ Source: *Biology by Numbers*

The praying mantis blends in with its environment.

FUEL EFFICIENCY In Exercises 58 and 59, use the following information.
The cost of fueling your car for one year can be calculated using this equation:

$$\text{Fuel cost for one year} = \frac{\text{Miles driven} \times \text{Price per gallon of fuel}}{\text{Fuel efficiency rate}}$$

58. Last year you drove 9000 miles, paid $1.10 per gallon of gasoline, and spent a total of $412.50 on gasoline. What is the fuel efficiency rate of your car?

59. How much would you have saved if your car's fuel efficiency rate were 25 miles per gallon?

Test Preparation

QUANTITATIVE COMPARISON In Exercises 60 and 61, choose the statement below that is true about the given quantities.

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
60.	The solution of $\frac{x^3 + 1}{x} = 2x^2$	The solution of $\frac{-2}{x + 3} = \frac{4}{x - 2}$
61.	The solution of $\frac{1}{x} + 3 = \frac{9}{2x}$	The solution of $\frac{1}{2} + \frac{3}{x} = \frac{43}{14}$

★ Challenge

62. **SCIENCE CONNECTION** You have 0.5 liter of an acid solution whose acid concentration is 16 moles per liter. To decrease the acid concentration to 12 moles per liter, you plan to add a certain amount of a second acid solution whose acid concentration is only 10 moles per liter. How many liters of the second acid solution should you add?

EXTRA CHALLENGE
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MIXED REVIEW

SLOPES OF LINES Find the slope of a line parallel to the given line and the slope of a line perpendicular to the given line. (Review 2.4 for 10.1)

63. $y = x + 3$
64. $y = 3x - 4$
65. $y = -\frac{2}{3}x + 15$

66. $y + 3 = 3x + 2$
67. $2y - x = 7$
68. $4x - 3y = 17$

PROPERTIES OF SQUARE ROOTS Simplify the expression. (Review 5.3 for 10.1)

69. $\sqrt{48}$
70. $\sqrt{18}$
71. $\sqrt{108}$
72. $\sqrt{432}$

73. $\sqrt{6} \cdot \sqrt{45}$
74. $\sqrt{\frac{16}{72}}$
75. $\sqrt{75} \cdot \sqrt{3}$
76. $\sqrt{\frac{8}{49}}$

77. **GEOLOGY** You can find the pH of a soil by using the formula

$$\text{pH} = -\log\,[\text{H}^+]$$

where $[\text{H}^+]$ is the soil's hydrogen ion concentration (in moles per liter). Find the pH of a layer of soil that has a hydrogen ion concentration of 1.6×10^{-7} moles per liter. (Review 8.4)

QUIZ 2

Self-Test for Lessons 9.4–9.6

Perform the indicated operation and simplify. **(Lessons 9.4 and 9.5)**

1. $\dfrac{3x^3y}{2xy^2} \cdot \dfrac{10x^4y^2}{9x}$

2. $\dfrac{x^2 - 3x - 40}{5x} \div (x + 5)$

3. $\dfrac{18x}{x^2 - 5x - 36} + \dfrac{2x}{x + 4}$

4. $\dfrac{8x^2}{25x^2 - 36} - \dfrac{1}{10x + 12}$

Simplify the complex fraction. **(Lesson 9.5)**

5. $\dfrac{\frac{8}{x} + 11}{\frac{1}{6x} - 1}$

6. $\dfrac{36 - \frac{1}{x^2}}{\frac{1}{6x^2} - 6}$

7. $\dfrac{\frac{2}{x^2 - 1} - \frac{1}{x + 1}}{\frac{1}{12x^2 - 3}}$

8. $\dfrac{\frac{1}{x - 5} - \frac{x}{x^2 - 25}}{\frac{5}{2x}}$

9. **AVERAGE COST** You bought a potholder weaving frame for \$10. A bag of potholder material costs \$4 and contains enough material to make a dozen potholders. How many dozens of potholders must you make before your average total cost per dozen falls to \$4.50? **(Lesson 9.6)**

MATH & History

Deep Water Diving

APPLICATION LINK www.mcdougallittell.com

THEN

IN 1530 the invention of the diving bell provided the first effective means of breathing underwater. Like many other diving devices, a diving bell uses air that is compressed by the pressure of the water. Because oxygen under high pressure (at great depths) can have toxic effects on the body, the percent of oxygen in the air must be adjusted. The recommended percent p of oxygen (by volume) in the air that a diver breathes is

$$p = \frac{660}{d + 33}$$

where d is the depth (in feet) at which the diver is working.

1. Graph the equation.
2. At what depth is the recommended percent of oxygen 5%?
3. What value does the recommended percent of oxygen approach as a diver's depth increases?

Sylvia Earle, marine biologist.

NOW

TODAY diving technology makes it easier for scientists like Dr. Sylvia Earle to study ocean life. Using one-person submarines, Earle has undertaken a five-year study of marine sanctuaries.