CHAPTER 8

Chapter Summary

What did you learn?	*Why did you learn it?*
Graph exponential functions.	
• exponential growth functions **(8.1)**	Estimate wind energy generated by turbines. **(p. 470)**
• exponential decay functions **(8.2)**	Find the depreciated value of a car. **(p. 476)**
• natural base functions **(8.3)**	Find the number of endangered species. **(p. 482)**
Evaluate and simplify expressions.	
• exponential expressions with base *e* **(8.3)**	Find air pressure on Mount Everest. **(p. 484)**
• logarithmic expressions **(8.4)**	Approximate distance traveled by a tornado. **(p. 491)**
Graph logarithmic functions. **(8.4)**	Estimate the average diameter of sand particles for a beach with given slope. **(p. 489)**
Use properties of logarithms. **(8.5)**	Compare loudness of sounds. **(p. 495)**
Solve exponential and logarithmic equations. **(8.6)**	Use Newton's law of cooling. **(p. 502)**
Model data with exponential and power functions. **(8.7)**	Model the number of U.S. stamps issued. **(p. 515)**
Evaluate and graph logistic growth functions. **(8.8)**	Model the height of a sunflower. **(p. 519)**
Use exponential, logarithmic, and logistic growth functions to model real-life situations. **(8.1–8.8)**	Model a telescope's limiting magnitude. **(p. 507)**

How does Chapter 8 fit into the BIGGER PICTURE of algebra?

In Chapter 2 you began your study of functions and learned that quantities that increase by the same *amount* over equal periods of time are modeled by linear functions. In Chapter 8 you saw that quantities that increase by the same *percent* over equal periods of time are modeled by exponential functions.

Exponential functions and logarithmic functions are two important "families" of functions. They model many real-life situations, and they are used in advanced mathematics topics such as calculus and probability.

STUDY STRATEGY

How did you study with a group?

Here is an example of a summary prepared for Lesson 8.4 and presented to the group, following the **Study Strategy** on page 464.

Study Group

Lesson 8.4 Summary

Definition of logarithm: $\log_b y = x$ if and only if $b^x = y$

Common logarithm (base 10): $\log_{10} x = \log x$

Natural logarithm (base e): $\log_e x = \ln x$

Inverse functions: $f(x) = b^x$ (exponential) and
$g(x) = \log_b x$ (logarithmic)

Graph of logarithmic function $f(x) = \log_b (x - h) + k$: asymptote $x = h$; domain $x > h$; range all real numbers; up $b > 0$; down $0 < b < 1$

CHAPTER 8

Chapter Review

VOCABULARY

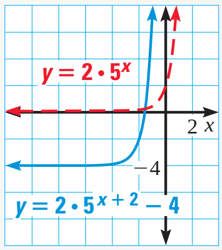
- exponential function, p. 465
- base of an exponential function, p. 465
- asymptote, p. 465
- exponential growth function, p. 466
- growth factor, p. 467
- exponential decay function, p. 474
- decay factor, p. 476
- natural base *e*, or Euler number, p. 480
- logarithm of *y* with base *b*, p. 486
- common logarithm, p. 487
- natural logarithm, p. 487
- change-of-base formula, p. 494
- logistic growth function, p. 517

8.1 EXPONENTIAL GROWTH

Examples on pp. 465–468

EXAMPLE An exponential growth function has the form $y = ab^x$ with $a > 0$ and $b > 1$.

To graph $y = 2 \cdot 5^{x+2} - 4$, first lightly sketch the graph of $y = 2 \cdot 5^x$, which passes through $(0, 2)$ and $(1, 10)$. Then translate the graph 2 units to the left and 4 units down. The graph passes through $(-2, -2)$ and $(-1, 6)$. The asymptote is the line $y = -4$. The domain is all real numbers, and the range is $y > -4$.



Graph the function. State the domain and range.

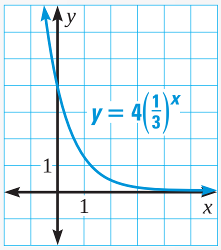
1. $y = -2^x + 4$ **2.** $y = 3 \cdot 2^x$ **3.** $y = 5 \cdot 3^{x-2}$ **4.** $y = 4^{x+3} - 1$

8.2 EXPONENTIAL DECAY

Examples on pp. 474–476

EXAMPLE An exponential decay function has the form $y = ab^x$ with $a > 0$ and $0 < b < 1$.

To graph $y = 4\left(\frac{1}{3}\right)^x$, plot $(0, 4)$ and $\left(1, \frac{4}{3}\right)$. From *right* to *left* draw a curve that begins just above the *x*-axis, passes through the two points, and moves up. The asymptote is the line $y = 0$. The domain is all real numbers, and the range is $y > 0$.



