6.1 Using Properties of Exponents

What you should learn

GOAL 1 Use properties of exponents to evaluate and simplify expressions involving powers.

GOAL 2 Use exponents and scientific notation to solve **real-life** problems, such as finding the per capita GDP of Denmark in **Example 4**.

Why you should learn it

▼ To simplify **real-life** expressions, such as the ratio of a state's park space to total area in **Ex. 57**.

REAL LIFE

Lake Clark National Park, Alaska

GOAL 1 PROPERTIES OF EXPONENTS

Recall that the expression a^n, where n is a positive integer, represents the product that you obtain when a is used as a factor n times. In the activity you will investigate two properties of exponents.

ACTIVITY
Developing Concepts

Products and Quotients of Powers

1. How many factors of 2 are there in the product $2^3 \cdot 2^4$? Use your answer to write the product as a single power of 2.
2. Write each product as a single power of 2 by counting the factors of 2. Use a calculator to check your answers.
   a. $2^2 \cdot 2^5$ **b.** $2^1 \cdot 2^6$ **c.** $2^3 \cdot 2^6$ **d.** $2^4 \cdot 2^4$
3. Complete this equation: $2^m \cdot 2^n = 2^{?}$
4. Write each quotient as a single power of 2 by first writing the numerator and denominator in "expanded form" (for example, $2^3 = 2 \cdot 2 \cdot 2$) and then canceling common factors. Use a calculator to check your answers.
   a. $\frac{2^3}{2^1}$ **b.** $\frac{2^5}{2^2}$ **c.** $\frac{2^7}{2^3}$ **d.** $\frac{2^6}{2^2}$
5. Complete this equation: $\frac{2^m}{2^n} = 2^{?}$

In the activity you may have discovered two of the following properties of exponents.

CONCEPT SUMMARY **PROPERTIES OF EXPONENTS**

Let a and b be real numbers and let m and n be integers.

PRODUCT OF POWERS PROPERTY	$a^m \cdot a^n = a^{m+n}$
POWER OF A POWER PROPERTY	$(a^m)^n = a^{mn}$
POWER OF A PRODUCT PROPERTY	$(ab)^m = a^m b^m$
NEGATIVE EXPONENT PROPERTY	$a^{-m} = \frac{1}{a^m}, a \neq 0$
ZERO EXPONENT PROPERTY	$a^0 = 1, a \neq 0$
QUOTIENT OF POWERS PROPERTY	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
POWER OF A QUOTIENT PROPERTY	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

The properties of exponents can be used to evaluate numerical expressions and to simplify algebraic expressions. In this book we assume that any base with a zero or negative exponent is nonzero. A simplified algebraic expression contains only positive exponents.

EXAMPLE 1 Evaluating Numerical Expressions

a. $(2^3)^4 = 2^{3 \cdot 4}$ Power of a power property

$= 2^{12}$ Simplify exponent.

$= 4096$ Evaluate power.

b. $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$ Power of a quotient property

$= \frac{9}{16}$ Evaluate powers.

c. $(-5)^{-6}(-5)^4 = (-5)^{-6+4}$ Product of powers property

$= (-5)^{-2}$ Simplify exponent.

$= \frac{1}{(-5)^2}$ Negative exponent property

$= \frac{1}{25}$ Evaluate power.

STUDENT HELP

Study Tip

When you multiply powers, do not multiply the bases. For example, $2^3 \cdot 2^5 \neq 4^8$.

EXAMPLE 2 Simplifying Algebraic Expressions

a. $\left(\frac{r}{s^{-5}}\right)^2 = \frac{r^2}{(s^{-5})^2}$ Power of a quotient property

$= \frac{r^2}{s^{-10}}$ Power of a power property

$= r^2 s^{10}$ Negative exponent property

b. $(7b^{-3})^2 b^5 b = 7^2 (b^{-3})^2 b^5 b$ Power of a product property

$= 49b^{-6} b^5 b$ Power of a power property

$= 49b^{-6+5+1}$ Product of powers property

$= 49b^0$ Simplify exponent.

$= 49$ Zero exponent property

c. $\frac{(xy^2)^2}{x^3 y^{-1}} = \frac{x^2 (y^2)^2}{x^3 y^{-1}}$ Power of a product property

$= \frac{x^2 y^4}{x^3 y^{-1}}$ Power of a power property

$= x^{2-3} y^{4-(-1)}$ Quotient of powers property

$= x^{-1} y^5$ Simplify exponents.

