# 6.2

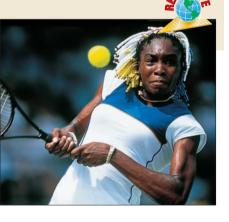
## What you should learn

**GOAL** Evaluate a polynomial function.

GOAL (2) Graph a polynomial function, as applied in **Example 5**.

## Why you should learn it

▼ To find values of **real-life** functions, such as the amount of prize money awarded at the U.S. Open Tennis Tournament in **Ex. 86**.



# **Evaluating and Graphing Polynomial Functions**



## **1** EVALUATING POLYNOMIAL FUNCTIONS

A **polynomial function** is a function of the form

 $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ 

where  $a_n \neq 0$ , the exponents are all whole numbers, and the coefficients are all real numbers. For this polynomial function,  $a_n$  is the **leading coefficient**,  $a_0$  is the **constant term**, and *n* is the **degree**. A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

You are already familiar with some types of polynomial functions. For instance, the linear function f(x) = 3x + 2 is a polynomial function of degree 1. The quadratic function  $f(x) = x^2 + 3x + 2$  is a polynomial function of degree 2. Here is a summary of common types of polynomial functions.

Degree	Туре	Standard form
0	Constant	$f(x) = a_0$
1	Linear	$f(x) = a_1 x + a_0$
2	Quadratic	$f(x) = a_2 x^2 + a_1 x + a_0$
3	Cubic	$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$
4	Quartic	$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

## EXAMPLE 1

## Identifying Polynomial Functions

Decide whether the function is a polynomial function. If it is, write the function in standard form and state its degree, type, and leading coefficient.

**a.**  $f(x) = \frac{1}{2}x^2 - 3x^4 - 7$  **b.**  $f(x) = x^3 + 3^x$  **c.**  $f(x) = 6x^2 + 2x^{-1} + x$ **d.**  $f(x) = -0.5x + \pi x^2 - \sqrt{2}$ 

## SOLUTION

- **a.** The function is a polynomial function. Its standard form is  $f(x) = -3x^4 + \frac{1}{2}x^2 7$ . It has degree 4, so it is a quartic function. The leading coefficient is -3.
- **b.** The function is not a polynomial function because the term  $3^x$  does not have a variable base and an exponent that is a whole number.
- **c.** The function is not a polynomial function because the term  $2x^{-1}$  has an exponent that is not a whole number.
- **d**. The function is a polynomial function. Its standard form is  $f(x) = \pi x^2 0.5x \sqrt{2}$ . It has degree 2, so it is a quadratic function. The leading coefficient is  $\pi$ .

One way to evaluate a polynomial function is to use direct substitution. For instance,  $f(x) = 2x^4 - 8x^2 + 5x - 7$  can be evaluated when x = 3 as follows.

$$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7$$
  
= 162 - 72 + 15 - 7  
= 98

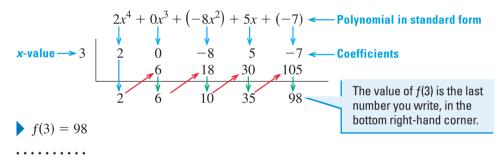
Another way to evaluate a polynomial function is to use **synthetic substitution**.

## **EXAMPLE 2** Using Synthetic Substitution

Use synthetic substitution to evaluate  $f(x) = 2x^4 - 8x^2 + 5x - 7$  when x = 3.

## SOLUTION

Write the value of x and the coefficients of f(x) as shown. Bring down the leading coefficient. Multiply by 3 and write the result in the next column. Add the numbers in that column and write the sum below the line. Continue to multiply and add, as shown.



