7.2 Properties of Rational Exponents

What *you should learn*

GOAL 1 Use properties of rational exponents to evaluate and simplify expressions.

GOAL 2 Use properties of rational exponents to solve **real-life** problems, such as finding the surface area of a mammal in **Example 8**.

Why *you should learn it*

▼ To model **real-life** quantities, such as the frequencies in the musical range of a trumpet for **Ex. 94**.

REAL LIFE

GOAL 1 PROPERTIES OF RATIONAL EXPONENTS AND RADICALS

The properties of integer exponents presented in Lesson 6.1 can also be applied to rational exponents.

CONCEPT SUMMARY — **PROPERTIES OF RATIONAL EXPONENTS**

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 323, but now apply to rational exponents as illustrated.

	PROPERTY	EXAMPLE
1.	$a^m \cdot a^n = a^{m+n}$	$3^{1/2} \cdot 3^{3/2} = 3^{(1/2 + 3/2)} = 3^2 = 9$
2.	$(a^m)^n = a^{mn}$	$(4^{3/2})^2 = 4^{(3/2 \cdot 2)} = 4^3 = 64$
3.	$(ab)^m = a^m b^m$	$(9 \cdot 4)^{1/2} = 9^{1/2} \cdot 4^{1/2} = 3 \cdot 2 = 6$
4.	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5.	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{5/2}}{6^{1/2}} = 6^{(5/2 - 1/2)} = 6^2 = 36$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{2}{3}$

If $m = \frac{1}{n}$ for some integer n greater than 1, the third and sixth properties can be written using radical notation as follows:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \textbf{Product property}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad \textbf{Quotient property}$$

EXAMPLE 1 *Using Properties of Rational Exponents*

Use the properties of rational exponents to simplify the expression.

a. $5^{1/2} \cdot 5^{1/4} = 5^{(1/2 + 1/4)} = 5^{3/4}$

b. $(8^{1/2} \cdot 5^{1/3})^2 = (8^{1/2})^2 \cdot (5^{1/3})^2 = 8^{(1/2 \cdot 2)} \cdot 5^{(1/3 \cdot 2)} = 8^1 \cdot 5^{2/3} = 8 \cdot 5^{2/3}$

c. $(2^4 \cdot 3^4)^{-1/4} = [(2 \cdot 3)^4]^{-1/4} = (6^4)^{-1/4} = 6^{[4 \cdot (-1/4)]} = 6^{-1} = \frac{1}{6}$

d. $\frac{7}{7^{1/3}} = \frac{7^1}{7^{1/3}} = 7^{(1 - 1/3)} = 7^{2/3}$

e. $\left(\frac{12^{1/3}}{4^{1/3}}\right)^2 = \left[\left(\frac{12}{4}\right)^{1/3}\right]^2 = (3^{1/3})^2 = 3^{(1/3 \cdot 2)} = 3^{2/3}$

STUDENT HELP

Look Back
For help with properties of exponents, see p. 324.

EXAMPLE 2 *Using Properties of Radicals*

Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4 \cdot 16} = \sqrt[3]{64} = 4$ **Use the product property.**

b. $\dfrac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\dfrac{162}{2}} = \sqrt[4]{81} = 3$ **Use the quotient property.**

. . . .

For a radical to be in **simplest form**, you must not only apply the properties of radicals, but also remove any perfect nth powers (other than 1) and rationalize any denominators.

EXAMPLE 3 *Writing Radicals in Simplest Form*

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Write the expression in simplest form.

a. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$ **Factor out perfect cube.**

$= \sqrt[3]{27} \cdot \sqrt[3]{2}$ **Product property**

$= 3\sqrt[3]{2}$ **Simplify.**

b. $\sqrt[5]{\dfrac{3}{4}} = \sqrt[5]{\dfrac{3 \cdot 8}{4 \cdot 8}}$ **Make the denominator a perfect fifth power.**

$= \sqrt[5]{\dfrac{24}{32}}$ **Simplify.**

$= \dfrac{\sqrt[5]{24}}{\sqrt[5]{32}}$ **Quotient property**

$= \dfrac{\sqrt[5]{24}}{2}$ **Simplify.**

. . . .

Two radical expressions are **like radicals** if they have the same index and the same radicand. For instance, $\sqrt[3]{2}$ and $4\sqrt[3]{2}$ are like radicals. To add or subtract like radicals, use the distributive property.