Tell whether the function represents *exponential growth* or *exponential decay*.

5. $f(x) = 5\left(\frac{3}{4}\right)^x$ **6.** $f(x) = 2\left(\frac{5}{4}\right)^x$ **7.** $f(x) = 3(6)^{-x}$ **8.** $f(x) = 4(3)^x$

Graph the function. State the domain and range.

9. $y = \left(\frac{1}{4}\right)^x$ **10.** $y = 2\left(\frac{3}{5}\right)^{x-1}$ **11.** $y = \left(\frac{1}{2}\right)^x - 5$ **12.** $y = -3\left(\frac{3}{4}\right)^x + 2$

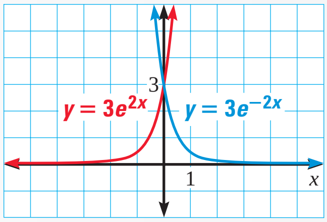
8.3 THE NUMBER *e*

Examples on pp. 480–482

EXAMPLES You can use e as the base of an exponential function. To graph such a function, use $e \approx 2.718$ and plot some points.

$f(x) = 3e^{2x}$ is an exponential growth function, since $2 > 0$.
$g(x) = 3e^{-2x}$ is an exponential decay function, since $-2 < 0$.

For both functions, the y-intercept is 3, the asymptote is $y = 0$, the domain is all real numbers, and the range is $y > 0$.



Graph the function. State the domain and range.

13. $y = e^{x+5}$ **14.** $y = 0.4e^{x} - 3$ **15.** $y = 4e^{-2x}$ **16.** $y = -e^{x} + 3$

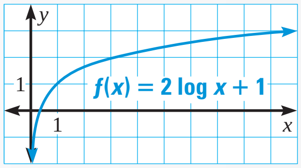
8.4 LOGARITHMIC FUNCTIONS

Examples on pp. 486–489

EXAMPLES You can use the definition of logarithm to evaluate expressions: $\log_b y = x$ if and only if $b^x = y$. The common logarithm has base 10 ($\log_{10} x = \log x$). The natural logarithm has base e ($\log_e x = \ln x$).

To evaluate $\log_8 4096$, write $\log_8 4096 = \log_8 8^4 = 4$.

To graph the logarithmic function $f(x) = 2 \log x + 1$, plot points such as $(1, 1)$ and $(10, 3)$. The vertical line $x = 0$ is an asymptote. The domain is $x > 0$, and the range is all real numbers.



Evaluate the expression without using a calculator.

17. $\log_4 64$ **18.** $\log_2 \frac{1}{8}$ **19.** $\log_3 \frac{1}{9}$ **20.** $\log_6 1$

Graph the function. State the domain and range.

21. $y = 3 \log_5 x$ **22.** $y = \log 4x$ **23.** $y = \ln x + 4$ **24.** $y = \log (x - 2)$

8.5 PROPERTIES OF LOGARITHMS

Examples on pp. 493–495

EXAMPLES You can use product, quotient, and power properties of logarithms.

Expand: $\log_2 \frac{3x}{y} = \log_2 3x - \log_2 y = \log_2 3 + \log_2 x - \log_2 y$

Condense: $3 \log_6 4 + \log_6 2 = \log_6 4^3 + \log_6 2 = \log_6 (64 \cdot 2) = \log_6 128$

Expand the expression.

25. $\log_3 6xy$ **26.** $\ln \frac{7x}{3}$ **27.** $\log 5x^3$ **28.** $\log \frac{x^5 y^{-2}}{2y}$

Condense the expression.

29. $2 \ln 3 - \ln 5$ **30.** $\log_4 3 + 3 \log_4 2$ **31.** $0.5 \log 4 + 2(\log 6 - \log 2)$

8.6 SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

Examples on pp. 501–504

EXAMPLES You can solve exponential equations by equating exponents or by taking the logarithm of each side. You can solve logarithmic equations by exponentiating each side of the equation.

$10^x = 4.3$

$\log 10^x = \log 4.3$ ← Take log of each side.

$x = \log 4.3 \approx 0.633$

$\log_4 x = 3$

$4^{\log_4 x} = 4^3$ ← Exponentiate each side.

$x = 4^3 = 64$

Solve the equation. Check for extraneous solutions.

32. $2(3)^{2x} = 5$ **33.** $3e^{-x} - 4 = 9$ **34.** $3 + \ln x = 8$ **35.** $5 \log (x - 2) = 11$

8.7 MODELING WITH EXPONENTIAL AND POWER FUNCTIONS

Examples on pp. 509–512

EXAMPLE You can write an exponential function of the form $y = ab^x$ or a power function of the form $y = ax^b$ that passes through two given points.

