CHAPTER 6

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Use properties of exponents to evaluate and simplify expressions. **(6.1)**	Use scientific notation to find the ratio of a state's park space to its total area. **(p. 328)**
Evaluate polynomial functions using direct or synthetic substitution. **(6.2)**	Estimate the amount of prize money awarded at a tennis tournament. **(p. 335)**
Sketch and analyze graphs of polynomial functions. **(6.2, 6.8)**	Find maximum or minimum values of a function such as oranges consumed in the U.S. **(p. 377)**
Add, subtract, and multiply polynomials. **(6.3)**	Write a polynomial model for the power needed to move a bicycle at a certain speed. **(p. 342)**
Factor polynomial expressions. **(6.4)**	Find the dimensions of a block discovered by archeologists. **(p. 347)**
Solve polynomial equations. **(6.4)**	Find the dimensions of a sculpture. **(p. 350)**
Divide polynomials using long division or synthetic division. **(6.5)**	Write a function for the average annual amount of money spent per person at the movies. **(p. 358)**
Find zeros of polynomial functions. **(6.6, 6.7)**	Find dimensions for a candle-wax model of the Louvre pyramid. **(p. 361)**
Use finite differences and cubic regression to find polynomial models for data. **(6.9)**	Write and use a polynomial model for the speed of a space shuttle. **(p. 385)**
Use polynomials to solve real-life problems. **(6.1–6.9)**	Find the maximum volume and dimensions of a box made from a piece of cardboard. **(p. 375)**

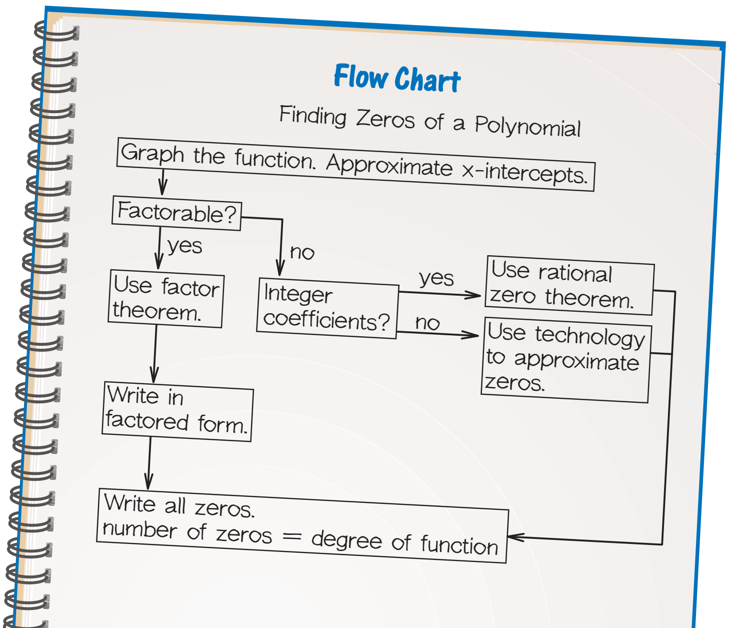
How does Chapter 6 fit into the BIGGER PICTURE of algebra?

Chapter 6 contains the fundamental theorem of algebra. Finding the solutions of a polynomial equation is the most classic problem in all of algebra. It is equivalent to finding the zeros of a polynomial function. Real-life situations have been modeled by polynomial functions for hundreds of years.

STUDY STRATEGY

How did you make and use a flow chart?

Here is a flow chart for finding all the zeros of a polynomial function, following the **Study Strategy** on page 322.



