4.4 Identity and Inverse Matrices

What you should learn

GOAL 1 Find and use inverse matrices.

GOAL 2 Use inverse matrices in **real-life** situations, such as encoding a message in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as decoding names of landmarks in **Exs. 44–48**.

REAL LIFE

The artist Jim Sanborn uses cryptograms in his work, such as *Kryptos* above.

GOAL 1 Using Inverse Matrices

The number 1 is the multiplicative identity for real numbers because $1 \cdot a = a$ and $a \cdot 1 = a$. For matrices, the $n \times n$ **identity matrix** is the matrix that has 1's on the main diagonal and 0's elsewhere.

2 × 2 IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $IA = A$ and $AI = A$.

Two $n \times n$ matrices are **inverses** of each other if their product (in *both* orders) is the $n \times n$ identity matrix. For example, matrices A and B below are inverses of each other.

$$AB = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad BA = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

The symbol used for the inverse of A is A^{-1}.

THE INVERSE OF A 2 X 2 MATRIX

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

EXAMPLE 1 *Finding the Inverse of a 2 × 2 Matrix*

Find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$.

Solution

$$A^{-1} = \frac{1}{6 - 4}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

✓**CHECK** You can check the inverse by showing that $AA^{-1} = I = A^{-1}A$.

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

STUDENT HELP

Look Back
For help with multiplicative inverses of real numbers, see p. 5.

EXAMPLE 2 *Solving a Matrix Equation*

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$$\overbrace{\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}}^{A} X = \overbrace{\begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix}}^{B}$$

SOLUTION

Begin by finding the inverse of A.

$$A^{-1} = \frac{1}{4-3}\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

To solve the equation for X, multiply both sides of the equation by A^{-1} *on the left.*

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix} \qquad A^{-1}AX = A^{-1}B$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & -2 \\ 0 & -3 \end{bmatrix} \qquad IX = A^{-1}B$$

$$X = \begin{bmatrix} 2 & -2 \\ 0 & -3 \end{bmatrix} \qquad X = A^{-1}B$$

✓**CHECK** You can check the solution by multiplying A and X to see if you get B.

....

Some matrices do not have an inverse. You can tell whether a matrix has an inverse by evaluating its determinant. If $\det A = 0$, then A does not have an inverse. If $\det A \neq 0$, then A has an inverse.

The inverse of a 3×3 matrix is difficult to compute by hand. A calculator that will compute inverse matrices is useful in this case.

EXAMPLE 3 *Finding the Inverse of a 3 × 3 Matrix*

STUDENT HELP

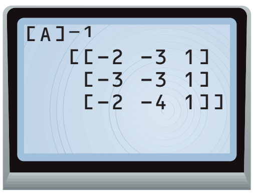
KEYSTROKE HELP
Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

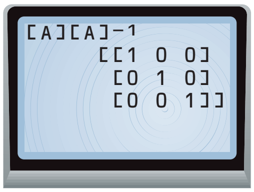
Use a graphing calculator to find the inverse of A. Then use the calculator to verify your result.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

SOLUTION

Enter the matrix A into the graphing calculator and calculate A^{-1}. Then compute AA^{-1} and $A^{-1}A$ to verify that you obtain the 3×3 identity matrix.





GOAL 2 USING INVERSE MATRICES IN REAL LIFE

A *cryptogram* is a message written according to a secret code. (The Greek word *kruptos* means *hidden* and the Greek word *gramma* means *letter*.) The following technique uses matrices to encode and decode messages.

First assign a number to each letter in the alphabet with 0 assigned to a blank space.

__ = 0	E = 5	J = 10	O = 15	T = 20	Y = 25
A = 1	F = 6	K = 11	P = 16	U = 21	Z = 26
B = 2	G = 7	L = 12	Q = 17	V = 22	
C = 3	H = 8	M = 13	R = 18	W = 23	
D = 4	I = 9	N = 14	S = 19	X = 24	

Then convert the message to numbers partitioned into 1×2 *uncoded row matrices.*

To *encode* a message, choose a 2×2 matrix A that has an inverse and multiply the uncoded row matrices by *A on the right* to obtain *coded row matrices.*

REAL LIFE
Cryptography

EXAMPLE 4 *Converting a Message*

Use the list above to convert the message GET HELP to row matrices.

SOLUTION

G E T __ H E L P

$[7 \quad 5][20 \quad 0][8 \quad 5][12 \quad 16]$

FOCUS ON APPLICATIONS

REAL LIFE **NAVAJO CODE** During World War II, a Marine Corps code based on the complex Navajo language was used to send messages.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

EXAMPLE 5 *Encoding a Message*

CRYPTOGRAPHY Use $A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ to encode the message GET HELP.