Using synthetic substitution is equivalent to evaluating the polynomial in nested form.

$$f(x) = 2x^{4} + 0x^{3} - 8x^{2} + 5x - 7$$
  
Write original function.  
$$= (2x^{3} + 0x^{2} - 8x + 5)x - 7$$
  
Factor x out of first 4 terms.  
$$= ((2x^{2} + 0x - 8)x + 5)x - 7$$
  
Factor x out of first 3 terms.  
$$= (((2x + 0)x - 8)x + 5)x - 7$$
  
Factor x out of first 2 terms.

## **EXAMPLE 3** Evaluating a Polynomial Function in Real Life

**PHOTOGRAPHY** The time t (in seconds) it takes a camera battery to recharge after flashing n times can be modeled by  $t = 0.000015n^3 - 0.0034n^2 + 0.25n + 5.3$ . Find the recharge time after 100 flashes. Source: Popular Photography

#### SOLUTION

100	0.000015	$-0.0034 \\ 0.0015$	0.25 -0.19	5.3 6
		-0.0019		

The recharge time is about 11 seconds.

#### STUDENT HELP

**Study Tip** In Example 2, note that the row of coefficients for f(x) must include a coefficient of 0 for the "missing"  $x^3$ -term.

#### FOCUS ON CAREERS



PHOTOGRAPHER Some photographers work in advertising, some work for newspapers, and some are self-employed. Others specialize in aerial, police, medical, or scientific photography.



## GOAL 2

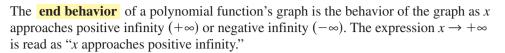
Concepts

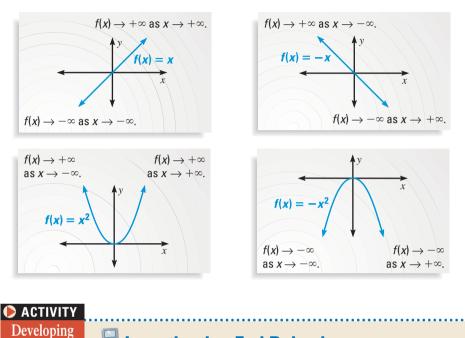
## 2 GRAPHING POLYNOMIAL FUNCTIONS

STUDENT HELP

## Look Back

For help with graphing functions, see pp. 69 and 250.





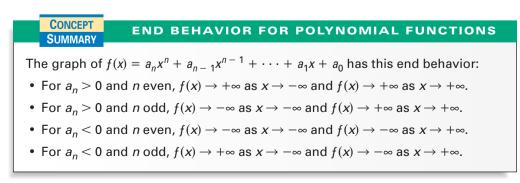
## 📙 Investigating End Behavior

**1** Use a graphing calculator to graph each function. Then complete these statements:  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow +\infty$ .

<b>a.</b> $f(x) = x^3$	<b>b.</b> $f(x) = x^4$	<b>c.</b> $f(x) = x^5$	<b>d.</b> $f(x) = x^6$
<b>e.</b> $f(x) = -x^3$	<b>f.</b> $f(x) = -x^4$	<b>g.</b> $f(x) = -x^5$	<b>h.</b> $f(x) = -x^6$

- 2 How does the sign of the leading coefficient affect the behavior of a polynomial function's graph as x → +∞?
- 3 How is the behavior of a polynomial function's graph as x → +∞ related to its behavior as x → -∞ when the function's degree is odd? when it is even?

In the activity you may have discovered that the end behavior of a polynomial function's graph is determined by the function's degree and leading coefficient.





## **EXAMPLE 4** Graphing Polynomial Functions

Graph (a)  $f(x) = x^3 + x^2 - 4x - 1$  and (b)  $f(x) = -x^4 - 2x^3 + 2x^2 + 4x$ .

#### SOLUTION

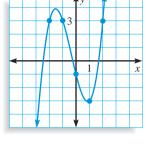
**a.** To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

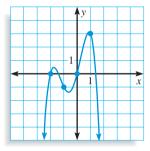
x	-3	-2	-1	0	1	2	3
<b>f</b> ( <b>x</b> )	-7	3	3	-1	-3	3	23

The degree is odd and the leading coefficient is positive, so  $f(x) \to -\infty$  as  $x \to -\infty$  and  $f(x) \to +\infty$  as  $x \to +\infty$ .