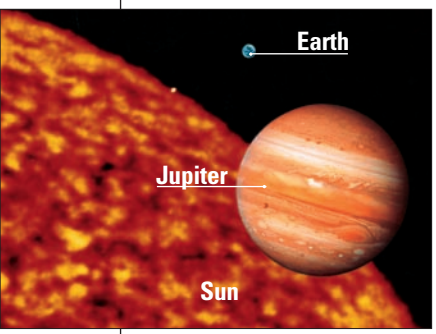
$= \frac{y^5}{x}$ Negative exponent property

STUDENT HELP

INTERNET HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

FOCUS ON APPLICATIONS



REAL LIFE **ASTRONOMY** Jupiter is the largest planet in the solar system. It has a radius of 71,400 km—over 11 times as great as Earth's, but only about one tenth as great as the sun's.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

GOAL 2 USING PROPERTIES OF EXPONENTS IN REAL LIFE

EXAMPLE 3 *Comparing Real-Life Volumes*

ASTRONOMY The radius of the sun is about 109 times as great as Earth's radius. How many times as great as Earth's volume is the sun's volume?

SOLUTION

Let r represent Earth's radius.

$$\frac{\text{Sun's volume}}{\text{Earth's volume}} = \frac{\frac{4}{3}\pi(109r)^3}{\frac{4}{3}\pi r^3}$$ The volume of a sphere is $\frac{4}{3}\pi r^3$.

$$= \frac{\frac{4}{3}\pi 109^3 r^3}{\frac{4}{3}\pi r^3}$$ Power of a product property

$$= 109^3 r^0$$ Quotient of powers property

$$= 109^3$$ Zero exponent property

$$= 1{,}295{,}029$$ Evaluate power.

▶ The sun's volume is about 1.3 million times as great as Earth's volume.

STUDENT HELP **Skills Review** For help with scientific notation, see p. 913.

A number is expressed in **scientific notation** if it is in the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer. For instance, the width of a molecule of water is about 2.5×10^{-8} meter, or 0.000000025 meter. When working with numbers in scientific notation, the properties of exponents listed on page 323 can help make calculations easier.

REAL LIFE *Economics*

EXAMPLE 4 *Using Scientific Notation in Real Life*

In 1997 Denmark had a population of 5,284,000 and a gross domestic product (GDP) of $131,400,000,000. Estimate the per capita GDP of Denmark.

INTERNET **DATA UPDATE** of UN/ECE Statistical Division data at www.mcdougallittell.com

SOLUTION

"Per capita" means per person, so divide the GDP by the population.

$$\frac{\text{GDP}}{\text{Population}} = \frac{131{,}400{,}000{,}000}{5{,}284{,}000}$$ Divide GDP by population.

$$= \frac{1.314 \times 10^{11}}{5.284 \times 10^6}$$ Write in scientific notation.

$$= \frac{1.314}{5.284} \times 10^5$$ Quotient of powers property

$$\approx 0.249 \times 10^5$$ Use a calculator.

$$= 24{,}900$$ Write in standard notation.

▶ The per capita GDP of Denmark in 1997 was about $25,000 per person.

GUIDED PRACTICE

Vocabulary Check ✓

1. State the name of the property illustrated.

a. $a^m \cdot a^n = a^{m+n}$ **b.** $(a^m)^n = a^{mn}$ **c.** $(ab)^m = a^m b^m$

Concept Check ✓

2. ERROR ANALYSIS Describe the mistake made in simplifying the expression.

a. $(-2)^2(-2)^3 = 4^5$ **b.** $\frac{x^8}{x^2} = x^4$ **c.** $x^4 \cdot x^3 = x^{12}$

Skill Check ✓

Evaluate the expression. Tell which properties of exponents you used.

3. $6 \cdot 6^2$ **4.** $(9^6)(9^2)^{-3}$ **5.** $(2^3)^2$

6. $\left(\frac{3}{2^{-2}}\right)\left(\frac{1}{2}\right)^2$ **7.** $\left(\frac{3}{5}\right)^{-2}$ **8.** $\frac{7^{-5}}{7^{-3}}$

Simplify the expression. Tell which properties of exponents you used.

