7.6 Solving Radical Equations

What *you should learn*

GOAL 1 Solve equations that contain radicals or rational exponents.

GOAL 2 Use radical equations to solve **real-life** problems, such as determining wind speeds that correspond to the Beaufort wind scale in **Example 6**.

Why *you should learn it*

▼ To solve **real-life** problems, such as determining which boats satisfy the rule for competing in the America's Cup sailboat race in **Ex. 68**.

REAL LIFE

GOAL 1 Solving a Radical Equation

To solve a *radical equation*—an equation that contains radicals or rational exponents—you need to eliminate the radicals or rational exponents and obtain a polynomial equation. The key step is to *raise each side of the equation to the same power*.

If $a = b$, then $a^n = b^n$. **Powers property of equality**

Then solve the new equation using standard procedures. Before raising each side of an equation to the same power, you should isolate the radical expression on one side of the equation.

EXAMPLE 1 *Solving a Simple Radical Equation*

Solve $\sqrt[3]{x} - 4 = 0$.

SOLUTION

$\sqrt[3]{x} - 4 = 0$ **Write original equation.**

$\sqrt[3]{x} = 4$ **Isolate radical.**

$(\sqrt[3]{x})^3 = 4^3$ **Cube each side.**

$x = 64$ **Simplify.**

▶ The solution is 64. Check this in the original equation.

EXAMPLE 2 *Solving an Equation with Rational Exponents*

Solve $2x^{3/2} = 250$.

SOLUTION

Because x is raised to the $\frac{3}{2}$ power, you should isolate the power and then raise each side of the equation to the $\frac{2}{3}$ power $\left(\frac{2}{3}\text{ is the reciprocal of }\frac{3}{2}\right)$.

$2x^{3/2} = 250$ **Write original equation.**

$x^{3/2} = 125$ **Isolate power.**

$(x^{3/2})^{2/3} = 125^{2/3}$ **Raise each side to $\frac{2}{3}$ power.**

$x = (125^{1/3})^2$ **Apply properties of roots.**

$x = 5^2 = 25$ **Simplify.**

▶ The solution is 25. Check this in the original equation.

STUDENT HELP

Study Tip
To solve an equation of the form $x^{m/n} = k$ where k is a constant, raise both sides of the equation to the $\frac{n}{m}$ power, because $(x^{m/n})^{n/m} = x^1 = x$.

EXAMPLE 3 *Solving an Equation with One Radical*

Solve $\sqrt{4x - 7} + 2 = 5$.

SOLUTION

$\sqrt{4x - 7} + 2 = 5$ **Write original equation.**

$\sqrt{4x - 7} = 3$ **Isolate radical.**

$(\sqrt{4x - 7})^2 = 3^2$ **Square each side.**

$4x - 7 = 9$ **Simplify.**

$4x = 16$ **Add 7 to each side.**

$x = 4$ **Divide each side by 4.**

✓**CHECK** Check $x = 4$ in the original equation.

$\sqrt{4x - 7} + 2 = 5$ **Write original equation.**

$\sqrt{4(4) - 7} \stackrel{?}{=} 3$ **Substitute 4 for *x*.**

$\sqrt{9} \stackrel{?}{=} 3$ **Simplify.**

$3 = 3$ ✓ **Solution checks.**

▶ The solution is 4.

Some equations have two radical expressions. Before raising both sides to the same power, you should rewrite the equation so that each side of the equation has only one radical expression.

EXAMPLE 4 *Solving an Equation with Two Radicals*

Solve $\sqrt{3x + 2} - 2\sqrt{x} = 0$.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

SOLUTION

$\sqrt{3x + 2} - 2\sqrt{x} = 0$ **Write original equation.**

$\sqrt{3x + 2} = 2\sqrt{x}$ **Add $2\sqrt{x}$ to each side.**

$(\sqrt{3x + 2})^2 = (2\sqrt{x})^2$ **Square each side.**

$3x + 2 = 4x$ **Simplify.**

$2 = x$ **Solve for *x*.**

✓**CHECK** Check $x = 2$ in the original equation.

$\sqrt{3x + 2} - 2\sqrt{x} = 0$ **Write original equation.**

$\sqrt{3(2) + 2} - 2\sqrt{2} \stackrel{?}{=} 0$ **Substitute 2 for *x*.**

$2\sqrt{2} - 2\sqrt{2} \stackrel{?}{=} 0$ **Simplify.**

$0 = 0$ ✓ **Solution checks.**

▶ The solution is 2.