EXAMPLE 4 *Adding and Subtracting Roots and Radicals*

Perform the indicated operation.

a. $7(6^{1/5}) + 2(6^{1/5}) = (7 + 2)(6^{1/5}) = 9(6^{1/5})$

b. $\sqrt[3]{16} - \sqrt[3]{2} = \sqrt[3]{8 \cdot 2} - \sqrt[3]{2}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2} - \sqrt[3]{2}$

$= 2\sqrt[3]{2} - \sqrt[3]{2}$

$= (2 - 1)\sqrt[3]{2}$

$= \sqrt[3]{2}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative or zero, sometimes absolute value is needed when simplifying a variable expression.

$\sqrt[n]{x^n} = x$ when n is odd $\qquad \sqrt[7]{2^7} = 2$ and $\sqrt[7]{(-2)^7} = -2$

$\sqrt[n]{x^n} = |x|$ when n is even $\qquad \sqrt[4]{5^4} = 5$ and $\sqrt[4]{(-5)^4} = 5$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 5 Simplifying Expressions Involving Variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{125y^6} = \sqrt[3]{5^3(y^2)^3} = 5y^2$

b. $(9u^2v^{10})^{1/2} = 9^{1/2}(u^2)^{1/2}(v^{10})^{1/2} = 3u^{(2 \cdot 1/2)}v^{(10 \cdot 1/2)} = 3uv^5$

c. $\sqrt[4]{\dfrac{x^4}{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \dfrac{x}{y^2}$

d. $\dfrac{6xy^{1/2}}{2x^{1/3}z^{-5}} = 3x^{(1-1/3)}y^{1/2}z^{-(-5)} = 3x^{2/3}y^{1/2}z^5$

EXAMPLE 6 Writing Variable Expressions in Simplest Form

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{5a^5b^9c^{13}} = \sqrt[5]{5a^5b^5b^4c^{10}c^3}$ Factor out perfect fifth powers.

$= \sqrt[5]{a^5b^5c^{10}} \cdot \sqrt[5]{5b^4c^3}$ Product property

$= abc^2\sqrt[5]{5b^4c^3}$ Simplify.

b. $\sqrt[3]{\dfrac{x}{y^7}} = \sqrt[3]{\dfrac{xy^2}{y^7y^2}}$ Make the denominator a perfect cube.

$= \sqrt[3]{\dfrac{xy^2}{y^9}}$ Simplify.

$= \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{y^9}}$ Quotient property

$= \dfrac{\sqrt[3]{xy^2}}{y^3}$ Simplify.

EXAMPLE 7 Adding and Subtracting Expressions Involving Variables

Perform the indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y} = (5 + 6)\sqrt{y} = 11\sqrt{y}$

b. $2xy^{1/3} - 7xy^{1/3} = (2 - 7)xy^{1/3} = -5xy^{1/3}$

c. $3\sqrt[3]{5x^5} - x\sqrt[3]{40x^2} = 3x\sqrt[3]{5x^2} - 2x\sqrt[3]{5x^2} = (3x - 2x)\sqrt[3]{5x^2} = x\sqrt[3]{5x^2}$

GOAL 2 PROPERTIES OF RATIONAL EXPONENTS IN REAL LIFE

REAL LIFE Biology

EXAMPLE 8 *Evaluating a Model Using Properties of Rational Exponents*

Biologists study characteristics of various living things. One way of comparing different animals is to compare their sizes. For example, a mammal's surface area S (in square centimeters) can be approximated by the model $S = km^{2/3}$ where m is the mass (in grams) of the mammal and k is a constant. The values of k for several mammals are given in the table.

Mammal	Mouse	Cat	Large dog	Cow	Rabbit	Human
k	9.0	10.0	11.2	9.0	9.75	11.0

Approximate the surface area of a cat that has a mass of 5 kilograms (5×10^3 grams). ▶ Source: *Scaling: Why Is Animal Size So Important?*

SOLUTION

$S = km^{2/3}$ — **Write model.**

$= 10.0(5 \times 10^3)^{2/3}$ — **Substitute for k and m.**

$= 10.0(5)^{2/3}(10^3)^{2/3}$ — **Power of a product property**

$\approx 10.0(2.92)(10^2)$ — **Power of a power property**

$= 2920$ — **Simplify.**

▶ The cat's surface area is approximately 3000 square centimeters.

EXAMPLE 9 *Using Properties of Rational Exponents with Variables*

BIOLOGY CONNECTION You are studying a Canadian lynx whose mass is twice the mass of an average house cat. Is its surface area also twice that of an average house cat?