**b.** To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	-3	-2	-1	0	1	2	3
<b>f</b> ( <b>x</b> )	-21	0	-1	0	3	-16	-105





The degree is even and the leading coefficient is negative, so  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$ .



## **EXAMPLE 5** Graphing a Polynomial Model

A rainbow trout can grow up to 40 inches in length. The weight y (in pounds) of a rainbow trout is related to its length x (in inches) according to the model  $y = 0.0005x^3$ . Graph the model. Use your graph to estimate the length of a 10 pound rainbow trout.

#### SOLUTION

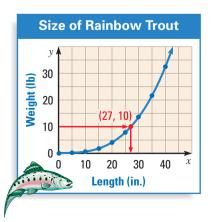
*Make* a table of values. The model makes sense only for positive values of *x*.

x	0	5	10	15	20	25	30	35	40
y	0	0.0625	0.5	1.69	4	7.81	13.5	21.4	32

**Plot** the points and connect them with a smooth curve, as shown at the right. Notice that the leading coefficient of the model is positive and the degree is odd, so the graph rises to the right.

**Read** the graph backwards to see that  $x \approx 27$  when y = 10.

• A 10 pound trout is approximately 27 inches long.



## **GUIDED PRACTICE**

Vocabulary Check 🗸	<b>1.</b> Identify the degree, type, lead polynomial function $f(x) = 5$	ding coefficient, and constant $5x - 2x^3$ .	nt term	of the		
Concept Check 🗸	<b>2.</b> Complete the synthetic substitute right. Describe each step	itution shown at $-2$ of the process.	3	1 ?	-9 ?	2
	<b>3</b> . Describe the graph of a const	ant function.	3	?	?	(
Skill Check 🗸	Decide whether each function i substitution to evaluate the fur	s a polynomial function. If notion when $x = -1$ .	it is, ι	ise sy	nthetic	
	<b>4.</b> $f(x) = x^4 \sqrt{5} - x$	<b>5.</b> $f(x) = x^3 + $			3	
	<b>6.</b> $f(x) = 6^{2x} - 12x$	<b>7.</b> $f(x) = 14 - 14$	$21x^2 -$	$+ 5x^4$		

Describe the end behavior of the graph of the polynomial function by completing the statements  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow +\infty$ .

<b>8.</b> $f(x) = x^3 - 5x$	<b>9.</b> $f(x) = -x^5 - 3x^3 + 2$	<b>10.</b> $f(x) = x^4 - 4x^2 + x$
<b>11.</b> $f(x) = x + 12$	<b>12.</b> $f(x) = -x^2 + 3x + 1$	<b>13.</b> $f(x) = -x^8 + 9x^5 - 2x^4$

 $\frac{2}{2}$ 

14. S VIDEO RENTALS The total revenue (actual and projected) from home video rentals in the United States from 1985 to 2005 can be modeled by

$$R = 1.8t^3 - 76t^2 + 1099t + 2600$$

where *R* is the revenue (in millions of dollars) and *t* is the number of years since 1985. Graph the function.  $\triangleright$  Source: *The Wall Street Journal Almanac* 

# PRACTICE AND APPLICATIONS

STUDENT HELP

► HOMEWORK HELP				
Example 1: Ex	s. 15–26			
Example 2: Ex	s. 37–46			

- STUDENT HELP

Example 2:	EXS. 37–40
Example 3:	Exs. 81, 82
Example 4:	Exs. 47–79
Example 5:	Exs. 83–86

**CLASSIFYING POLYNOMIALS** Decide whether the function is a polynomial function. If it is, write the function in standard form and state the degree, type, and leading coefficient.