If you try to solve $\sqrt{x} = -1$ by squaring both sides, you get $x = 1$. But $x = 1$ is not a valid solution of the original equation. This is an example of an **extraneous** (or false) **solution.** Raising both sides of an equation to the same power may introduce extraneous solutions. So, when you use this procedure it is critical that you check each solution in the *original* equation.

EXAMPLE 5 *An Equation with an Extraneous Solution*

Solve $x - 4 = \sqrt{2x}$.

STUDENT HELP

Look Back
For help with factoring, see p. 256.

SOLUTION

$x - 4 = \sqrt{2x}$	Write original equation.
$(x-4)^2 = (\sqrt{2x})^2$	Square each side.
$x^2 - 8x + 16 = 2x$	Expand left side; simplify right side.
$x^2 - 10x + 16 = 0$	Write in standard form.
$(x-2)(x-8) = 0$	Factor.
$x - 2 = 0$ or $x - 8 = 0$	Zero product property
$x = 2$ or $x = 8$	Simplify.

✓**CHECK** Check $x = 2$ in the original equation.

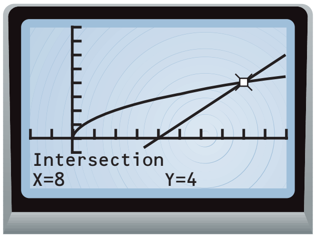
$x - 4 = \sqrt{2x}$	Write original equation.
$2 - 4 \stackrel{?}{=} \sqrt{2(2)}$	Substitute 2 for x.
$-2 \stackrel{?}{=} \sqrt{4}$	Simplify.
$-2 \neq 2$	Solution does not check.

✓**CHECK** Check $x = 8$ in the original equation.

$x - 4 = \sqrt{2x}$	Write original equation.
$8 - 4 \stackrel{?}{=} \sqrt{2(8)}$	Substitute 8 for x.
$4 \stackrel{?}{=} \sqrt{16}$	Simplify.
$4 = 4$ ✓	Solution checks.

▶ The only solution is 8.

If you graph each side of the equation in Example 5, as shown, you can see that the graphs of $y = x - 4$ and $y = \sqrt{2x}$ intersect only at $x = 8$. This confirms that $x = 8$ is a solution of the equation, but that $x = 2$ is not.



In general, all, some, or none of the apparent solutions of a radical equation can be extraneous. When all of the apparent solutions of a radical equation are extraneous, the equation has *no solution*.

FOCUS ON APPLICATIONS

REAL LIFE
BEAUFORT WIND SCALE
The Beaufort wind scale was developed by Rear-Admiral Sir Francis Beaufort in 1805 so that sailors could detect approaching storms. Today the scale is used mainly by meteorologists.

INTERNET APPLICATION LINK
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GOAL 2 SOLVING RADICAL EQUATIONS IN REAL LIFE

EXAMPLE 6 *Using a Radical Model*

BEAUFORT WIND SCALE The Beaufort wind scale was devised to measure wind speed. The Beaufort numbers B, which range from 0 to 12, can be modeled by $B = 1.69\sqrt{s + 4.45} - 3.49$ where s is the speed (in miles per hour) of the wind. Find the wind speed that corresponds to the Beaufort number $B = 11$.

Beaufort Wind Scale

Beaufort number	Force of wind	Effects of wind
0	Calm	Smoke rises vertically.
1	Light air	Direction shown by smoke.
2	Light breeze	Leaves rustle; wind felt on face.
3	Gentle breeze	Leaves move; flags extend.
4	Moderate breeze	Small branches sway; paper blown about.
5	Fresh breeze	Small trees sway.
6	Strong breeze	Large branches sway; umbrellas difficult to use.
7	Moderate gale	Large trees sway; walking difficult.
8	Fresh gale	Twigs break; walking hindered.
9	Strong gale	Branches scattered about; slight damage to buildings.
10	Whole gale	Trees uprooted; severe damage to buildings.
11	Storm	Widespread damage.
12	Hurricane	Devastation.

SOLUTION

$B = 1.69\sqrt{s + 4.45} - 3.49$ Write model.

$11 = 1.69\sqrt{s + 4.45} - 3.49$ Substitute 11 for B.

$14.49 = 1.69\sqrt{s + 4.45}$ Add 3.49 to each side.

$8.57 \approx \sqrt{s + 4.45}$ Divide each side by 1.69.

$73.4 \approx s + 4.45$ Square each side.

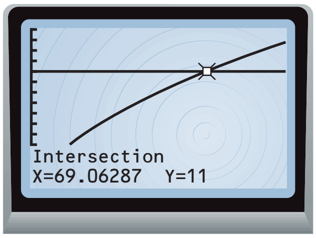
$69.0 \approx s$ Subtract 4.45 from each side.