SOLUTION

Let m be the mass of an average house cat. Then the mass of the Canadian lynx is $2m$. The surface areas of the house cat and the Canadian lynx can be approximated by:

$$S_{\text{cat}} = 10.0m^{2/3} \qquad S_{\text{lynx}} = 10.0(2m)^{2/3}$$

To compare the surface areas look at their ratio.

$\dfrac{S_{\text{lynx}}}{S_{\text{cat}}} = \dfrac{10.0(2m)^{2/3}}{10.0m^{2/3}}$ — **Write ratio of surface areas.**

$= \dfrac{10.0(2^{2/3})(m^{2/3})}{10.0m^{2/3}}$ — **Power of a product property**

$= 2^{2/3}$ — **Simplify.**

≈ 1.59 — **Evaluate.**

▶ The surface area of the Canadian lynx is about one and a half times that of an average house cat, not twice as much.

FOCUS ON APPLICATIONS

REAL LIFE **BIOLOGY** The average mass of a Canadian lynx is about 9.1 kilograms. The average mass of a house cat is about 4.8 kilograms.

Guided Practice

Vocabulary Check ✓

1. List three pairs of like radicals.

Concept Check ✓

2. If you know that $46{,}656{,}000 = 2^9 \cdot 3^6 \cdot 5^3$, what is the cube root of 46,656,000? Explain your reasoning.

ERROR ANALYSIS Explain the error made in simplifying the expression.

3. $\cancel{3\sqrt[4]{5} + 2\sqrt[4]{5} = 5\sqrt[4]{10}}$

4. $\cancel{\left(\frac{x}{y^8}\right)^{1/3} = \frac{x^{1/3}}{(y^8)^{1/3}} = \frac{x^{1/3}}{y^2}}$

Skill Check ✓

Simplify the expression.

5. $3^{1/4} \cdot 3^{3/4}$
6. $(5^{1/3})^6$
7. $\sqrt[3]{16} \cdot \sqrt[3]{4}$
8. $4^{-1/2}$
9. $\sqrt[4]{\frac{16}{81}}$
10. $\sqrt[3]{\frac{1}{4}}$
11. $8^{1/7} + 2(8^{1/7})$
12. $\sqrt{200} - 3\sqrt{2}$

Simplify the expression. Assume all variables are positive.

13. $x^{2/3} \cdot x^{4/3}$
14. $(y^{1/6})^3$
15. $\sqrt{4a^6}$
16. $b^{-1/3}$
17. $\sqrt[5]{\frac{x^{10}}{y^5}}$
18. $\sqrt[3]{\frac{x^2}{z}}$
19. $2a^{1/5} - 6a^{1/5}$
20. $x\sqrt[3]{y^6} + y^2\sqrt[3]{x^3}$

21. **BIOLOGY CONNECTION** The average mass of a rabbit is 1.6 kilograms. Use the information given in Example 8 to approximate the surface area of a rabbit.

Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 949.

PROPERTIES OF RATIONAL EXPONENTS Simplify the expression.

22. $3^{5/3} \cdot 3^{1/3}$
23. $(5^{2/3})^{1/2}$
24. $4^{1/4} \cdot 64^{1/4}$
25. $\frac{1}{36^{-1/2}}$
26. $\frac{7^{1/5}}{7^{3/5}}$
27. $\frac{70^{1/3}}{14^{1/3}}$
28. $(2^{1/4} \cdot 2^{1/3})^6$
29. $\left(\frac{5^2}{8^2}\right)^{-1/2}$
30. $\frac{6^{2/3} \cdot 4^{2/3}}{3^{2/3}}$
31. $\frac{125^{2/9} \cdot 125^{1/9}}{5^{1/4}}$
32. $\frac{12^{10/8}}{12^{-3/8}}$
33. $(10^{3/4} \cdot 4^{3/4})^{-4}$

PROPERTIES OF RADICALS Simplify the expression.

