

7.3

Power Functions and Function Operations

What you should learn

GOAL 1 Perform operations with functions including power functions.

GOAL 2 Use power functions and function operations to solve **real-life** problems, such as finding the proportion of water loss in a bird's egg in **Example 4**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the height of a dinosaur in **Ex. 56**.



GOAL 1 PERFORMING FUNCTION OPERATIONS

In Chapter 6 you learned how to add, subtract, multiply, and divide polynomial functions. These operations can be defined for any functions.

CONCEPT SUMMARY

OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations (addition, subtraction, multiplication, and division) on f and g .

Operation	Definition	Example: $f(x) = 2x$, $g(x) = x + 1$
ADDITION	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x + 1) = 3x + 1$
SUBTRACTION	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x + 1) = x - 1$
MULTIPLICATION	$h(x) = f(x) \cdot g(x)$	$h(x) = (2x)(x + 1) = 2x^2 + 2x$
DIVISION	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{2x}{x + 1}$

The domain of h consists of the x -values that are in the domains of both f and g . Additionally, the domain of a quotient does not include x -values for which $g(x) = 0$.

So far you have studied various types of functions, including linear functions, quadratic functions, and polynomial functions of higher degree. Another common type of function is a **power function**, which has the form $y = ax^b$ where a is a real number and b is a rational number.

Note that when b is a positive integer, a power function is simply a type of polynomial function. For example, $y = ax^b$ is a linear function when $b = 1$, a quadratic function when $b = 2$, and a cubic function when $b = 3$.

EXAMPLE 1 Adding and Subtracting Functions

Let $f(x) = 2x^{1/2}$ and $g(x) = -6x^{1/2}$. Find (a) the sum of the functions, (b) the difference of the functions, and (c) the domains of the sum and difference.

SOLUTION

$$\text{a. } f(x) + g(x) = 2x^{1/2} + (-6x^{1/2}) = (2 - 6)x^{1/2} = -4x^{1/2}$$

$$\text{b. } f(x) - g(x) = 2x^{1/2} - (-6x^{1/2}) = [2 - (-6)]x^{1/2} = 8x^{1/2}$$

c. The functions f and g each have the same domain—all nonnegative real numbers. So, the domains of $f + g$ and $f - g$ also consist of all nonnegative real numbers.

EXAMPLE 2 Multiplying and Dividing Functions

STUDENT HELP

Look Back

For help with function operations, see p. 338.

Let $f(x) = 3x$ and $g(x) = x^{1/4}$. Find (a) the product of the functions, (b) the quotient of the functions, and (c) the domains of the product and quotient.

SOLUTION

- a. $f(x) \cdot g(x) = (3x)(x^{1/4}) = 3x^{(1 + 1/4)} = 3x^{5/4}$
- b. $\frac{f(x)}{g(x)} = \frac{3x}{x^{1/4}} = 3x^{(1 - 1/4)} = 3x^{3/4}$
- c. The domain of f consists of all real numbers and the domain of g consists of all nonnegative real numbers. So, the domain of $f \cdot g$ consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $\frac{f}{g}$ is restricted to all *positive* real numbers.

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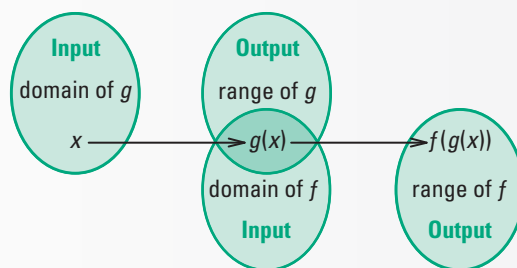
A fifth operation that can be performed with two functions is *composition*.

COMPOSITION OF TWO FUNCTIONS

The **composition** of the function f with the function g is:

$$h(x) = f(g(x))$$

The domain of h is the set of all x -values such that x is in the domain of g and $g(x)$ is in the domain of f .



As with subtraction and division of functions, you need to pay attention to the order of functions when they are composed. In general, $f(g(x))$ is not equal to $g(f(x))$.