▶ The wind speed is about 69 miles per hour.

✓ **ALGEBRAIC CHECK** Substitute 69 for s into the model and evaluate.

$$1.69\sqrt{69 + 4.45} - 3.49 \approx 1.69(8.57) - 3.49$$
$$\approx 11 \checkmark$$

✓ **GRAPHIC CHECK** You can use a graphing calculator to graph the model, and then use the *Intersect* feature to check that $x \approx 69$ when $y = 11$.



GUIDED PRACTICE

Vocabulary Check ✓

1. What is an extraneous solution?

Concept Check ✓

2. Marcy began solving $x^{2/3} = 5$ by cubing each side. What will she have to do next? What could she have done to solve the equation in just one step?

3. Zach was asked to solve $\sqrt{5x-2} - \sqrt{7x-4} = 0$. His first step was to square each side. While trying to isolate x, he gave up in frustration. What could Zach have done to avoid this situation?

Skill Check ✓

Solve the rational exponent equation. Check for extraneous solutions.

4. $3x^{1/4} = 4$
5. $(2x+7)^{3/2} = 27$
6. $x^{4/3} + 9 = 25$
7. $4x^{2/3} - 6 = 10$
8. $5(x-8)^{3/4} = 40$
9. $(x+9)^{5/2} - 1 = 31$

Solve the radical equation. Check for extraneous solutions.

10. $\sqrt[4]{x} = 3$
11. $\sqrt[3]{3x} + 6 = 10$
12. $\sqrt[5]{2x+1} + 5 = 9$
13. $\sqrt{x-2} = x - 2$
14. $\sqrt[3]{x+4} = \sqrt[3]{2x-5}$
15. $6\sqrt{x} - \sqrt{x-1} = 0$
16. **BEAUFORT WIND SCALE** Use the information in Example 6 to determine the wind speed that corresponds to the Beaufort number $B = 2$.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 950.

CHECKING SOLUTIONS **Check whether the given x-value is a solution of the equation.**

17. $\sqrt{x} - 3 = 6;\ x = 81$
18. $4(x-5)^{1/2} = 28;\ x = 12$
19. $(x+7)^{3/2} - 20 = 7;\ x = 2$
20. $\sqrt[3]{4x} + 11 = 5;\ x = -54$
21. $2\sqrt{5x+4} + 10 = 10;\ x = 0$
22. $\sqrt{4x-3} - \sqrt{3x} = 0;\ x = 3$

SOLVING RATIONAL EXPONENT EQUATIONS **Solve the equation. Check for extraneous solutions.**

23. $x^{5/2} = 32$
24. $x^{1/3} - \frac{2}{5} = 0$
25. $x^{2/3} + 15 = 24$
26. $-\frac{1}{2}x^{1/5} = 10$
27. $4x^{3/4} = 108$
28. $(x-4)^{3/2} = -6$
29. $(2x+5)^{1/2} = 4$
30. $3(x+1)^{4/3} = 48$
31. $-(x-5)^{1/4} + \frac{7}{3} = 2$

SOLVING RADICAL EQUATIONS **Solve the equation. Check for extraneous solutions.**

32. $\sqrt{x} = \frac{1}{9}$
33. $\sqrt[3]{x} + 10 = 16$
34. $\sqrt[4]{2x} - 13 = -9$
35. $\sqrt{x+56} = 16$
36. $\sqrt[3]{x+40} = -5$
37. $\sqrt{6x-5} + 10 = 3$
38. $\frac{2}{5}\sqrt{10x+6} = 12$
39. $2\sqrt{7x+4} - 1 = 7$
40. $-2\sqrt[5]{2x-1} + 4 = 0$
41. $x - 12 = \sqrt{16x}$
42. $\sqrt[4]{x^4+1} = 3x$
43. $\sqrt{x^2+5} = x + 3$
44. $\sqrt[3]{x} = x - 6$
45. $\sqrt{8x+1} = x + 2$
46. $\sqrt{2x + \frac{1}{6}} = x + \frac{5}{6}$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 17–22, 32–46
Example 2: Exs. 17–22, 23–31
Example 3: Exs. 17–22, 32–46
Example 4: Exs. 17–22, 47–54
Example 5: Exs. 23–54
Example 6: Exs. 63–69

SOLVING EQUATIONS WITH TWO RADICALS **Solve the equation. Check for extraneous solutions.**