34. $\sqrt{64} \cdot \sqrt[3]{64}$
35. $\sqrt[4]{8} \cdot \sqrt[4]{2}$
36. $\sqrt[4]{5} \cdot \sqrt[4]{5}$
37. $(\sqrt[4]{6} \cdot \sqrt[4]{6})^{12}$
38. $\frac{\sqrt{7}}{\sqrt[5]{7}}$
39. $\frac{\sqrt[3]{4}}{\sqrt[3]{32}}$
40. $\frac{\sqrt[6]{8} \cdot \sqrt[6]{16}}{\sqrt[6]{2}}$
41. $\frac{\sqrt[3]{9} \cdot \sqrt[3]{6}}{\sqrt[6]{2} \cdot \sqrt[6]{2}}$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 22–33
Example 2: Exs. 34–41
Example 3: Exs. 42–49
Example 4: Exs. 50–55
Example 5: Exs. 56–67
Example 6: Exs. 68–75
Example 7: Exs. 76–81
Example 8: Exs. 90–93
Example 9: Exs. 94–97

SIMPLEST FORM Write the expression in simplest form.

42. $\sqrt{50}$
43. $\sqrt[5]{1215}$
44. $\sqrt[3]{18} \cdot \sqrt[3]{15}$
45. $3\sqrt[4]{24} \cdot 5\sqrt[4]{2}$
46. $\sqrt[3]{\frac{1}{7}}$
47. $\frac{2}{\sqrt[6]{81}}$
48. $\sqrt[4]{\frac{80}{9}}$
49. $\frac{\sqrt[3]{4}}{\sqrt[5]{8}}$

COMBINING ROOTS AND RADICALS Perform the indicated operation.

50. $\sqrt[5]{6} + 5\sqrt[5]{6}$
51. $5(5)^{1/7} - 7(5)^{1/7}$
52. $-\sqrt[8]{4} + 5\sqrt[8]{4}$
53. $160^{1/2} - 10^{1/2}$
54. $\sqrt[3]{375} + \sqrt[3]{81}$
55. $2\sqrt[4]{176} + 5\sqrt[4]{11}$

VARIABLE EXPRESSIONS **Simplify the expression. Assume all variables are positive.**

56. $x^{1/3} \cdot x^{1/5}$ **57.** $(y^3)^{1/6}$ **58.** $\sqrt[5]{32x^5}$ **59.** $\dfrac{1}{x^{-5/4}}$

60. $\dfrac{x^{3/7}}{x^{1/3}}$ **61.** $\sqrt[4]{\dfrac{x^{12}}{y^4}}$ **62.** $\dfrac{x^{5/3}y}{xy^{-1/2}}$ **63.** $(y \cdot y^{1/4})^{4/3}$

64. $(\sqrt[4]{x^3} \cdot \sqrt[4]{x^5})^{-2}$ **65.** $\dfrac{x^{3/4}yz^{-1/3}}{x^{1/4}z^{2/3}}$ **66.** $\dfrac{2\sqrt{x} \cdot \sqrt{x^3}}{\sqrt{9x^{10}}}$ **67.** $\dfrac{\sqrt[3]{y^6}}{\sqrt[3]{27y} \cdot \sqrt[3]{y^{11}}}$

SIMPLEST FORM **Write the expression in simplest form. Assume all variables are positive.**

68. $\sqrt{36x^3}$ **69.** $\sqrt[4]{10x^5y^8z^{10}}$ **70.** $\sqrt[5]{8xy^7} \cdot \sqrt[5]{6x^6}$ **71.** $\sqrt{xyz} \cdot \sqrt{2y^3z^4}$

72. $\dfrac{4}{\sqrt[3]{x}}$ **73.** $\sqrt[3]{\dfrac{x^3}{y^2}}$ **74.** $\sqrt{\dfrac{9x^2y}{32z^3}}$ **75.** $\dfrac{\sqrt[5]{x^3}}{\sqrt[7]{x^4}}$

COMBINING VARIABLE EXPRESSIONS **Perform the indicated operation. Assume all variables are positive.**

76. $2\sqrt[5]{y} + 7\sqrt[5]{y}$ **77.** $9x^{1/5} - 2x^{1/5}$ **78.** $-\sqrt[4]{x} + 2\sqrt[4]{x}$

79. $(x^9y)^{1/3} + (xy^{1/9})^3$ **80.** $\sqrt{4x^5} - x\sqrt{x^3}$ **81.** $y\sqrt[3]{24x^5} + \sqrt[3]{-3x^2y^3}$

EXTENSION: IRRATIONAL EXPONENTS **The properties you studied in this lesson can also be applied to irrational exponents. Simplify the expression. Assume all variables are positive.**

