CHAPTER 7

Chapter Summary

WHAT did you learn?	WHY did you learn it?
Evaluate *n*th roots of real numbers. (7.1)	Find the number of reptile and amphibian species that Puerto Rico can support. (p. 405)
Use properties of rational exponents to evaluate and simplify expressions. (7.2)	Model frequencies in the musical range of a trumpet. (p. 413)
Perform function operations. (7.3)	Find the height of a dinosaur. (p. 419)
Find inverses of linear and nonlinear functions. (7.4)	Find your bowling average. (p. 428)
Graph square root and cube root functions. (7.5)	Find the age of an African elephant. (p. 433)
Solve equations that contain radicals or rational exponents. (7.6)	Determine which boats satisfy the rule for competing in the America's Cup. (p. 443)
Use roots and rational exponents in real-life problems. (7.1–7.6)	Find surface areas of mammals. (p. 410)
Use power functions, inverse functions, and radical functions to solve real-life problems. (7.3–7.6)	Find wind speeds that correspond to Beaufort wind scale numbers. (p. 440)
Use measures of central tendency and measures of dispersion to describe data sets. (7.7)	Analyze data sets such as the free-throw percentages for the players in the WNBA. (pp. 445 and 446)
Represent data graphically with box-and-whisker plots and histograms. (7.7)	Graph data sets such as the ages of the Presidents and Vice Presidents of the United States. (p. 451)

How does Chapter 7 fit into the BIGGER PICTURE of algebra?

In Chapter 7 you saw the familiar ideas of squares and square roots extended. This was a significant step in your study of powers and roots as you used exponents that were *not* whole numbers in expressions, functions, and many real-life problems. You will continue to build on these ideas as long as you study mathematics.

STUDY STRATEGY

How did you quiz yourself?

Here is an example of a quiz that was written for Lesson 7.3 and used before a class quiz was given, following the **Study Strategy** on page 400.

Quiz Yourself

Let $f(x) = -2x$ and $g(x) = x - 4$. Perform the indicated operation and state the domain. (Lesson 7.3)

1. $f(x) + g(x)$
2. $f(x) - g(x)$
3. $f(x) \cdot g(x)$
4. $\dfrac{f(x)}{g(x)}$
5. $g(f(x))$
6. $f(f(x))$

CHAPTER 7

Chapter Review

VOCABULARY

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- index, p. 401
- simplest form, p. 408
- like radicals, p. 408
- power function, p. 415
- composition, p. 416
- inverse relation, p. 422
- inverse function, p. 422
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- statistics, p. 445
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- mean, p. 445
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- measure of dispersion, p. 446
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- lower quartile, p. 447
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- histogram, p. 448
- frequency, p. 448
- frequency distribution, p. 448

7.1 *N*TH ROOTS AND RATIONAL EXPONENTS

Examples on pp. 401–403

EXAMPLES You can evaluate *n*th roots using radicals or rational exponents.

Radical notation: $27^{-2/3} = \dfrac{1}{27^{2/3}} = \dfrac{1}{(\sqrt[3]{27})^2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

Rational exponent notation: $27^{-2/3} = \dfrac{1}{27^{2/3}} = \dfrac{1}{(27^{1/3})^2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

Evaluate the expression without using a calculator.

1. $\sqrt[4]{16}$ **2.** $(\sqrt[3]{64})^2$ **3.** $9^{-5/2}$ **4.** $216^{1/3}$ **5.** $\sqrt[5]{-32}$

6. Find the real *n*th root(s) of *a* if $n = 4$ and $a = 81$.

7. Find the real *n*th root(s) of *a* if $n = 5$ and $a = -1$.

8. Find the real *n*th root(s) of *a* if $n = 7$ and $a = 0$.

7.2 PROPERTIES OF RATIONAL EXPONENTS

Examples on pp. 407–410

EXAMPLES You can use properties of rational exponents to simplify expressions.

$\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{12 \cdot 4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = \sqrt[3]{8} \cdot \sqrt[3]{6} = 2\sqrt[3]{6}$

$\dfrac{(x^{1/2}y)^2}{x^{1/2}y^{3/4}} = \dfrac{x^{(1/2 \cdot 2)}y^2}{x^{1/2}y^{3/4}} = \dfrac{xy^2}{x^{1/2}y^{3/4}} = x^{(1 - 1/2)}y^{(2 - 3/4)} = x^{1/2}y^{5/4}$

Simplify the expression. Assume all variables are positive.

