EXPLORING DATA AND STATISTICS

4.1

Matrix Operations

What you should learn

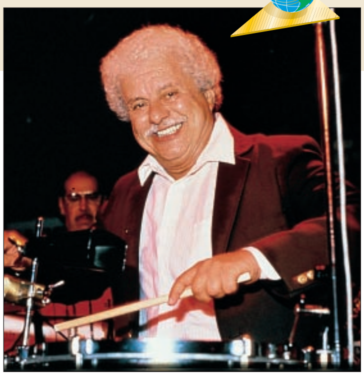
GOAL 1 Add and subtract matrices, multiply a matrix by a scalar, and solve matrix equations.

GOAL 2 Use matrices in **real-life** situations, such as organizing data about health care plans in **Example 5**.

Why you should learn it

▼ To organize **real-life** data, such as the data for Hispanic CD, cassette, and music video sales in **Exs. 39–41**.

REAL LIFE



GOAL 1 USING MATRIX OPERATIONS

A **matrix** is a rectangular arrangement of numbers in rows and columns. For instance, matrix A below has two rows and three columns. The **dimensions** of this matrix are 2×3 (read "2 by 3"). The numbers in a matrix are its **entries**. In matrix A, the entry in the second row and third column is **5**.

$$A = \underbrace{\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}}_{\textbf{3 columns}} \Bigg\} \textbf{2 rows}$$

Some *matrices* (the plural of *matrix*) have special names because of their dimensions or entries.

NAME	DESCRIPTION	EXAMPLE
Row matrix	A matrix with only 1 row	$\begin{bmatrix} 3 & -2 & 0 & 4 \end{bmatrix}$
Column matrix	A matrix with only 1 column	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
Square matrix	A matrix with the same number of rows and columns	$\begin{bmatrix} 4 & -1 & 5 \\ 2 & 0 & 1 \\ 1 & -3 & 6 \end{bmatrix}$
Zero matrix	A matrix whose entries are all zeros	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Two matrices are **equal** if their dimensions are the same and the entries in corresponding positions are equal.

EXAMPLE 1 *Comparing Matrices*

a. The following matrices are equal because corresponding entries are equal.

$$\begin{bmatrix} 5 & 0 \\ -\frac{4}{4} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -1 & 0.75 \end{bmatrix}$$

b. The following matrices are not equal because corresponding entries in the second row are not equal.

$$\begin{bmatrix} -2 & 6 \\ 0 & -3 \end{bmatrix} \neq \begin{bmatrix} -2 & 6 \\ 3 & 0 \end{bmatrix}$$

To add or subtract matrices, you simply add or subtract corresponding entries. You can add or subtract matrices only if they have the same dimensions.

EXAMPLE 2 *Adding and Subtracting Matrices*

Perform the indicated operation, if possible.

a. $\begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ **b.** $\begin{bmatrix} 8 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -7 \\ 6 & -1 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

SOLUTION

a. Since the matrices have the same dimensions, you can add them.

$$\begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3+1 \\ -4+0 \\ 7+3 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 10 \end{bmatrix}$$

b. Since the matrices have the same dimensions, you can subtract them.

$$\begin{bmatrix} 8 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -7 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} 8-2 & 3-(-7) \\ 4-6 & 0-(-1) \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -2 & 1 \end{bmatrix}$$

c. Since the dimensions of $\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$ are 2×2 and the dimensions of $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ are 2×1, you cannot add the matrices.

In matrix algebra, a real number is often called a **scalar**. To multiply a matrix by a scalar, you multiply each entry in the matrix by the scalar. This process is called **scalar multiplication**.

EXAMPLE 3 *Multiplying a Matrix by a Scalar*

STUDENT HELP

Study Tip

The order of operations for matrix expressions is similar to that for real numbers. In particular, you perform scalar multiplication before matrix addition and subtraction, as shown in part (b) of Example 3.

Perform the indicated operation(s), if possible.

a. $3\begin{bmatrix} -2 & 0 \\ 4 & -7 \end{bmatrix}$ **b.** $-2\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$

SOLUTION

a. $3\begin{bmatrix} -2 & 0 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} 3(-2) & 3(0) \\ 3(4) & 3(-7) \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 12 & -21 \end{bmatrix}$

b.
$$-2\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} -2(1) & -2(-2) \\ -2(0) & -2(3) \\ -2(-4) & -2(5) \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 4 \\ 0 & -6 \\ 8 & -10 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} -6 & 9 \\ 6 & -14 \\ 6 & -4 \end{bmatrix}$$

You can use what you know about matrix operations and matrix equality to solve a matrix equation.