CHAPTER 6

Chapter Review

VOCABULARY

- scientific notation, p. 325
- polynomial function, p. 329
- leading coefficient, p. 329
- constant term, p. 329
- degree of a polynomial function, p. 329
- standard form of a polynomial function, p. 329
- synthetic substitution, p. 330
- end behavior, p. 331
- factor by grouping, p. 346
- quadratic form, p. 346
- polynomial long division, p. 352
- remainder theorem, p. 353
- synthetic division, p. 353
- factor theorem, p. 354
- rational zero theorem, p. 359
- fundamental theorem of algebra, p. 366
- repeated solution, p. 366
- local maximum, p. 374
- local minimum, p. 374
- finite differences, p. 380

6.1 Using Properties of Exponents

Examples on pp. 323–325

EXAMPLE You can use properties of exponents to evaluate numerical expressions and to simplify algebraic expressions.

$$\frac{(3x^2y)^5}{9x^{10}y^6} = \frac{3^5x^{2\cdot 5}y^5}{9x^{10}y^6} = \frac{243}{9}x^{10-10}y^{5-6} = 27x^0y^{-1} = \frac{27}{y}$$

all positive exponents

Simplify the expression. Tell which properties of exponents you used.

1. $\left(\frac{2}{3}\right)^2 \cdot (6xy^{-1})^3$

2. $x^4(x^{-5}x^3)^2$

3. $\frac{-63xy^9}{18x^{-2}y^3}$

4. $\frac{5x^2}{y^{-2}} \cdot \frac{1}{25x^2y}$

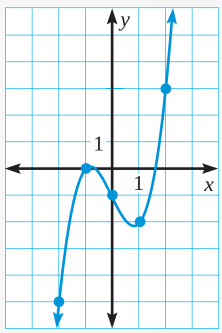
6.2 Evaluating and Graphing Polynomial Functions

Examples on pp. 329–332

EXAMPLES Use direct or synthetic substitution to evaluate a polynomial function.

Evaluate $f(x) = x^3 - 2x - 1$ when $x = 3$ (synthetic substitution):

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -2 & -1 \\ & & 3 & 9 & 21 \\ \hline & 1 & 3 & 7 & 20 \end{array} \leftarrow f(3) = 20$$



To graph, make a table of values, plot points, and identify end behavior.

x	-3	-2	-1	0	1	2	3
$f(x)$	-22	-5	0	-1	-2	3	20

The leading coefficient is positive and the degree is odd, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Use synthetic substitution to evaluate the polynomial function for the given value of x.

5. $f(x) = x^3 + 3x^2 - 12x + 7, x = 3$

6. $f(x) = x^4 - 5x^3 - 3x^2 + x - 5, x = -1$

Graph the polynomial function.

7. $f(x) = -x^3 + 2$ **8.** $f(x) = x^4 - 3$ **9.** $f(x) = x^3 - 4x + 1$

6.3 ADDING, SUBTRACTING, AND MULTIPLYING POLYNOMIALS

Examples on pp. 338–340

EXAMPLES You can add, subtract, or multiply polynomials.

$$\begin{array}{rr} & 4x^3 + 2x^2 \quad\;\; + 1 \\ - & (x^2 + x - 5) \\ \hline & 4x^3 + \;\; x^2 - x + 6 \end{array}$$

$$\begin{aligned}(x-3)(x^2+5x-1) &= (x-3)(x^2) + (x-3)(5x) + (x-3)(-1) \\ &= x^3 - 3x^2 + 5x^2 - 15x - x + 3 \\ &= x^3 + 2x^2 - 16x + 3\end{aligned}$$

Perform the indicated operation.

10. $(3x^3 + x^2 + 1) - (x^3 + 3)$ **11.** $(x - 3)(x^2 + x - 7)$ **12.** $(x + 3)(x - 5)(2x + 1)$

6.4 FACTORING AND SOLVING POLYNOMIAL EQUATIONS

Examples on pp. 345–347

EXAMPLES You can solve some polynomial equations by factoring.