<b>15.</b> $f(x) = 12 - 5x$	<b>16.</b> $f(x) = 2x + \frac{3}{5}x^4 + 9$	<b>17.</b> $f(x) = x + \pi$
<b>18.</b> $f(x) = x^2 \sqrt{2} + x - 5$	<b>19.</b> $f(x) = x - 3x^{-2} - 2x^3$	<b>20.</b> $f(x) = -2$
<b>21.</b> $f(x) = x^2 - x + 1$	<b>22.</b> $f(x) = 22 - 19x + 2^x$	<b>23.</b> $f(x) = 36x^2 - x^3 + x^4$
<b>24.</b> $f(x) = 3x^2 - 2x^{-x}$	<b>25.</b> $f(x) = 3x^3$	<b>26.</b> $f(x) = -6x^2 + x - \frac{3}{x}$

**DIRECT SUBSTITUTION** Use direct substitution to evaluate the polynomial function for the given value of *x*.

<b>27.</b> $f(x) = 2x^3 + 5x^2 + 4x + 8, x = -2$	<b>28.</b> $f(x) = 2x^3 - x^4 + 5x^2 - x, x = 3$
<b>29.</b> $f(x) = x + \frac{1}{2}x^3, x = 4$	<b>30.</b> $f(x) = x^2 - x^5 + 1, x = -1$
<b>31.</b> $f(x) = 5x^4 - 8x^3 + 7x^2, x = 1$	<b>32.</b> $f(x) = x^3 + 3x^2 - 2x + 5, x = -3$
<b>33.</b> $f(x) = 11x^3 - 6x^2 + 2, x = 0$	<b>34.</b> $f(x) = x^4 - 2x + 7, x = 2$
<b>35.</b> $f(x) = 7x^3 + 9x^2 + 3x, x = 10$	<b>36.</b> $f(x) = -x^5 - 4x^3 + 6x^2 - x, x = -2$

Extra Practice to help you master skills is on p. 947.

# **SYNTHETIC SUBSTITUTION** Use synthetic substitution to evaluate the polynomial function for the given value of *x*.

**37.**  $f(x) = 5x^3 + 4x^2 + 8x + 1, x = 2$  **39.**  $f(x) = x^3 + 3x^2 + 6x - 11, x = -5$  **41.**  $f(x) = -4x^3 + 3x - 5, x = 2$  **43.**  $f(x) = 2x^4 + x^3 - 3x^2 + 5x, x = -1$ **45.**  $f(x) = 2x^3 - x^2 + 6x, x = 5$ 

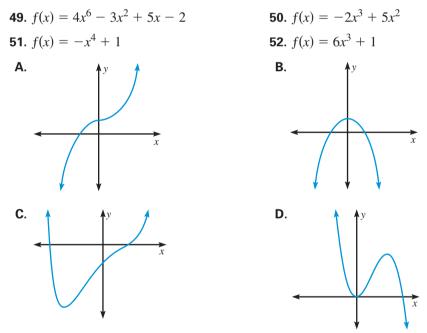
**38.** 
$$f(x) = -3x^3 + 7x^2 - 4x + 8, x = 3$$
  
**40.**  $f(x) = x^3 - x^2 + 12x + 15, x = -1$   
**42.**  $f(x) = -x^4 + x^3 - x + 1, x = -3$   
**44.**  $f(x) = 3x^5 - 2x^2 + x, x = 2$   
**46.**  $f(x) = -x^4 + 8x^3 + 13x - 4, x = -2$ 

**END BEHAVIOR PATTERNS** Graph each polynomial function in the table. Then copy and complete the table to describe the end behavior of the graph of each function.

47.	Function	As	As
		$x \rightarrow -\infty$	$x \rightarrow +\infty$
	$f(x) = -5x^3$	?	?
	$f(x)=-x^3+1$	?	?
	$f(x)=2x-3x^3$	?	?
	$f(x)=2x^2-x^3$	?	?