EXAMPLE 4 *Solving a Matrix Equation*

Solve the matrix equation for x and y: $2\left(\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}\right) = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$

SOLUTION

Simplify the left side of the equation.

$$2\left(\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}\right) = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

$$2\begin{bmatrix} 3x + 4 & 0 \\ 6 & 5 - y \end{bmatrix} = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 6x + 8 & 0 \\ 12 & 10 - 2y \end{bmatrix} = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

Equate corresponding entries and solve the two resulting equations.

$6x + 8 = 26$ $\qquad$ $10 - 2y = 8$

$x = 3$ $\qquad$ $y = 1$

STUDENT HELP

Look Back
For help with properties of real numbers, see p. 5.

In Example 4, you could have distributed the scalar 2 to each matrix inside the parentheses before adding the matrices.

$$2\left(\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}\right) = 2\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + 2\begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}$$

$$= \begin{bmatrix} 6x & -2 \\ 16 & 10 \end{bmatrix} + \begin{bmatrix} 8 & 2 \\ -4 & -2y \end{bmatrix}$$

$$= \begin{bmatrix} 6x + 8 & 0 \\ 12 & 10 - 2y \end{bmatrix}$$

This illustrates one of several properties of matrix operations stated below.

CONCEPT SUMMARY — PROPERTIES OF MATRIX OPERATIONS

Let A, B, and C be matrices with the same dimensions and let c be a scalar.

When adding matrices, you can regroup them and change their order without affecting the result.

ASSOCIATIVE PROPERTY OF ADDITION $\quad (A + B) + C = A + (B + C)$

COMMUTATIVE PROPERTY OF ADDITION $\quad A + B = B + A$

Multiplication of a sum or difference of matrices by a scalar obeys the distributive property.

DISTRIBUTIVE PROPERTY OF ADDITION $\quad c(A + B) = cA + cB$

DISTRIBUTIVE PROPERTY OF SUBTRACTION $\quad c(A - B) = cA - cB$

GOAL 2 USING MATRICES IN REAL LIFE

Real Life: Health Care

EXAMPLE 5 *Using Matrices to Organize Data*

Use matrices to organize the following information about health care plans.

This Year *For individuals, Comprehensive, HMO Standard, and HMO Plus cost \$694.32, \$451.80, and \$489.48, respectively. For families, the Comprehensive, HMO Standard, and HMO Plus plans cost \$1725.36, \$1187.76, and \$1248.12.*

Next Year *For individuals, Comprehensive, HMO Standard, and HMO Plus will cost \$683.91, \$463.10, and \$499.27, respectively. For families, the Comprehensive, HMO Standard, and HMO Plus plans will cost \$1699.48, \$1217.45, and \$1273.08.*

SOLUTION

One way to organize the data is to use 3 × 2 matrices, as shown.

$$\begin{array}{r c c} & \textbf{This Year } (A) & \textbf{Next Year } (B) \\ & \begin{array}{cc}\textbf{Individual} & \textbf{Family}\end{array} & \begin{array}{cc}\textbf{Individual} & \textbf{Family}\end{array} \\ \begin{array}{r}\textbf{Comprehensive}\\ \textbf{HMO Standard}\\ \textbf{HMO Plus}\end{array} & \begin{bmatrix} \$694.32 & \$1725.36 \\ \$451.80 & \$1187.76 \\ \$489.48 & \$1248.12 \end{bmatrix} & \begin{bmatrix} \$683.91 & \$1699.48 \\ \$463.10 & \$1217.45 \\ \$499.27 & \$1273.08 \end{bmatrix} \end{array}$$

You can also organize the data using 2 × 3 matrices where the row labels are levels of coverage (individual and family) and the column labels are the types of plans (Comprehensive, HMO Standard, and HMO Plus).

EXAMPLE 6 *Using Matrix Operations*

HEALTH CARE A company offers the health care plans in Example 5 to its employees. The employees receive monthly paychecks from which health care payments are deducted. Use the matrices in Example 5 to write a matrix that shows the monthly changes in health care payments from this year to next year.