Factor $8x^3 - 125$.

$$\begin{aligned}8x^3 - 125 &= (2x)^3 - 5^3 \\ &= (2x - 5)\big((2x)^2 + (2x \cdot 5) + 5^2\big) \\ &= (2x - 5)(4x^2 + 10x + 25)\end{aligned}$$

Solve $x^3 - 3x^2 - 5x + 15 = 0$.

$$\begin{aligned}x^2(x - 3) - 5(x - 3) &= 0 \\ (x - 3)(x^2 - 5) &= 0 \\ x = 3 \text{ or } x &= \pm\sqrt{5}\end{aligned}$$

Find the real-number solutions of the equation.

13. $x^3 + 64 = 0$ **14.** $x^4 - 6x^2 = 27$ **15.** $x^3 + 3x^2 - x - 3 = 0$

6.5 THE REMAINDER AND FACTOR THEOREMS

Examples on pp. 352–355

EXAMPLES You can use polynomial long division, and in some cases synthetic division, to divide polynomials.

$$\begin{array}{r} x^2 - \;\; 7x + \;\; 6 \\ x + 9\overline{)x^3 + 2x^2 - 57x + 54} \\ \underline{x^3 + 9x^2} \qquad\qquad\quad \\ -7x^2 - 57x \qquad \\ \underline{-7x^2 - 63x} \qquad \\ 6x + 54 \\ \underline{6x + 54} \\ 0 \end{array}$$

$$\frac{x^3 + 2x^2 - 57x + 54}{x + 9} = x^2 - 7x + 6$$

Divide $3x^3 + 2x^2 - x + 4$ by $x + 5$.

$$\begin{array}{r|rrrr} -5 & 3 & 2 & -1 & 4 \\ & & -15 & 65 & -320 \\ \hline & 3 & -13 & 64 & -316 \end{array}$$

$$\frac{3x^3 + 2x^2 - x + 4}{x + 5} = 3x^2 - 13x + 64 + \frac{-316}{x + 5}$$

Divide. Use synthetic division if possible.

16. $(x^4 + 5x^3 - x^2 - 3x - 1) \div (x - 1)$ **17.** $(2x^3 - 5x^2 + 5x + 4) \div (2x - 5)$

6.6–6.7 FINDING ZEROS OF POLYNOMIAL FUNCTIONS

Examples on pp. 359–361 and pp. 366–368

EXAMPLE You can use the rational zero theorem and the fundamental theorem of algebra to find all the zeros of a polynomial function.

$f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$ Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 11, \pm 22}{1}$

Using synthetic division, you can find that the rational zeros are 1 and 2. The degree of f is 4, so f has 4 zeros. To find the other two zeros, write in factored form: $f(x) = (x - 1)(x - 2)(x^2 + 6x + 11)$. Solve $x^2 + 6x + 11 = 0$: $x = -3 \pm \sqrt{2}i$. So the zeros of $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$ are $1, 2, -3 + \sqrt{2}i, -3 - \sqrt{2}i$.

Find all the real zeros of the function.

18. $f(x) = x^3 + 12x^2 + 21x + 10$

19. $f(x) = x^4 + x^3 - x^2 + x - 2$

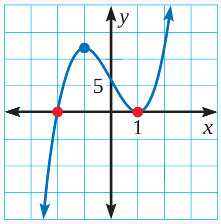
6.8 ANALYZING GRAPHS OF POLYNOMIAL FUNCTIONS

Examples on pp. 373–375

EXAMPLE You can identify x-intercepts and turning points when you analyze the graph of a polynomial function.

The graph of $f(x) = 3x^3 - 9x + 6$ has

- two x-intercepts, -2 and 1.
- a local maximum at $(-1, 12)$.
- a local minimum at $(1, 0)$.



Graph the polynomial function. Identify the x-intercepts and the points where the local maximums and local minimums occur.

20. $f(x) = (x - 2)^2(x + 2)$

21. $f(x) = x^3 - 3x^2$

22. $f(x) = 3x^4 + 4x^3$

6.9 MODELING WITH POLYNOMIALS

Examples on pp. 380–382

EXAMPLE Sometimes you can use finite differences or cubic regression to find a polynomial model for a set of data.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
−1	2	7	14	23	34	**function values**
3	5	7	9	11		**first-order differences**
2	2	2	2			**second-order differences**

Since second-order differences are nonzero and constant, the data set can be modeled by a polynomial function of degree 2. The function is $f(x) = x^2 - 2$.