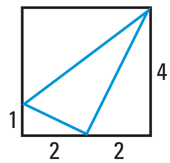
82. $x^2 \cdot x^{\sqrt{3}}$ **83.** $(y^{\sqrt{2}})^{\sqrt{2}}$ **84.** $(xy)^{\pi}$ **85.** $4^{-\sqrt{7}}$

86. $\dfrac{x^{2\sqrt{5}}}{x^{\sqrt{5}}}$ **87.** $\left(\dfrac{x^{1/\pi}}{y^{2/\pi}}\right)^{\pi}$ **88.** $3x^{\sqrt{2}} + x^{\sqrt{2}}$ **89.** $xy^{\sqrt{11}} - 3xy^{\sqrt{11}}$

STUDENT HELP

Skills Review
For help with perimeter and area, see p. 914.

90. **GEOMETRY CONNECTION** Find a radical expression for the perimeter of the triangle. Simplify the expression.

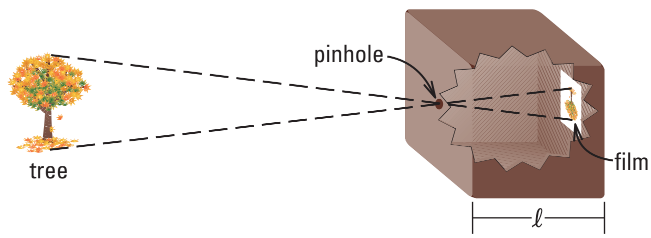


Ex. 90

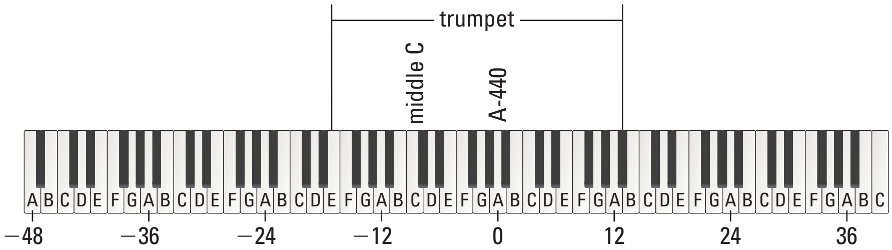
91. **GEOMETRY CONNECTION** The areas of two circles are 15 square centimeters and 20 square centimeters. Find the exact ratio of the radius of the smaller circle to the radius of the larger circle.

92. **BIOLOGY CONNECTION** Look back at Example 8. Approximate the surface area of a human that has a mass of 68 kilograms.

93. **PINHOLE CAMERA** A pinhole camera is made out of a light-tight box with a piece of film attached to one side and a pinhole on the opposite side. The optimum diameter d (in millimeters) of the pinhole can be modeled by $d = 1.9[(5.5 \times 10^{-4})\ell]^{1/2}$, where ℓ is the length of the camera box (in millimeters). What is the optimum diameter for a pinhole camera if the camera box has a length of 10 *centimeters*?



MUSIC In Exercises 94 and 95, use the following information.
The musical note A-440 (the A above middle C) has a frequency of 440 vibrations per second. The frequency f of any note can be found using $f = 440 \cdot 2^{n/12}$ where n represents the number of black and white keys the given note is above or below A-440. For notes above A-440, $n > 0$, and for notes below A-440, $n < 0$.



94. Find the highest and lowest frequencies in the musical range of a trumpet. What is the exact ratio of these two frequencies?

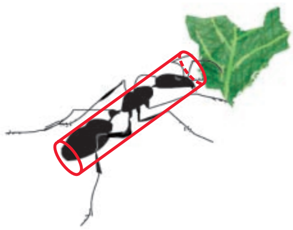
95. LOGICAL REASONING Describe the pattern of the frequencies of successive notes with the same letter.

96. DISTANCE OF AN OBJECT The maximum horizontal distance d that an object can travel when launched at an optimum angle of projection from an initial height h_0 can be modeled by $d = \frac{v_0\sqrt{(v_0)^2 + 2gh_0}}{g}$ where v_0 is the initial speed and g is the acceleration due to gravity. Simplify the model when $h_0 = 0$.

97. BALLOONS You have filled two round balloons with air. One balloon has twice as much air as the other balloon. The formula for the surface area S of a sphere in terms of its volume V is $S = (4\pi)^{1/3}(3V)^{2/3}$. By what factor is the surface area of the larger balloon greater than that of the smaller balloon?