FOCUS ON CAREERS

HEALTH SERVICES MANAGER
Health services managers in health maintenance organizations (HMOs) plan and organize the delivery of health care.

INTERNET CAREER LINK
www.mcdougallittell.com

SOLUTION

Begin by subtracting matrix A from matrix B to determine the yearly changes in health care payments. Then multiply the result by $\frac{1}{12}$ and round answers to the nearest cent to find the monthly changes.

$$\frac{1}{12}(B - A) = \frac{1}{12}\left(\begin{bmatrix} 683.91 & 1699.48 \\ 463.10 & 1217.45 \\ 499.27 & 1273.08 \end{bmatrix} - \begin{bmatrix} 694.32 & 1725.36 \\ 451.80 & 1187.76 \\ 489.48 & 1248.12 \end{bmatrix}\right)$$

$$= \frac{1}{12}\begin{bmatrix} -10.41 & -25.88 \\ 11.30 & 29.69 \\ 9.79 & 24.96 \end{bmatrix}$$

$$\approx \begin{bmatrix} -\$.87 & -\$2.16 \\ \$.94 & \$2.47 \\ \$.82 & \$2.08 \end{bmatrix}$$

▸ The monthly deductions for the Comprehensive plan will decrease, but the monthly deductions for the other two plans will increase.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is a matrix? Describe and give an example of a row matrix, a column matrix, and a square matrix.

Concept Check ✓

2. Are the two matrices equal? Explain. $\begin{bmatrix} -6 & \frac{1}{2} \\ 4 & -5 \\ 3 & 5 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} -6 & 0.5 \\ 4 & -5 \\ 3 & 5 \end{bmatrix}$

3. To add or subtract two matrices, what must be true?

4. Use the matrices at the right to find $-2(A + B)$. Is your answer the same as that for part (b) of Example 3? Explain. $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$

5. Rework Example 5 by organizing the data using 2×3 matrices.

Skill Check ✓

Perform the indicated operation(s), if possible.

6. $\begin{bmatrix} 20 \\ -22 \\ 9 \end{bmatrix} - \begin{bmatrix} -11 \\ -10 \\ -6 \end{bmatrix}$

7. $\begin{bmatrix} -6 & -7 & 4 \\ -4 & 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -5 & 8 \\ 9 & 12 & -9 \end{bmatrix}$

8. $-4\begin{bmatrix} 2 & 0 \\ -4 & -5 \end{bmatrix}$

9. $6\begin{bmatrix} -5 & -1 \\ 2 & 0 \end{bmatrix} - 5\begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix}$

10. **HEALTH CARE** In Example 5, suppose the annual health care costs given in matrix B increase by 4% the following year. Write a matrix that shows the new *monthly* payment.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on p. 944.

COMPARING MATRICES **Tell whether the matrices are *equal* or *not equal*.**

11. $\begin{bmatrix} 5 & -1 & 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 7 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 0 & -8 \\ 8 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -8 \\ 8 & 0 & 1 \end{bmatrix}$

13. $\begin{bmatrix} 4 & 0 \\ 2 & -4 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ -2 & -4 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1.5 & 4.25 \\ 0.5 & -0.5 & 0 \end{bmatrix}, \begin{bmatrix} 2 & \frac{3}{2} & \frac{17}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$

ADDING AND SUBTRACTING MATRICES **Perform the indicated operation, if possible. If not possible, state the reason.**

15. $\begin{bmatrix} 1 & -4 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ -5 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 4 & -2 \\ 0 & -6 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

17. $\begin{bmatrix} -8 & -2 \\ 6 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}$

18. $\begin{bmatrix} -3 & 5 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ -4 & 9 \end{bmatrix}$

19. $\begin{bmatrix} 1.2 & 3.5 \\ 0.2 & 5.1 \end{bmatrix} + \begin{bmatrix} 4.1 & 8.7 \\ 2.6 & 5.3 \end{bmatrix}$

20. $\begin{bmatrix} 7 & -1 & 4 \\ 11 & -9 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 6 & 3 \\ 10 & 1 & -5 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 6 \\ -1 & -6 \\ 2 & 8 \end{bmatrix} - \begin{bmatrix} 7 & -3 & 9 \\ -2 & -7 & 9 \\ 11 & -1 & 2 \end{bmatrix}$