9. $5^{1/4} \cdot 5^{-9/4}$ **10.** $(100^{1/3})^{3/4}$ **11.** $\sqrt[3]{\dfrac{16}{1000}}$ **12.** $5\sqrt[3]{17} - 4\sqrt[3]{17}$

13. $(81x)^{1/4}$ **14.** $\dfrac{(4x)^2}{(4x)^{1/2}}$ **15.** $\sqrt[6]{6x^6y^7z^{10}}$ **16.** $\sqrt[3]{4a^6} + a\sqrt[3]{108a^3}$

7.3 Power Functions and Function Operations

Examples on pp. 415–417

EXAMPLES You can add, subtract, multiply, or divide any two functions f and g. You can also find the composition of any two functions.

Let $f(x) = 2x^{1/2}$ and $g(x) = x^4$

Addition: $f(x) + g(x) = 2x^{1/2} + x^4$

Multiplication: $f(x) \cdot g(x) = 2x^{1/2} \cdot x^4 = 2x^{9/2}$

Composition: $f(g(x)) = f(x^4) = 2(x^4)^{1/2} = 2x^2$

Let $f(x) = 2x - 4$ and $g(x) = x - 2$. Perform the indicated operation.

17. $f(x) + g(x)$ **18.** $f(x) - g(x)$ **19.** $f(x) \cdot g(x)$ **20.** $\frac{f(x)}{g(x)}$ **21.** $f(g(x))$

7.4 Inverse Functions

Examples on pp. 422–425

EXAMPLES You can find the inverse relation of any function. To verify that two functions are inverses of each other, show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

$$f(x) = y = 2x - 5$$
$$x = 2y - 5$$
$$x + 5 = 2y$$
$$\frac{1}{2}x + \frac{5}{2} = y = f^{-1}(x)$$

$$f(f^{-1}(x)) = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5 = x + 5 - 5 = x$$
$$f^{-1}(f(x)) = \frac{1}{2}(2x - 5) + \frac{5}{2} = x - \frac{5}{2} + \frac{5}{2} = x$$

Find the inverse function.

22. $f(x) = -2x + 1$ **23.** $f(x) = -x^4, x \ge 0$ **24.** $f(x) = 5x^3 + 7$

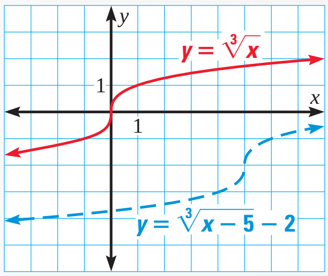
25. Verify that $f(x) = -2x^5$ and $g(x) = \sqrt[5]{-\frac{x}{2}}$ are inverse functions.

7.5 Graphing Square Root and Cube Root Functions

Examples on pp. 431–433

EXAMPLE You can graph a square root function by starting with the graph of $y = \sqrt{x}$. You can graph a cube root function by starting with the graph of $y = \sqrt[3]{x}$.

To graph $y = \sqrt[3]{x - 5} - 2$, first sketch $y = \sqrt[3]{x}$ (shown in red). Then shift the graph right 5 units and down 2 units. From the graph of $y = \sqrt[3]{x - 5} - 2$, you can see that the domain and range of the function are both all real numbers.



Graph the function. Then state the domain and range.

26. $y = (x - 7)^{1/3}$ **27.** $y = \sqrt{x} + 6$ **28.** $y = -2(x - 3)^{1/2}$ **29.** $y = 3\sqrt[3]{x + 4} - 9$

7.6 SOLVING RADICAL EQUATIONS

Examples on pp. 437–440

EXAMPLES You can solve equations that contain radicals or rational exponents by raising each side of the equation to the same power.

$$\sqrt{x-4} = 6 \qquad\qquad 4x^{2/3} = 100$$

$$(\sqrt{x-4})^2 = 6^2 \quad \text{Square each side.} \qquad x^{2/3} = 25$$

$$x - 4 = 36 \qquad\qquad (x^{2/3})^{3/2} = 25^{3/2} \quad \text{Raise each side to } \tfrac{3}{2} \text{ power.}$$

$$x = 40 \qquad\qquad x = 125$$

Solve the equation. Check for extraneous solutions.

30. $3(x+1)^{1/5} + 5 = 11$ **31.** $\sqrt[3]{5x+3} - \sqrt[3]{4x} = 0$ **32.** $\sqrt{4x} = x - 8$

7.7 STATISTICS AND STATISTICAL GRAPHS

Examples on pp. 445–448

EXAMPLES The table shows the normal daily high temperatures (in degrees Fahrenheit) for Phoenix, Arizona, from 1961 to 1990.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
65.9	70.7	75.5	84.5	93.6	103.5	105.9	103.7	98.3	88.1	74.9	66.2

MEAN Find the average of the numbers: $\dfrac{65.9 + 70.7 + \cdots + 66.2}{12} = \dfrac{1030.8}{12} = 85.9$