47. $\sqrt{2x-1} = \sqrt{x+4}$

48. $\sqrt[4]{6x-5} = \sqrt[4]{x+10}$

49. $-\sqrt{8x+\frac{4}{3}} = \sqrt{2x+\frac{1}{3}}$

50. $2\sqrt[3]{10-3x} = \sqrt[3]{2-x}$

51. $\sqrt[4]{2x} + \sqrt[4]{x+3} = 0$

52. $\sqrt{x-6} - \sqrt{\frac{1}{3}x} = 0$

53. $\sqrt{2x+10} - 2\sqrt{x} = 0$

54. $\sqrt[3]{2x+15} - \frac{3}{2}\sqrt[3]{x} = 0$

SOLVING EQUATIONS **Use the *Intersect* feature on a graphing calculator to solve the equation.**

55. $\frac{3}{4}x^{1/3} = -2$

56. $2(x+19)^{2/5} - 1 = 17$

57. $(3.5x+1)^{2/7} = (6.4x+0.7)^{2/7}$

58. $\left(\frac{1}{5}x\right)^{3/4} = x - \frac{3}{8}$

59. $\sqrt{6.7x+14} = 9.4$

60. $\sqrt[3]{70-2x} - 10 = -6$

61. $\sqrt[4]{x-\frac{1}{6}} = 2\sqrt[4]{3x}$

62. $\sqrt{1.1x+2.4} = 19x - 4.2$

63. **NAILS** The length l (in inches) of a standard nail can be modeled by

$$l = 54d^{3/2}$$

where d is the diameter (in inches) of the nail. What is the diameter of a standard nail that is 3 inches long?

64. **SCIENCE CONNECTION** Scientists have found that the body mass m (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (1.6 \times 10^{-4})C^{273/100}$$

where C is the circumference (in millimeters) of the dinosaur's femur. Scientists have estimated that the mass of a *Tyrannosaurus rex* might have been 4500 kilograms. What size femur would have led them to this conclusion?
▶ Source: The Zoological Society of London

FOCUS ON PEOPLE

REAL LIFE **DR. ALEXA CANADY** was the first African-American woman to become a neurosurgeon in the United States. She received her MD degree, discussed in Ex. 65, in 1975.

65. **WOMEN IN MEDICINE** For 1970 through 1995, the percent p of Doctor of Medicine (MD) degrees earned each year by women can be modeled by

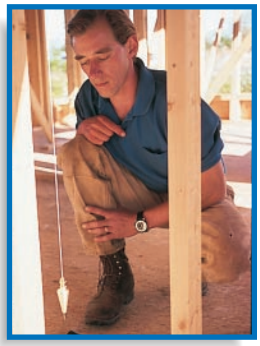
$$p = (0.867t^2 + 39.2t + 57.1)^{1/2}$$

where t is the number of years since 1970. In what year were about 36% of the degrees earned by women? ▶ Source: *Statistical Abstract of the United States*

66. **PLUMB BOBS** You work for a company that manufactures plumb bobs. The same mold is used to cast plumb bobs of different sizes. The equation

$$h = 1.5\sqrt[3]{t},\ 0 \le h \le 3$$

models the relationship between the height h (in inches) of the plumb bob and the time t (in seconds) that metal alloy is poured into the mold. How long should you pour the alloy into the mold to cast a plumb bob with a height of 2 inches?



67. **BEAUFORT WIND SCALE** Recall from Example 6 that the Beaufort number B from the Beaufort wind scale can be modeled by

$$B = 1.69\sqrt{s + 4.45} - 3.49$$

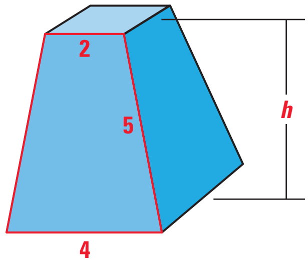
where s is the speed (in miles per hour) of the wind. Find the wind speed that corresponds to the Beaufort number $B = 7$.

68. **AMERICA'S CUP** In order to compete in the America's Cup sailboat race, a boat must satisfy the rule

$$\frac{l + 1.25\sqrt{s} - 9.8\sqrt[3]{d}}{0.679} \le 24$$

where l is the length (in meters) of the boat, s is the area (in square meters) of the sails, and d is the volume (in cubic meters) of water displaced by the boat. If a boat has a length of 20 meters and a sail area of 300 square meters, what is the minimum allowable value for d? ▶ Source: America's Cup

69. **GEOMETRY CONNECTION** You are trying to determine the height of a truncated pyramid that cannot be measured directly. The height h and slant height l of a truncated pyramid are related by the formula



$$l = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2}$$

where b_1 and b_2 are the lengths of the upper and lower bases of the pyramid, respectively. If $l = 5$, $b_1 = 2$, and $b_2 = 4$, what is the height of the pyramid?