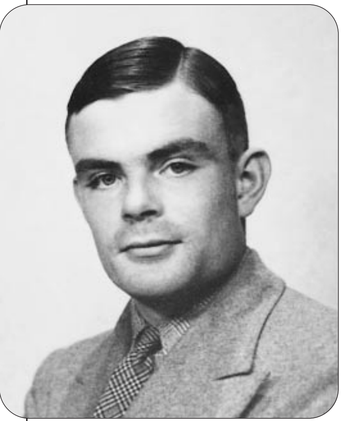
SOLUTION

The coded row matrices are obtained by multiplying each of the uncoded row matrices from Example 4 by the matrix *A on the right.*

UNCODED ROW MATRIX	ENCODING MATRIX *A*	CODED ROW MATRIX
$[7 \quad 5]$	$\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$	$= [9 \quad 11]$
$[20 \quad 0]$	$\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$	$= [40 \quad 60]$
$[8 \quad 5]$	$\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$	$= [11 \quad 14]$
$[12 \quad 16]$	$\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$	$= [8 \quad 4]$

▶ The coded message is 9, 11, 40, 60, 11, 14, 8, 4.

FOCUS ON PEOPLE



REAL LIFE **ALAN TURING,** an English mathematician, helped break codes used by the German military during World War II.

DECODING USING MATRICES Decoding the cryptogram created in Example 5 would be difficult for people who do not know the matrix A. When larger coding matrices are used, decoding is even more difficult. But for an authorized receiver who knows the matrix A, decoding is simple. The receiver only needs to multiply the coded row matrices by A^{-1} *on the right* to retrieve the uncoded row matrices.

EXAMPLE 6 *Decoding a Message*

CRYPTOGRAPHY Use the inverse of $A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ to decode this message:

−4, 3, −23, 12, −26, 13, 15, −5, 31, −5,
−38, 19, −21, 12, 20, 0, 75, −25

SOLUTION

First find A^{-1}: $A^{-1} = \frac{1}{3-2}\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

To decode the message, partition it into groups of two numbers to form coded row matrices. Then multiply each coded row matrix by A^{-1} *on the right* to obtain the uncoded row matrices.

CODED ROW MATRIX	DECODING MATRIX A^{-1}		UNCODED ROW MATRIX
$\begin{bmatrix} -4 & 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 2 & 5 \end{bmatrix}$
$\begin{bmatrix} -23 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 1 & 13 \end{bmatrix}$
$\begin{bmatrix} -26 & 13 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 0 & 13 \end{bmatrix}$
$\begin{bmatrix} 15 & -5 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 5 & 0 \end{bmatrix}$
$\begin{bmatrix} 31 & -5 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 21 & 16 \end{bmatrix}$
$\begin{bmatrix} -38 & 19 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 0 & 19 \end{bmatrix}$
$\begin{bmatrix} -21 & 12 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 3 & 15 \end{bmatrix}$
$\begin{bmatrix} 20 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 20 & 20 \end{bmatrix}$
$\begin{bmatrix} 75 & -25 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$	=	$\begin{bmatrix} 25 & 0 \end{bmatrix}$

From the uncoded row matrices you can read the message as follows.

$$\begin{matrix} [2 & 5] & [1 & 13] & [0 & 13] & [5 & 0] & [21 & 16] & [0 & 19] & [3 & 15] & [20 & 20] & [25 & 0] \\ B & E & A & M & \_ & M & E & \_ & U & P & \_ & S & C & O & T & T & Y & \_ \end{matrix}$$

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the identity matrix for 2 × 2 matrices? for 3 × 3 matrices?

Concept Check ✓

2. For two 2 × 2 matrices A and B to be inverses of each other, what must be true of AB and BA?
3. Explain how to find the inverse of a 2 × 2 matrix.
4. How do you know that the matrix X in Example 2 must be 2 × 2?
5. If $B = \begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix}$, does B have an inverse? Explain.

Skill Check ✓

Find the inverse of the matrix.

6. $\begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$
7. $\begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$
8. $\begin{bmatrix} -1 & 0 \\ 6 & 4 \end{bmatrix}$
9. $\begin{bmatrix} \frac{1}{2} & 4 \\ -2 & \frac{1}{4} \end{bmatrix}$
10. $\begin{bmatrix} 0.5 & 3 \\ 2.5 & 4 \end{bmatrix}$
11. $\begin{bmatrix} 1.6 & 2 \\ 3.2 & 0.2 \end{bmatrix}$
12. **DECODING A MESSAGE** Use the coding information on pages 225 and 226 and the inverse of the matrix D to decode the following message.

$D = \begin{bmatrix} -5 & 3 \\ -2 & 1 \end{bmatrix}$

−71, 39, −35, 20, −118, 69, −84, 49, −95, 57

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 945.