MEDIAN Write the numbers in increasing order and locate the middle number(s): 65.9, 66.2, 70.7, 74.9, 75.5, **84.5**, **88.1**, 93.6, 98.3, 103.5, 103.7, 105.9
There are two middle numbers, so find their mean: $\dfrac{84.5 + 88.1}{2} = 86.3$

MODE Find the number(s) that occur most frequently: none

RANGE Find the difference between the greatest and least numbers: $105.9 - 65.9 = 40$

STANDARD DEVIATION Use the formula: $\sqrt{\dfrac{(65.9-85.9)^2 + (70.7-85.9)^2 + \cdots + (66.2-85.9)^2}{12}} \approx 14.4$

BOX-AND-WHISKER PLOT Find the quartiles: $\dfrac{70.7 + 74.9}{2} = 72.8$ and $\dfrac{98.3 + 103.5}{2} = 100.9$
Plot the minimum, maximum, median, and quartiles. Then draw the box and the whiskers (not shown).

HISTOGRAM Using five intervals beginning with 60–69, tally the data values for each interval. Then draw a histogram of the data set (not shown).

In Exercises 33 and 34, use the following data set of employees' ages at a small company: 21, 25, 30, 36, 39, 40, 44, 45, 46, 51, 51, 63.

33. Find the mean, median, mode, range, and standard deviation of the data set.

34. Draw a box-and-whisker plot and a histogram of the data set. For the histogram, use five intervals beginning with 20–29.

CHAPTER 7

Chapter Test

Evaluate the expression without using a calculator.

1. $\sqrt[3]{-1000}$ **2.** $4^{5/2}$ **3.** $(-64)^{2/3}$ **4.** $243^{-1/5}$ **5.** $\sqrt[4]{16}$

Simplify the expression. Assume all variables are positive.

6. $(2^{1/3} \cdot 5^{1/2})^4$ **7.** $\sqrt[3]{27x^3y^6z^9}$ **8.** $\dfrac{3xy^{-1}}{12x^{1/2}y}$ **9.** $\left(\dfrac{81x^2}{y}\right)^{3/4}$ **10.** $\sqrt{18} + \sqrt{200}$

Perform the indicated operation and state the domain.

11. $f + g$; $f(x) = x - 8$, $g(x) = 3x$

12. $f - g$; $f(x) = 2x^{1/4}$, $g(x) = 5x^{1/4}$

13. $f \cdot g$; $f(x) = 5x + 7$, $g(x) = x - 9$

14. $\dfrac{f}{g}$; $f(x) = x^{-1/5}$, $g(x) = x^{3/5}$

15. $f(g(x))$; $f(x) = 4x^2 - 5$, $g(x) = -x$

16. $g(f(x))$; $f(x) = x^2 + 3x$, $g(x) = 2x + 1$

Find the inverse function.

17. $f(x) = \frac{1}{3}x - 4$ **18.** $f(x) = -5x + 5$ **19.** $f(x) = \frac{3}{4}x^2, x \ge 0$ **20.** $f(x) = x^5 - 2$

Graph the function. Then state the domain and range.

21. $f(x) = \sqrt{x - 6}$ **22.** $f(x) = \sqrt[3]{x} + 3$ **23.** $f(x) = 3(x + 4)^{1/3} - 2$ **24.** $f(x) = -2x^{1/2} + 4$

Solve the equation. Check for extraneous solutions.

25. $x^{5/2} - 10 = 22$ **26.** $(x + 8)^{1/4} + 1 = 0$ **27.** $\sqrt[3]{7x - 9} + 11 = 14$ **28.** $\sqrt{4x + 15} - 3\sqrt{x} = 0$

29. **BIOLOGY CONNECTION** Some biologists study the structure of animals. By studying a series of antelopes, biologists have found that the length l (in millimeters) of an antelope's bone can be modeled by

$$l = 24.1d^{2/3}$$

where d is the midshaft diameter of the bone (in millimeters). If the bone of an antelope has a midshaft diameter of 20 millimeters, what is the length of the bone? ▶ Source: *On Size and Life*

ACADEMY AWARDS **In Exercises 30–33, use the tables below which give the ages of the Academy Award winners for best actress and for best actor from 1980 to 1998.**

Best actress
21, 25, 26, 29, 31, 33, 33, 34, 34, 38, 39, 41, 42, 45, 49, 49, 61, 72, 80

Best actor
30, 32, 35, 37, 37, 38, 39, 42, 43, 45, 45, 46, 51, 52, 52, 54, 60, 61, 76

30. Find the mean, median, mode, range, and standard deviation of each data set.

31. Draw a box-and-whisker plot of each data set.

32. Make a frequency distribution of each data set using six intervals beginning with 21–30. Then draw a histogram of each data set.

33. *Writing* Compare the ages of the best actresses with the ages of the best actors. Use statistics and statistical graphs to support your statements.