<b>48</b> .	Function	As	As
		$\chi \rightarrow -\infty$	$x \rightarrow +\infty$
	$f(x) = x^4 + 3x^3$	?	?
	$f(x)=x^4+2$	?	?
	$f(x)=x^4-2x-1$	?	?
	$f(x)=3x^4-5x^2$	?	?

**MATCHING** Use what you know about end behavior to match the polynomial function with its graph.



**DESCRIBING END BEHAVIOR** Describe the end behavior of the graph of the polynomial function by completing these statements:  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow \underline{?}$  as  $x \rightarrow +\infty$ .

**53.**  $f(x) = -5x^4$ **54.**  $f(x) = -x^2 + 1$ **55.** f(x) = 2x**56.**  $f(x) = -10x^3$ **57.**  $f(x) = -x^6 + 2x^3 - x$ **58.**  $f(x) = x^5 + 2x^2$ **59.**  $f(x) = -3x^5 - 4x^2 + 3$ **60.**  $f(x) = x^7 - 3x^3 + 2x$ **61.**  $f(x) = 3x^6 - x - 4$ **62.**  $f(x) = 3x^8 - 4x^3$ **63.**  $f(x) = -6x^3 + 10x$ **64.**  $f(x) = x^4 - 5x^3 + x - 1$ 

**GRAPHING POLYNOMIALS** Graph the polynomial function.

<b>65.</b> $f(x) = -x^3$	<b>66.</b> $f(x) = -x^4$	<b>67.</b> $f(x) = x^5 + 2$
<b>68.</b> $f(x) = x^4 - 4$	<b>69.</b> $f(x) = x^4 + 6x^2 - 5$	<b>70.</b> $f(x) = 2 - x^3$
<b>71.</b> $f(x) = x^5 - 2$	<b>72.</b> $f(x) = -x^4 + 3$	<b>73.</b> $f(x) = -x^3 + 3x$
<b>74.</b> $f(x) = -x^3 + 2x^2 - 4$	<b>75.</b> $f(x) = -x^5 + x^2 + 1$	<b>76.</b> $f(x) = x^3 - 3x - 1$
<b>77.</b> $f(x) = x^5 + 3x^3 - x$	<b>78.</b> $f(x) = x^4 - 2x - 3$	<b>79.</b> $f(x) = -x^4 + 2x - 1$

- **80. CRITICAL THINKING** Give an example of a polynomial function *f* such that  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$ .
- **81. SHOPPING** The retail space in shopping centers in the United States from 1972 to 1996 can be modeled by

$$S = -0.0068t^3 - 0.27t^2 + 150t + 1700$$

where *S* is the amount of retail space (in millions of square feet) and *t* is the number of years since 1972. How much retail space was there in 1990?

**82. Solution** Sector Sector

$$R = -0.0036t^3 + 0.13t^2 - 0.073t + 7.7$$

where *R* is the monthly rate (in dollars) and *t* is the number of years since 1980. What was the monthly rate in 1983?

#### SURSING In Exercises 83 and 84, use the following information.

From 1985 to 1995, the number of graduates from nursing schools in the United States can be modeled by

$$y = -0.036t^4 + 0.605t^3 - 1.87t^2 - 4.67t + 82.5$$

where y is the number of graduates (in thousands) and t is the number of years since 1985.  $\triangleright$  Source: Statistical Abstract of the United States

- **83.** Describe the end behavior of the graph of the function. From the end behavior, would you expect the number of nursing graduates in the year 2010 to be more than or less than the number of nursing graduates in 1995? Explain.
- **84.** Graph the function for  $0 \le t \le 10$ . Use the graph to find the first year in which there were over 82,500 nursing graduates.

