ACTIVITY 4.4

Developing Concepts

Group Activity for use with Lesson 4.4

Investigating Identity and Inverse Matrices

GROUP ACTIVITY
Work with a partner.

MATERIALS
- pencil
- paper

QUESTION What are some properties of identity and inverse matrices?

EXPLORING THE CONCEPT

1 Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix}$. Consider the 2×2 *identity matrix* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find AI, BI, and CI. What do you notice?

2 Find IA, IB, and IC using the matrices from **Step 1**. Is multiplication by the identity matrix commutative?

3 Let $D = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$. The *inverse* of D is $E = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$. Find DE and ED. What do you notice?

4 Use matrix multiplication to decide which of the following is the inverse of the matrix A in **Step 1**: $\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}$, $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$, or $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$.

DRAWING CONCLUSIONS

1. For any 2×2 matrix A, what is true of the products AI and IA where I is the 2×2 identity matrix? Justify your answer mathematically.

   (*Hint:* Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and compute AI and IA.)

2. How is the relationship between $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and other 2×2 matrices similar to the relationship between 1 and other real numbers?

3. What do you think is the identity matrix for the set of 3×3 matrices? Check your answer by multiplying your proposed identity matrix by several 3×3 matrices.

4. What is the relationship between a matrix, its inverse, and the identity matrix? How is this relationship like the one that exists between a nonzero real number, its reciprocal, and 1?

5. Does every nonzero matrix have an inverse? Explain. (*Hint:* Consider a 2×2 matrix whose first row contains all nonzero entries and whose second row contains all zero entries.)

6. Find the inverse of $F = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$ by finding values of a, b, c, and d such that $\begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.