EXAMPLE 3 Finding the Composition of Functions

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

- a. $f(g(x))$ b. $g(f(x))$ c. $f(f(x))$ d. the domain of each composition

SOLUTION

- a. $f(g(x)) = f(2x - 1) = 3(2x - 1)^{-1} = \frac{3}{2x - 1}$
- b. $g(f(x)) = g(3x^{-1}) = 2(3x^{-1}) - 1 = 6x^{-1} - 1 = \frac{6}{x} - 1$
- c. $f(f(x)) = f(3x^{-1}) = 3(3x^{-1})^{-1} = 3(3^{-1}x) = 3^0x = x$
- d. The domain of $f(g(x))$ consists of all real numbers except $x = \frac{1}{2}$ because $g\left(\frac{1}{2}\right) = 0$ is not in the domain of f . The domains of $g(f(x))$ and $f(f(x))$ consist of all real numbers except $x = 0$, because 0 is not in the domain of f . Note that $f(f(x))$ simplifies to x , but that result is not what determines the domain.

STUDENT HELP

Study Tip

When you are writing the composition of f with g , you may want to first rewrite $f(x) = 3x^{-1}$ as $f(\square) = 3(\square)^{-1}$ and then substitute $g(x) = 2x - 1$ everywhere there is a box.

FOCUS ON APPLICATIONS



BIOLOGY The largest bird egg is an ostrich's egg, which has a mass of about 1.65 kilograms. For comparison, an average chicken egg has a mass of only 56.7 grams.

GOAL 2 USING FUNCTION OPERATIONS IN REAL LIFE

EXAMPLE 4 Using Function Operations

BIOLOGY CONNECTION You are doing a science project and have found research indicating that the incubation time I (in days) of a bird's egg can be modeled by $I(m) = 12m^{0.217}$ where m is the egg's mass (in grams). You have also found that during incubation the egg's rate of water loss R (in grams per day) can be modeled by $R(m) = 0.015m^{0.742}$.

You conjecture that the proportion of water loss during incubation is about the same for any size egg. Show how you can use the two power function models to verify your conjecture. ▶ Source: *Biology by Numbers*

SOLUTION

VERBAL MODEL	Proportion of water loss = $\frac{\text{Total water loss}}{\text{Egg's mass}} = \frac{\text{Daily water loss} \cdot \text{Number of days}}{\text{Egg's mass}}$
LABELS	Daily water loss = $R(m)$ Number of days = $I(m)$ Egg's mass = m
ALGEBRAIC MODEL	$\frac{R(m) \cdot I(m)}{m} = \frac{(0.015m^{0.742})(12m^{0.217})}{m}$ $= \frac{0.18m^{0.959}}{m}$ $= 0.18m^{-0.041}$

Because $m^{-0.041}$ is approximately m^0 , the proportion of water loss can be treated as $0.18m^0 = (0.18)(1) = 0.18$. So, the proportion of water loss is about 18% for any size bird's egg, and your conjecture is correct.



EXAMPLE 5 Using Composition of Functions

A clothing store advertises that it is having a 25% off sale. For one day only, the store advertises an additional savings of 10%.

- Use composition of functions to find the total percent discount.
- What would be the sale price of a \$40 sweater?

SOLUTION

- Let x represent the price. The sale price for a 25% discount can be represented by the function $f(x) = x - 0.25x = 0.75x$. The reduced sale price for an additional 10% discount can be represented by the function $g(x) = x - 0.10x = 0.90x$.

$$g(f(x)) = g(0.75x) = 0.90(0.75x) = 0.675x$$

- ▶ The total percent discount is $100\% - 67.5\% = 32.5\%$.

- Let $x = 40$. Then $g(f(x)) = g(f(40)) = 0.675(40) = 27$.

- ▶ The sale price of the sweater is \$27.

STUDENT HELP

Skills Review

For help with calculating percents, see p. 907.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: The function $y = ax^b$ is a(n) function where a is a(n) number and b is a(n) number.

Concept Check ✓

2. **LOGICAL REASONING** Tell whether the sum of two power functions is *sometimes*, *always*, or *never* a power function.

ERROR ANALYSIS Let $f(x) = x^2 + 2$ and $g(x) = 3x$. What is wrong with the composition shown? Explain.

3.

$$\begin{aligned} f(g(x)) &= f(x^2 + 2) \\ &= 3(x^2 + 2) \\ &= 3x^2 + 6 \end{aligned}$$

4.

$$\begin{aligned} f(g(x)) &= f(3x) \\ &= 3x^2 + 2 \end{aligned}$$

Skill Check ✓

Let $f(x) = 4x$ and $g(x) = x - 1$. Perform the indicated operation and state the domain.