#### S TENNIS In Exercises 85 and 86, use the following information.

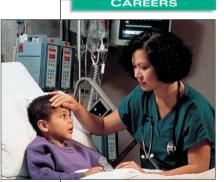
The amount of prize money for the women's U.S. Open Tennis Tournament from 1970 to 1997 can be modeled by

$$P = 1.141t^2 - 5.837t + 14.31$$

where *P* is the prize money (in thousands of dollars) and *t* is the number of years since 1970.  $\triangleright$  Source: U.S. Open

- **85.** Describe the end behavior of the graph of the function. From the end behavior, would you expect the amount of prize money in the year 2005 to be more than or less than the amount in 1995? Explain.
- **86.** Graph the function for  $0 \le t \le 40$ . Use the graph to estimate the amount of prize money in the year 2005.

FOCUS ON



NURSE Although the majority of nurses work in hospitals, nurses also work in doctors' offices, in private homes, at nursing homes, and in other community settings.

CAREER LINK



**87. MULTI-STEP PROBLEM** To determine whether a Holstein heifer's height is normal, a veterinarian can use the cubic functions

$$L = 0.0007t^3 - 0.061t^2 + 2.02t + 30$$

 $H = 0.001t^3 - 0.08t^2 + 2.3t + 31$ 

where L is the minimum normal height (in inches), H is the maximum normal height (in inches), and t is the age (in months).

- Source: Journal of Dairy Science
- **a.** What is the normal height range for an 18-month-old Holstein heifer?
- **b**. Describe the end behavior of each function's graph.
- **c**. Graph the two height functions.



A heifer is a young cow that has not yet had calves.

**d.** *Writing* Suppose a veterinarian examines a Holstein heifer that is 43 inches tall. About how old do you think the cow is? How did you get your answer?

## ★ Challenge

**EXAMINING END BEHAVIOR** Use a spreadsheet or a graphing calculator to evaluate the polynomial functions  $f(x) = x^3$  and  $g(x) = x^3 - 2x^2 + 4x + 5$  for the given values of x.

- **88.** Copy and complete the table.
- **89.** Use the results of Exercise 88 to complete this statement:

As 
$$x \to +\infty$$
,  $\frac{f(x)}{g(x)} \to \underline{?}$ 

Explain how this statement shows that the functions *f* and *g* have the same end behavior as  $x \to +\infty$ .

x	<b>f</b> (x)	<b>g</b> (x)	$\frac{f(x)}{g(x)}$
50	?	?	?
100	?	?	?
500	?	?	?
1000	?	?	?
5000	?	?	?

EXTRA CHALLENGE
 www.mcdougallittell.com

# **MIXED REVIEW**

#### SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 1.2 for 6.3)

**90.** x + 3 - 2x - x + 2 **91.**  $-2x^2 + 3x + 4x + 2x^2$  **92.**  $-3x^2 + 1 - (x^2 + 2)$  **93.**  $x^2 + x + 1 + 3(x - 4)$  **94.**  $4x - 2x^2 + 3 - x^2 - 4$ **95.**  $x^2 - 1 - (2x^2 + x - 3)$ 

**STANDARD FORM** Write the quadratic function in standard form. (Review 5.1 for 6.3)

<b>96.</b> $y = -4(x-2)^2 + 5$	<b>97.</b> $y = -2(x+6)(x-5)$	<b>98.</b> $y = 2(x - 7)(x + 4)$
<b>99.</b> $y = 4(x - 3)^2 - 24$	<b>100.</b> $y = -(x+5)^2 + 12$	<b>101.</b> $y = -3(x - 5)^2 + 3$

#### **SOLVING QUADRATIC EQUATIONS** Solve the equation. (Review 5.4)

**102.**  $x^2 = -9$ **103.**  $x^2 = -5$ **104.**  $-3x^2 + 1 = 7$ **105.**  $4x^2 + 15 = 3$ **106.**  $6x^2 + 5 = 2x^2 + 1$ **107.**  $x^2 = 7x^2 + 1$ **108.**  $x^2 - 4 = -3x^2 - 24$ **109.**  $3x^2 + 5 = 5x^2 + 10$ **110.**  $5x^2 + 2 = -2x^2 + 1$