FINDING INVERSES **Find the inverse of the matrix.**

13. $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$
14. $\begin{bmatrix} 6 & 2 \\ 8 & 3 \end{bmatrix}$
15. $\begin{bmatrix} 1 & 8 \\ 1 & 7 \end{bmatrix}$
16. $\begin{bmatrix} -6 & 17 \\ 1 & -3 \end{bmatrix}$
17. $\begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$
18. $\begin{bmatrix} -7 & -2 \\ -4 & 1 \end{bmatrix}$
19. $\begin{bmatrix} -6 & -7 \\ 2 & 2 \end{bmatrix}$
20. $\begin{bmatrix} 5 & -4 \\ -4 & 4 \end{bmatrix}$
21. $\begin{bmatrix} 11 & -3 \\ -9 & 3 \end{bmatrix}$
22. $\begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ -2 & 1 \end{bmatrix}$
23. $\begin{bmatrix} 2.2 & 2.5 \\ 8 & 10 \end{bmatrix}$
24. $\begin{bmatrix} \frac{4}{5} & \frac{3}{4} \\ -1 & \frac{5}{2} \end{bmatrix}$

SOLVING EQUATIONS **Solve the matrix equation.**

25. $\begin{bmatrix} -5 & -13 \\ 0 & 5 \end{bmatrix}X = \begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$
26. $\begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix}X = \begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$
27. $\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 4 & 0 & 6 \\ 3 & -1 & 5 \end{bmatrix}$
28. $\begin{bmatrix} -5 & -3 \\ 4 & 1 \end{bmatrix}X = \begin{bmatrix} -12 & -5 & 18 \\ 4 & -3 & -13 \end{bmatrix}$
29. $\begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}X + \begin{bmatrix} 8 & 5 \\ 1 & 15 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & -9 \end{bmatrix}$
30. $\begin{bmatrix} -7 & -9 \\ 4 & 5 \end{bmatrix}X + \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 6 & -6 \end{bmatrix}$
31. $\begin{bmatrix} -1 & 2 \\ -4 & 6 \end{bmatrix}X - \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$
32. $\begin{bmatrix} 4 & -3 \\ 6 & -2 \end{bmatrix}X - \begin{bmatrix} -1 & 1 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 2 \end{bmatrix}$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–24, 33
Example 2: Exs. 25–32
Example 3: Exs. 34–39
Examples 4, 5: Exs. 40–43
Example 6: Exs. 44–48

IDENTIFYING INVERSES **Tell whether the matrices are inverses of each other.**

33. $\begin{bmatrix} 10 & -3 \\ 3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 3 & -10 \end{bmatrix}$

34. $\begin{bmatrix} 0 & 2 & -1 \\ 5 & 2 & 3 \\ 7 & 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} -2 & -10 & 8 \\ 11 & 7 & -5 \\ 1 & 12 & -10 \end{bmatrix}$

35. $\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$ and $\begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix}$

36. $\begin{bmatrix} 10 & 2 & -25 \\ 4 & 1 & -10 \\ -9 & -2 & 23 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ -2 & 5 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

FINDING INVERSES **Use a graphing calculator to find the inverse of the matrix A. Check the result by showing that $AA^{-1} = I$ and $A^{-1}A = I$.**

37. $A = \begin{bmatrix} -3 & 4 & 5 \\ 1 & 5 & 0 \\ 5 & 2 & 2 \end{bmatrix}$

38. $A = \begin{bmatrix} -7 & 0 & -6 \\ -4 & 1 & 3 \\ 11 & -3 & -9 \end{bmatrix}$

39. $A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$

ENCODING **Use the code on page 225 and the matrix to encode the message.**

40. JOB WELL DONE

$A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

41. STAY THERE

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

42. COME TO DINNER

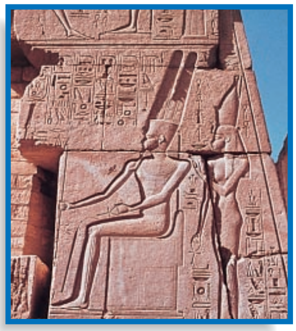
$A = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

43. HAPPY BIRTHDAY

$A = \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix}$

TRAVEL **In Exercises 44–48, use the following information.**

Your friend is traveling abroad and is sending you postcards with encoded messages. You must decipher what landmarks your friend has visited. Use the inverse of matrix D to decode each message. Each message represents a landmark in the country where your friend is traveling. Use the coding information on pages 225 and 226 to help you.



$$D = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

44. −1, 4, 30, −41, 39, −58, 22, −33, 31, −46, 23, −34, 1, 1

45. 21, −31, 22, −26, −9, 19, −20, 40, −3, 11, 20, −24, 10, −15

46. 39, −58, −2, 12, 0, 9, −19, 38, 13, −9, −16, 33, 10, −15

47. 32, −44, 10, −15, −4, 15, 9, −13, 40, −60, 22, −25, 7, −6, 4, 6

48. Using the decoded messages, tell what country your friend is visiting.

STUDENT HELP

Skills Review
For help with transformations, see p. 921.