5. $f(x) + g(x)$

6. $f(x) - g(x)$

7. $f(x) \cdot g(x)$

8. $\frac{f(x)}{g(x)}$

9. $f(g(x))$

10. $g(f(x))$

11. **SALES BONUS** You are a sales representative for a clothing manufacturer. You are paid an annual salary plus a bonus of 2% of your sales over \$200,000. Consider two functions: $f(x) = x - 200,000$ and $g(x) = 0.02x$. If $x > \$200,000$, which composition, $f(g(x))$ or $g(f(x))$, represents your bonus? Explain.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 949.

ADDING AND SUBTRACTING FUNCTIONS Let $f(x) = x^2 - 5x + 8$ and $g(x) = x^2 - 4$. Perform the indicated operation and state the domain.

12. $f(x) + g(x)$

13. $g(x) + f(x)$

14. $f(x) - f(x)$

15. $g(x) + g(x)$

16. $f(x) - g(x)$

17. $g(x) - f(x)$

18. $f(x) - f(x)$

19. $g(x) - g(x)$

MULTIPLYING AND DIVIDING FUNCTIONS Let $f(x) = 2x^{2/3}$ and $g(x) = 3x^{1/2}$. Perform the indicated operation and state the domain.

20. $f(x) \cdot g(x)$

21. $g(x) \cdot f(x)$

22. $f(x) \cdot f(x)$

23. $g(x) \cdot g(x)$

24. $\frac{f(x)}{g(x)}$

25. $\frac{g(x)}{f(x)}$

26. $\frac{f(x)}{f(x)}$

27. $\frac{g(x)}{g(x)}$

COMPOSITION OF FUNCTIONS Let $f(x) = 4x^{-5}$ and $g(x) = x^{3/4}$. Perform the indicated operation and state the domain.

28. $f(g(x))$

29. $g(f(x))$

30. $f(f(x))$

31. $g(g(x))$

FUNCTION OPERATIONS Let $f(x) = 10x$ and $g(x) = x + 4$. Perform the indicated operation and state the domain.

32. $f(x) + g(x)$

33. $f(x) - g(x)$

34. $f(x) \cdot g(x)$

35. $\frac{f(x)}{g(x)}$

36. $f(g(x))$

37. $g(f(x))$

38. $f(f(x))$

39. $g(g(x))$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–19,
32–51

Example 2: Exs. 20–27,
32–51


Example 3: Exs. 28–51

Example 4: Exs. 52, 53

Example 5: Exs. 54–56

FUNCTION OPERATIONS Perform the indicated operation and state the domain.


40. $f + g$; $f(x) = x + 3$, $g(x) = 5x$ 41. $f + g$; $f(x) = 3x^{1/2}$, $g(x) = -2x^{1/2}$
 42. $f - g$; $f(x) = -x^{2/3}$, $g(x) = x^{2/3}$ 43. $f - g$; $f(x) = x^2 - 3$, $g(x) = x + 5$
 44. $f \cdot g$; $f(x) = 7x^{2/5}$, $g(x) = -2x^3$ 45. $f \cdot g$; $f(x) = x - 4$, $g(x) = 4x^2$
 46. $\frac{f}{g}$; $f(x) = 9x^{-1}$, $g(x) = x^{1/4}$ 47. $\frac{f}{g}$; $f(x) = x^2 - 5x$, $g(x) = x$
 48. $f(g(x))$; $f(x) = 6x^{-1}$, $g(x) = 5x - 2$ 49. $g(f(x))$; $f(x) = x^2 - 3$, $g(x) = x^2 + 1$
 50. $f(f(x))$; $f(x) = 2x^{1/5}$ 51. $g(g(x))$; $g(x) = 9x - 2$

52.  **HEART RATE** For a mammal, the heart rate r (in beats per minute) and the life span s (in minutes) are related to body mass m (in kilograms) by these formulas:

$$r(m) = 241m^{-0.25} \qquad s(m) = (6 \times 10^6)m^{0.2}$$

Find the relationship between body mass and average number of heartbeats in a lifetime by calculating $r(m) \cdot s(m)$. Explain the results.