23. Show that the third-order differences for the function $f(n) = n^3 + 1$ are nonzero and constant.

24. Write a cubic function whose graph passes through points $(1, 0)$, $(-1, 0)$, $(4, 0)$, and $(2, -12)$. Use cubic regression on a graphing calculator to verify your answer.

CHAPTER 6

Chapter Test

Simplify the expression. Tell which properties of exponents you used.

1. $x^7 \cdot \frac{1}{x^2}$ **2.** $(3^2x^6)^3$ **3.** $\frac{x^9}{x^{-2}}$ **4.** $(8x^3y^2)^{-3}$ **5.** $\frac{15x^2y}{6x^4y^5} \cdot \frac{6x^3y^2}{5xy}$

Describe the end behavior of the graph of the polynomial function. Then evaluate the function for $x = -4, -3, -2, \ldots, 4$. Then graph the function.

6. $y = x^4 - 2x^2 - x - 1$ **7.** $y = -3x^3 - 6x^2$ **8.** $y = (x - 3)(x + 1)(x + 2)$

Perform the indicated operation.

9. $(3x^2 - 5x + 7) - (2x^2 + 9x - 1)$ **10.** $(2x - 3)(5x^2 - x + 6)$ **11.** $(x - 4)(x + 1)(x + 3)$

Factor the polynomial.

12. $64x^3 + 343$ **13.** $400x^2 - 25$ **14.** $x^4 + 8x^2 - 9$ **15.** $2x^3 - 3x^2 + 4x - 6$

Solve the equation.

16. $3x^4 - 11x^2 - 20 = 0$ **17.** $81x^4 = 16$ **18.** $4x^3 - 8x^2 - x + 2 = 0$

Divide. Use synthetic division if possible.

19. $(8x^4 + 5x^3 + 4x^2 - x + 7) \div (x + 1)$ **20.** $(12x^3 + 31x^2 - 17x - 6) \div (x + 3)$

List all the possible rational zeros of f using the rational zero theorem. Then find all the zeros of the function.

21. $f(x) = x^3 - 5x^2 - 14x$ **22.** $f(x) = x^3 + 4x^2 + 9x + 36$ **23.** $f(x) = x^4 + x^3 - 2x^2 + 4x - 24$

Write a polynomial function of least degree that has real coefficients, the given zeros, and a leading coefficient of 1.

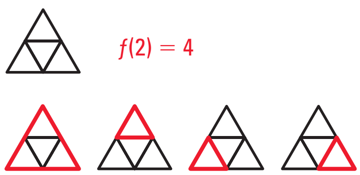
24. $1, -3, 4$ **25.** $2, 2, -1, 0$ **26.** $5, 2i, -2i$ **27.** $3, -3, 2 - i$

28. Use technology to approximate the real zeros of $f(x) = 0.25x^3 - 7x^2 + 15$.

29. Identify the x-intercepts, local maximum, and local minimum of the graph of $f(x) = \frac{1}{9}(x - 3)^2(x + 3)^2$. Then describe the end behavior of the graph.

30. Show that $f(x) = x^4 - 2x + 8$ has nonzero constant fourth-order differences.

31. The table gives the number of triangles that point upward that you can find in a large triangle that is n units on a side and divided into triangles that are each one unit on a side. Find a polynomial model for $f(n)$.



n	1	2	3	4	5	6	7
$f(n)$	1	4	10	20	35	56	84

32. **CELLS** An adult human body contains about 75,000,000,000,000 cells. Each is about 0.001 inch wide. If the cells were laid end to end to form a chain, about how long would the chain be in miles? Give your answer in scientific notation.