9. $z^{-2} \cdot z^{-4} \cdot z^6$ **10.** $yz^{-2}(x^2y)^3z$ **11.** $(4x^3)^{-2}$

12. $\left(\frac{2}{x^{-3}}\right)^6$ **13.** $\frac{3y^6}{y^3}$ **14.** $\frac{(xy)^4}{xy^{-1}}$

15. ASTRONOMY Earth has a radius of about 6.38×10^3 kilometers. The sun has a radius of about 6.96×10^5 kilometers. Use the formula for the volume of a sphere given on page 325 to calculate the volume of the sun and the volume of Earth. Divide the volumes. Do you get the same result as in Example 3?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 947.

EVALUATING NUMERICAL EXPRESSIONS **Evaluate the expression. Tell which properties of exponents you used.**

16. $4^2 \cdot 4^4$ **17.** $(5^{-2})^3$ **18.** $(-9)(-9)^3$ **19.** $(8^2)^3$

20. $\frac{5^2}{5^5}$ **21.** $\left(\frac{3}{7}\right)^3$ **22.** $\left(\frac{5}{9}\right)^{-3}$ **23.** $11^{-2} \cdot 11^0$

24. $\frac{4^{-2}}{4^{-3}}$ **25.** $\left(\frac{1}{8}\right)^{-4}$ **26.** $(2^{-4})^{-2}$ **27.** $\frac{2^2}{2^{-9}}$

28. $\frac{6^2}{(6^{-2} \cdot 5^1)^{-2}}$ **29.** $6^0 \cdot 6^3 \cdot 6^{-4}$ **30.** $\left(\frac{1}{10}\right)^3\left(\frac{1}{10}\right)^{-3}$ **31.** $\left(\left(\frac{2}{5}\right)^{-3}\right)^2$

SIMPLIFYING ALGEBRAIC EXPRESSIONS **Simplify the expression. Tell which properties of exponents you used.**

32. $x^8 \cdot \frac{1}{x^3}$ **33.** $(2^3x^2)^5$ **34.** $(x^2y^2)^{-1}$ **35.** $\frac{x^5}{x^{-2}}$

36. $\frac{x^5y^2}{x^4y^0}$ **37.** $(x^4y^7)^{-3}$ **38.** $\frac{x^{11}y^{10}}{x^{-3}y^{-1}}$ **39.** $-3x^{-4}y^0$

40. $(10x^3y^5)^{-3}$ **41.** $\frac{x^{-1}y}{xy^{-2}}$ **42.** $(4x^2y^5)^{-2}$ **43.** $\frac{2x^2y}{6xy^{-1}}$

44. $\frac{5x^3y^9}{20x^2y^{-2}}$ **45.** $\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{21x^5}$ **46.** $\frac{y^{10}}{2x^3} \cdot \frac{20x^{14}}{xy^6}$ **47.** $\frac{12xy}{7x^4} \cdot \frac{7x^5y^2}{4y}$

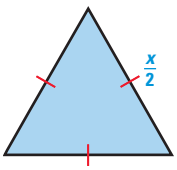
STUDENT HELP

HOMEWORK HELP

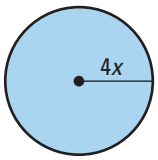
Example 1: Exs. 16–31
Example 2: Exs. 32–51
Examples 3, 4: Exs. 52–56

GEOMETRY CONNECTION **Write an expression for the area or volume of the figure in terms of *x*.**

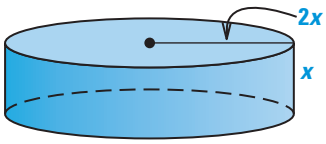
48. $A = \frac{\sqrt{3}}{4}s^2$



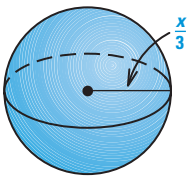
49. $A = \pi r^2$



50. $V = \pi r^2 h$



51. $V = \frac{4}{3}\pi r^3$



SCIENTIFIC NOTATION **In Exercises 52–56, use scientific notation.**

52. **NATIONAL DEBT** On June 8, 1999, the national debt of the United States was about $5,608,000,000,000. The population of the United States at that time was about 273,000,000. Suppose the national debt was divided evenly among everyone in the United States. How much would each person owe?