► Source: *Physiology by Numbers*

53.  **BREATHING RATE** For a mammal, the volume b (in milliliters) of air breathed in and the volume d (in milliliters) of the dead space (the portion of the lungs not filled with air) are related to body weight w (in grams) by these formulas:

$$b(w) = 0.007w \qquad d(w) = 0.002w$$


The relationship between breathing rate r (in breaths per minute) and body weight is:

$$r(w) = \frac{1.1w^{0.734}}{b(w) - d(w)}$$

Simplify $r(w)$ and calculate the breathing rate for body weights of 6.5 grams, 12,300 grams, and 70,000 grams. ► Source: *Respiration*

54.  **COAT SALE** In Exercises 54 and 55, use the following information.

A clothing store is having a sale in which you can take \$50 off the cost of any coat in the store. The store also offers 10% off your entire purchase if you open a charge account. You decide to open a charge account and buy a coat.

54. Use composition of functions to find the sale price of a \$175 coat when \$50 is subtracted before the 10% discount is applied.
 55. **CRITICAL THINKING** Why doesn't the store apply the 10% discount before subtracting \$50?
 56.  **PALEONTOLOGY** The height at the hip h (in centimeters) of an ornithomimid, a type of dinosaur, can be modeled by

$$h(l) = 3.49l^{1.02}$$

where l is the length (in centimeters) of the dinosaur's instep. The length of the instep can be modeled by

$$l(f) = 1.5f$$

where f is the footprint length (in centimeters). Use composition of functions to find the relationship between height and footprint length. Then find the height of an ornithomimid with a footprint length of 30 centimeters. ► Source: *Dinosaur Tracks*

STUDENT HELP



HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
 for help with problem
 solving in Ex. 53.

**FOCUS ON
CAREERS**



PALEONTOLOGIST

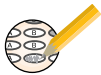
A paleontologist is a scientist who studies fossils of dinosaurs and other prehistoric life forms. Most paleontologists work as college professors.



CAREER LINK

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Test Preparation



57. **Writing** Explain how to perform the function operations $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, $\frac{f(x)}{g(x)}$, and $f(g(x))$ for any two functions f and g .

QUANTITATIVE COMPARISON In Exercise 58–61, choose the statement that is true about the given quantities.

- (A) The quantity in column A is greater.
 (B) The quantity in column B is greater.
 (C) The two quantities are equal.
 (D) The relationship cannot be determined from the given information.

	Column A	Column B
58.	$f(g(4)); f(x) = 6x, g(x) = 3x^2$	$f(g(2)); f(x) = x^{2/3}, g(x) = -2x$
59.	$g(f(-1)); f(x) = 5x^{-2}, g(x) = x$	$g(f(0)); f(x) = 2x + 5, g(x) = x^2$
60.	$f(f(3)); f(x) = 3x - 7$	$f(f(-2)); f(x) = 10x^3$
61.	$g(g(5)); g(x) = 16x^{-1/4}$	$g(g(7)); g(x) = x^2 + 8$

★ Challenge

FUNCTION COMPOSITION Find functions f and g such that $f(g(x)) = h(x)$.

62. $h(x) = (6x - 5)^3$
 63. $h(x) = \sqrt[3]{x + 2}$
 64. $h(x) = \frac{\sqrt[4]{x}}{2}$
 65. $h(x) = 3x^2 + 7$
 66. $h(x) = |2x + 9|$
 67. $h(x) = 21x$

EXTRA CHALLENGE

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MIXED REVIEW

REWRITING EQUATIONS Solve the equation for y . (Review 1.4 for 7.4)

68. $y - 3x = 10$
 69. $2x + 3y = -8$
 70. $x = -2y + 6$
 71. $xy + 2 = 7$
 72. $\frac{1}{2}x - \frac{2}{3}y = 1$
 73. $ax + by = c$

GRAPHING FUNCTIONS Graph the function. (Review 2.1 for 7.4)

74. $y = x - 2$
 75. $y = 4x - 3$
 76. $y = 5x - \frac{2}{3}$
 77. $y = -2x - 4$
 78. $y = -\frac{1}{2}x + 7$
 79. $y = -8$

SOLVING EQUATIONS Find the real-number solutions of the equation. (Review 6.4)

80. $3x^3 - 2x^2 = 0$
 81. $2x^3 - 6x^2 + x = 3$
 82. $5x^4 + 19x^2 - 4 = 0$
 83. $x^4 + 6x^3 + 8x + 48 = 0$
 84. **CRYPTOGRAPHY** Use the inverse of $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ and the code on page 225 to decode the message. (Review 4.4)

45, 21, 84, 35, 92, 37, 142, 61
 62, 25, 118, 49, 103, 44, 95, 38