Test Preparation

98. MULTI-STEP PROBLEM A common ant absorbs oxygen at a rate of about 6.2 milliliters per second per square centimeter of exoskeleton. It needs about 24 milliliters of oxygen per second per cubic centimeter of its body. An ant is basically cylindrical in shape, so its surface area S and volume V can be approximated by the formulas for the surface area and volume of a cylinder:



$$S = 2\pi rh + 2\pi r^2 \qquad V = \pi r^2 h$$

a. Approximate the surface area and volume of an ant that is 8 millimeters long and has a radius of 1.5 millimeters. Would this ant have a surface area large enough to meet its oxygen needs?

b. Consider a "giant" ant that is 8 meters long and has a radius of 1.5 meters. Would this ant have a surface area large enough to meet its oxygen needs?

c. *Writing* Substitute $1000r$ for r and $1000h$ for h into the formulas for surface area and volume. How does increasing the radius and height by a factor of 1000 affect surface area? How does it affect volume? Use the results to explain why "giant" ants do not exist.

★ Challenge

99. CRITICAL THINKING Substitute different combinations of odd and even positive integers for m and n in the expression $\sqrt[n]{x^m}$. Do *not* assume that x is always positive. When is absolute value needed in simplifying the expression?

Mixed Review

Completing the Square **Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.** (Review 5.5)

100. $x^2 + 14x + c$ **101.** $x^2 - 21x + c$ **102.** $x^2 - 7.6x + c$

103. $x^2 + 9.9x + c$ **104.** $x^2 + \frac{2}{3}x + c$ **105.** $x^2 - \frac{1}{4}x + c$

Polynomial Operations **Perform the indicated operation.** (Review 6.3 for 7.3)

106. $(-3x^3 + 6x) - (8x^3 + x^2 - 4x)$ **107.** $(50x - 3) + (8x^3 + 9x^2 + 2x + 4)$

108. $20x^2(x - 9)$ **109.** $(2x + 7)^2$

Long Division **Divide using long division.** (Review 6.5 for 7.3)

110. $(x^3 - 28x - 48) \div (x + 4)$ **111.** $(4x^2 + 3x - 3) \div (x + 1)$

112. $(4x^2 - 6x) \div (x - 2)$ **113.** $(x^4 - 2x^3 - 70x + 20) \div (x - 5)$

Quiz 1

Self-Test for Lessons 7.1 and 7.2

Evaluate the expression without using a calculator. (Lesson 7.1)

1. $8^{2/3}$ **2.** $32^{-3/5}$ **3.** $-(81^{1/4})$ **4.** $(-64)^{2/3}$

Solve the equation. Round your answer to two decimal places. (Lesson 7.1)

5. $x^5 = 10$ **6.** $-9x^6 = -18$ **7.** $x^4 - 4 = 9$ **8.** $(x + 2)^3 = -15$

Write the expression in simplest form. (Lesson 7.2)

9. $\dfrac{1}{4^{-1/4}}$ **10.** $\sqrt[4]{\dfrac{16}{3}}$ **11.** $\dfrac{512^{1/3}}{8^{1/3}}$

12. $\sqrt{45}$ **13.** $\sqrt[3]{7} \cdot \sqrt[3]{49}$ **14.** $8^{1/5} + 2(8^{1/5})$

Write the expression in simplest form. Assume all variables are positive. (Lesson 7.2)

15. $\sqrt[3]{x^2} \cdot \sqrt[4]{x}$ **16.** $(x^{1/5})^{5/2}$ **17.** $\dfrac{xy^{1/2}}{x^{3/4}y^{-2}}$

18. $\sqrt[3]{5x^3y^5}$ **19.** $\sqrt{\dfrac{36x}{y^3}}$ **20.** $x(9y)^{1/2} - (x^2y)^{1/2}$

21. **Generating Power** As a rule of thumb, the power P (in horsepower) that a ship needs can be modeled by $P = \dfrac{d^{2/3} \cdot s^3}{c}$ where d is the ship's displacement (in tons), s is the normal speed (in knots), and c is the Admiralty coefficient. If a ship displaces 30,090 tons, has a normal speed of 22.5 knots, and has an Admiralty coefficient of 370, how much power does it need? (Lesson 7.1)

22. **Biology Connection** The surface area S (in square centimeters) of a large dog can be approximated by the model $S = 11.2m^{2/3}$ where m is the mass (in grams) of the dog. A Labrador retriever's mass is about three times the mass of a Scottish terrier. Is its surface area also three times that of a Scottish terrier? (Lesson 7.2)