INTERNET **DATA UPDATE** of Bureau of the Public Debt and U.S. Census Bureau data at www.mcdougallittell.com

53. **SOCIAL STUDIES CONNECTION** The table shows the population and gross domestic product (GDP) in 1997 for each of six different countries. Calculate the per capita GDP for each country.

INTERNET **DATA UPDATE** of UN/ECE Statistical Division data at www.mcdougallittell.com

Country	Population	GDP (U.S. dollars)
France	58,607,000	1,249,600,000,000
Germany	82,061,000	1,839,300,000,000
Ireland	3,661,000	71,300,000,000
Luxembourg	420,000	13,600,000,000
The Netherlands	15,600,000	333,400,000,000
Sweden	8,849,000	177,300,000,000

FOCUS ON CAREERS

REAL LIFE **ORNITHOLOGIST** An ornithologist is a scientist who studies the history, classification, biology, and behavior of birds.

INTERNET **CAREER LINK** www.mcdougallittell.com

54. **BIOLOGY CONNECTION** A red blood cell has a diameter of approximately 0.00075 centimeter. Suppose one of the arteries in your body has a diameter of 0.0456 centimeter. How many red blood cells could fit across the artery?

55. **SPACE EXPLORATION** On February 17, 1998, *Voyager 1* became the most distant manmade object in space, at a distance of 10,400,000,000 kilometers from Earth. How long did it take *Voyager 1* to travel this distance given that it traveled an average of 1,390,000 kilometers per day? ▶ Source: NASA

56. **ORNITHOLOGY** Some scientists estimate that there are about 8600 species of birds in the world. The mean number of birds per species is approximately 12,000,000. About how many birds are there in the world?

Test Preparation

57. MULTI-STEP PROBLEM Suppose you live in a state that has a total area of 5.38×10^7 acres and 4.19×10^5 acres of park space. You think that the state should set aside more land for parks. The table shows the total area and the amount of park space for several states.

State	Total area (acres)	Amount of park space (acres)
Alaska	393,747,200	3,250,000
California	101,676,000	1,345,000
Connecticut	3,548,000	176,000
Kansas	52,660,000	29,000
Ohio	28,690,000	204,000
Pennsylvania	29,477,000	283,000

▶ Source: *Statistical Abstract of the United States*

a. Write the total area and the amount of park space for each state in scientific notation.

b. For each state, divide the amount of park space by the total area.

c. *Writing* You want to ask the state legislature to increase the amount of park space in your state. Use your results from parts (a) and (b) to write a letter that explains why your state needs more park space.

Challenge

LOGICAL REASONING **In Exercises 58 and 59, refer to the properties of exponents on page 323.**

58. Show how the negative exponent property can be derived from the quotient of powers property and the zero exponent property.

EXTRA CHALLENGE
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59. Show how the quotient of powers property can be derived from the product of powers property and the negative exponent property.

MIXED REVIEW

GRAPHING **Graph the equation.** (Review 2.3, 5.1 for 6.2)

60. $y = -4$ **61.** $y = -x - 3$ **62.** $y = 3x + 1$

63. $y = -2x + 5$ **64.** $y = 3x^2 + 2$ **65.** $y = -2x(x + 6)$

66. $y = x^2 - 2x - 6$ **67.** $y = 2x^2 - 4x + 10$ **68.** $y = -2(x - 3)^2 + 8$

SOLVING QUADRATIC EQUATIONS **Solve the equation.** (Review 5.3)

69. $2x^2 = 32$ **70.** $-3x^2 = -24$ **71.** $25x^2 = 16$

72. $3x^2 - 8 = 100$ **73.** $13 - 5x^2 = 8$ **74.** $4x^2 - 5 = 9$

75. $-x^2 + 9 = 2x^2 - 6$ **76.** $12 + 2x^2 = 5x^2 - 8$ **77.** $-2x^2 + 7 = x^2 - 2$

OPERATIONS WITH COMPLEX NUMBERS **Write the expression as a complex number in standard form.** (Review 5.4)

78. $(9 + 4i) + (9 - i)$ **79.** $(-5 + 3i) - (-2 - i)$ **80.** $(10 - i) - (4 + 7i)$

81. $-i(7 + 2i)$ **82.** $-11i(5 + i)$ **83.** $(3 + i)(9 + i)$