49. **GEOMETRY CONNECTION** Use the matrices shown. The columns of matrix T give the coordinates of the vertices of a triangle. Matrix A is a transformation matrix.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$$

a. Find AT and AAT. Then draw the original triangle and the two transformed triangles. What transformation does A represent?

b. Suppose you start with the triangle determined by AAT and want to reverse the transformation process to produce the triangle determined by AT and then the triangle determined by T. Describe how you can do this.

Test Preparation

50. *Writing* Describe the process used to solve a matrix equation.

51. MULTIPLE CHOICE What is the inverse of $\begin{bmatrix} -2 & -2 \\ 7 & 6 \end{bmatrix}$?

(A) $\begin{bmatrix} \frac{3}{13} & -\frac{1}{13} \\ \frac{7}{26} & -\frac{1}{13} \end{bmatrix}$ (B) $\begin{bmatrix} -3 & 1 \\ -\frac{7}{2} & 1 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{3}{13} & -\frac{1}{13} \\ -\frac{7}{26} & \frac{1}{13} \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 1 \\ -\frac{7}{2} & -1 \end{bmatrix}$

52. MULTIPLE CHOICE What is the solution of $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}X = \begin{bmatrix} 4 & 43 \\ 2 & 25 \end{bmatrix}$?

(A) $\begin{bmatrix} 0 & 7 \\ 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 7 \\ -2 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 2 \\ 7 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 4 \\ 2 & 7 \end{bmatrix}$ (E) $\begin{bmatrix} 0 & 7 \\ 4 & 2 \end{bmatrix}$

★ Challenge

53. CODE BREAKER You are a code breaker and intercept the encoded message 45, −35, 38, −30, 18, −18, 35, −30, 81, −60, 42, −28, 75, −55, 2, −2, 22, −21, 15, −10 that you know is being sent to someone named John. You can conclude that $\begin{bmatrix} 45 & -35 \end{bmatrix}A^{-1} = \begin{bmatrix} 10 & 15 \end{bmatrix}$ and $\begin{bmatrix} 38 & -30 \end{bmatrix}A^{-1} = \begin{bmatrix} 8 & 14 \end{bmatrix}$ where A^{-1} is the inverse of the encoding matrix A, 10 represents J, 15 represents O, 8 represents H, and 14 represents N.

Let $A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

a. Write and solve two systems of equations to find w, x, y, and z.

b. Find A^{-1}, and decode the rest of the message.

EXTRA CHALLENGE
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MIXED REVIEW

SOLVING SYSTEMS **Solve the system of linear equations using any algebraic method.** (Review 3.2, 3.6 for 4.5)

54. $3x + 5y = 12$
$x + 4y = 11$

55. $4x - 12y = 2$
$-2x + 6y = -1$

56. $-5x + 7y = 33$
$4x - 9y = -40$

57. $7x + y + 3z = 22$
$2x - 2y + 9z = -10$
$-3x - 5y - 10z = 8$

58. $x + 3z = 6$
$-2x + 3y + z = -11$
$3x - y + 2z = 13$

59. $2x + y - 4z = 4$
$4x - 3y + 8z = -8$
$-2x + 7y - 12z = 24$

MATRIX OPERATIONS **Perform the indicated operation, if possible. If not possible, state the reason.** (Review 4.1)

60. $\begin{bmatrix} -4 & 2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -3 \\ -2 & 0 \end{bmatrix}$

61. $\begin{bmatrix} 8 & -6 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -2 & -3 & -5 \\ 2 & -1 & 3 \end{bmatrix}$

62. $-8\begin{bmatrix} -1 & 3 & 4 \\ -6 & 8 & 0 \end{bmatrix}$

63. $\begin{bmatrix} 7 & -5 & 8 \\ -9 & 13 & 16 \end{bmatrix} - \begin{bmatrix} -10 & -2 & 9 \\ -9 & -12 & -15 \end{bmatrix}$

64. $\begin{bmatrix} 6 & -2 & -1 \\ -3 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 4 & -1 \\ -2 & 8 & -9 \end{bmatrix}$

65. $\frac{1}{2}\begin{bmatrix} 4 & 10 & 2 \\ 6 & 8 & 16 \end{bmatrix}$

66. CATERING You are in charge of catering for a school function. To limit the cost, you will serve only two entrees. One is a vegetarian dish that costs \$6 and the other is a chicken dish that costs \$8. If there will be 150 people at the function and your budget for the food is \$1000, how many of each type of entree will be served? (Review 3.1, 3.2)