70. **CRITICAL THINKING** Look back at Example 5. Solve $x - 4 = -\sqrt{2x}$ instead of $x - 4 = \sqrt{2x}$. How does changing $\sqrt{2x}$ to $-\sqrt{2x}$ change the solution(s) of the equation?

Test Preparation

71. **MULTIPLE CHOICE** What is the solution of $\sqrt{6x - 4} = 3$?

Ⓐ $-\frac{1}{6}$ Ⓑ $\frac{5}{6}$ Ⓒ $\frac{7}{6}$ Ⓓ $\frac{5}{3}$ Ⓔ $\frac{13}{6}$

72. **MULTIPLE CHOICE** What is (are) the solution(s) of $\sqrt{2x - 3} = \frac{1}{2}x$?

Ⓐ 2 Ⓑ 2, 6 Ⓒ $\frac{18}{7}$ Ⓓ $\frac{21}{4}$ Ⓔ none

73. **MULTIPLE CHOICE** What is the solution of $\sqrt[3]{x - 7} = \sqrt[3]{\frac{3}{4}x + 1}$?

Ⓐ -6 Ⓑ $-\frac{24}{7}$ Ⓒ -4 Ⓓ 2 Ⓔ 32

★ Challenge

SOLVING EQUATIONS WITH TWO RADICALS **Solve the equation. Check for extraneous solutions. (*Hint:* To solve these equations you will need to square each side of the equation two separate times.)**

74. $\sqrt{x + 5} = 5 - \sqrt{x}$

75. $\sqrt{2x + 3} = 3 - \sqrt{2x}$

76. $\sqrt{x + 3} - \sqrt{x - 1} = 1$

77. $\sqrt{2x + 4} + \sqrt{3x - 5} = 4$

78. $\sqrt{3x - 2} = 1 + \sqrt{2x - 3}$

79. $\frac{1}{2}\sqrt{2x - 5} - \frac{1}{2}\sqrt{3x + 4} = 1$

EXTRA CHALLENGE
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MIXED REVIEW

USING ORDER OF OPERATIONS **Evaluate the expression.** (Review 1.2 for 7.7)

80. $6 + 24 \div 3$ **81.** $3 \cdot 5 + 10 \div 2$ **82.** $27 - 4 \cdot 16 \div 8$

83. $2 - (10 \cdot 2)^2 \div 5$ **84.** $8 + (3 \cdot 10) \div 6 - 1$ **85.** $11 - 8 \div 2 + 48 \div 4$

USING GRAPHS **Graph the polynomial function. Identify the *x*-intercepts, local maximums, and local minimums.** (Review 6.8)

86. $f(x) = x^3 - 4x^2 + 3$ **87.** $f(x) = 3x^3 - 2.5x^2 + 1.25x + 6$

88. $f(x) = \frac{1}{2}x^4 - \frac{1}{2}$ **89.** $f(x) = x^5 + x^3 - 6x$

90. **PRINTING RATES** The cost C (in dollars) of printing x announcements (in hundreds) is given by the function shown. Graph the function. (Review 2.7)

$$C = \begin{cases} 62 + 22(x - 1), & \text{if } 1 \le x \le 5 \\ 150 + 14(x - 5), & \text{if } x > 5 \end{cases}$$

MATH & History — Tsunamis

APPLICATION LINK www.mcdougallittell.com

THEN

IN AUGUST OF 1883, a volcano erupted on the island of Krakatau, Indonesia. The eruption caused a *tsunami* (a type of wave) to form and travel into the Indian Ocean and into the Java Sea. The speed s (in kilometers per hour) that a tsunami travels can be modeled by $s = 356\sqrt{d}$ where d is the depth (in kilometers) of the water.

1. A tsunami from Krakatau hit Jakarta traveling about 60 kilometers per hour. What is the average depth of the water between Krakatau and Jakarta?
2. After 15 hours and 12 minutes a tsunami from Krakatau hit Port Elizabeth, South Africa, 7546 kilometers away. Find the average speed of the tsunami.
3. Based on your answer to Exercise 2, what is the average depth of the Indian Ocean between Krakatau and Port Elizabeth?

NOW

AFTER A TRAGIC TSUNAMI hit the Aleutian Islands in 1946, scientists began work on a tsunami warning system. Today that system is operated 24 hours a day at the Honolulu Observatory and effectively warns people when a tsunami might arrive.

c. 1800 — Famous tsunami art created by Hokusai.

1883 — Krakatau erupts.

1957 — Alaskan earthquake causes Pacific-wide tsunami.

1995 — Prototype of tsunami real-time reporting system developed.