To find a power function given (3, 2) and (9, 12), substitute the coordinates into $y = ax^b$ to get the equations $2 = a \cdot 3^b$ and $12 = a \cdot 9^b$. Solve the system of equations by substitution: $a \approx 0.333$ and $b \approx 1.631$. So, the function is $y = 0.333x^{1.631}$.

Find an exponential function of the form $y = ab^x$ whose graph passes through the given points.

36. (2, 6), (3, 8) **37.** (2, 8.9), (4, 20) **38.** (2, 4.2), (4, 3.6)

Find a power function of the form $y = ax^b$ whose graph passes through the given points.

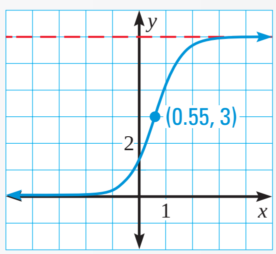
39. (2, 3.4), (6, 7.3) **40.** (2, 12.5), (4, 33.2) **41.** (0.5, 1), (10, 150)

8.8 LOGISTIC GROWTH FUNCTIONS

Examples on pp. 517–519

EXAMPLE You can graph logistic growth functions by plotting points and identifying important characteristics of the graph.

The graph of $y = \dfrac{6}{1 + 3e^{-2x}}$ is shown. It has asymptotes $y = 0$ and $y = 6$. The y-intercept is 1.5. The point of maximum growth is $\left(\dfrac{\ln 3}{2}, \dfrac{6}{2}\right) \approx (0.55, 3)$.



Graph the function. Identify the asymptotes, y-intercept, and point of maximum growth.

42. $y = \dfrac{2}{1 + e^{-2x}}$ **43.** $y = \dfrac{4}{1 + 2e^{-3x}}$ **44.** $y = \dfrac{3}{1 + 0.5e^{-0.5x}}$

CHAPTER 8

Chapter Test

Graph the function. State the domain and range.

1. $y = 2\left(\frac{1}{6}\right)^x$ **2.** $y = 4^{x-2} - 1$ **3.** $y = \frac{1}{2}e^x + 1$ **4.** $y = e^{-0.4x}$

5. $y = \log_{1/2} x$ **6.** $y = \ln x - 4$ **7.** $y = \log(x + 6)$ **8.** $y = \frac{2}{1 + 2e^{-x}}$

Simplify the expression.

9. $(2e^{-1})(3e^2)$ **10.** $\frac{-4e^x}{2e^{5x}}$ **11.** $e^6 \cdot e^x \cdot e^{-3x}$ **12.** $\log 1000^2$ **13.** $8^{\log_8 x}$

Evaluate the expression without using a calculator.

14. $\log_4 0.25$ **15.** $\log_{1/3} 27$ **16.** $\log 1$ **17.** $\ln e^{-2}$ **18.** $\log_3 243^2$

Solve the equation. Check for extraneous solutions.

19. $12 = 10^{x+5} - 7$ **20.** $5 - \ln x = 7$ **21.** $\log_2 4x = \log_2(x + 15)$ **22.** $\frac{4}{1 + 2.5e^{-4x}} = 3.3$

23. Tell whether the function $f(x) = 10(0.87)^x$ represents *exponential growth* or *exponential decay*.

24. Find the inverse of the function $y = \log_6 x$.

25. Use $\log_2 5 \approx 2.322$ to approximate $\log_2 50$ and $\log_2 0.4$.

26. Condense the expression $3 \log_4 14 - 3 \log_4 42$.

27. Expand the expression $\ln 2y^2x$.

28. Use the change-of-base formula to evaluate the expression $\log_7 15$.

29. Find an exponential function of the form $y = ab^x$ whose graph passes through the points (4, 6) and (7, 10).

30. Find a power function of the form $y = ax^b$ whose graph passes through the points (2, 3) and (10, 21).

31. **CAR DEPRECIATION** The value of a new car purchased for \$24,900 decreases by 10% per year. Write an exponential decay model for the value of the car. After about how many years will the car be worth half its purchase price?

32. **EARNING INTEREST** You deposit \$4000 in an account that pays 7% annual interest compounded continuously. Find the balance at the end of 5 years.

33. **COD WEIGHT** The table gives the mean weight w (in kilograms) and age x (in years) of Atlantic cod from the Gulf of Maine.

x	1	2	3	4	5	6	7	8
w	0.751	1.079	1.702	2.198	3.438	4.347	7.071	11.518

a. Draw a scatter plot of $\ln w$ versus x. Is an exponential model a good fit for the original data?

b. Find an exponential model for the original data. Estimate the weight of a cod that is 9 years old.