22. $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & \frac{3}{4} \\ \frac{1}{2} & 5 \end{bmatrix}$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 11–14
Example 2: Exs. 15–22
Example 3: Exs. 23–32
Example 4: Exs. 33–36
Examples 5, 6: Exs. 37–47

MULTIPLYING BY A SCALAR **Perform the indicated operation.**

23. $-4\begin{bmatrix} -1 & -3 & 7 \\ -4 & 0 & 6 \end{bmatrix}$

24. $5\begin{bmatrix} -2 & -6 \\ 3 & 1 \end{bmatrix}$

25. $4\begin{bmatrix} 1 & 3 & 9 \\ -5 & 5 & 15 \\ -3 & -5 & -11 \end{bmatrix}$

26. $-9\begin{bmatrix} 0 & 0 \\ -2 & -2 \\ \frac{1}{3} & \frac{4}{9} \end{bmatrix}$

27. $\frac{1}{2}\begin{bmatrix} -2 & -2 & 4 \\ \frac{1}{4} & \frac{6}{11} & -10 \end{bmatrix}$

28. $2.5\begin{bmatrix} -8.6 & 3.4 \\ 1.2 & -5.1 \\ -4.8 & 4.4 \\ 10 & -8 \end{bmatrix}$

COMBINING MATRIX OPERATIONS **Perform the indicated operations.**

29. $\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + 4\begin{bmatrix} -1 & 0 \\ 3 & -2 \\ -4 & 5 \end{bmatrix}$

30. $2\begin{bmatrix} -6 & -10 & 2 \\ 4 & -7 & -4 \end{bmatrix} - \begin{bmatrix} -1 & 5 & 13 \\ -3 & -6 & 19 \end{bmatrix}$

31. $2\begin{bmatrix} 7 & -7 \\ -1 & 3 \end{bmatrix} + 4\begin{bmatrix} 2 & -4 \\ -5 & -6 \end{bmatrix}$

32. $3\begin{bmatrix} -7 & 1 & 0 \\ 8 & -6 & -2 \end{bmatrix} - 2\begin{bmatrix} 4 & -1 & -7 \\ -3 & -5 & 5 \end{bmatrix}$

STUDENT HELP

INTERNET

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 33–36.

SOLVING MATRIX EQUATIONS **Solve the matrix equation for *x* and *y*.**

33. $\begin{bmatrix} -2x & -8 \\ -10 & -9 \end{bmatrix} = \begin{bmatrix} 6 & y \\ -10 & -9 \end{bmatrix}$

34. $\begin{bmatrix} 3x & -2 \\ -1 & 8 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ -7 & -8 \end{bmatrix} = \begin{bmatrix} -16 & -2 \\ y & 0 \end{bmatrix}$

35. $2x\begin{bmatrix} -3 & 4 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} 12 & -16 \\ y & -20 \end{bmatrix}$

36. $\begin{bmatrix} 4 & -3 \\ 8 & -7 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -5 & x \\ -7 & 7 \\ 4 & -9 \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ y & 0 \\ 5 & -7 \end{bmatrix}$

BASEBALL STATISTICS **In Exercises 37 and 38, use the following information about three Major League Baseball teams' wins and losses in 1998 before and after the All-Star Game.** ▶ Source: CNN/SI

Before *The Atlanta Braves had 59 wins and 29 losses, the Seattle Mariners had 37 wins and 51 losses, and the Chicago Cubs had 48 wins and 39 losses.*

After *The Atlanta Braves had 47 wins and 27 losses, the Seattle Mariners had 39 wins and 34 losses, and the Chicago Cubs had 42 wins and 34 losses.*

37. Use matrices to organize the information.

38. Using your matrices from Exercise 37, write a matrix that shows the total numbers of wins and losses for the three teams in 1998.

HISPANIC MUSIC **In Exercises 39–41, use the following information.** The figures below give the number (in millions) of Hispanic CD, cassette, and music video units shipped to all market channels and the dollar value (in millions) of those shipments (at suggested list prices). ▶ Source: Recording Industry Association of America

1996 *Number of units—CDs: 20,779; cassettes: 15,299; and music videos: 45.*
Dollar value—CDs: 268,441; cassettes: 122,329; and music videos: 916.

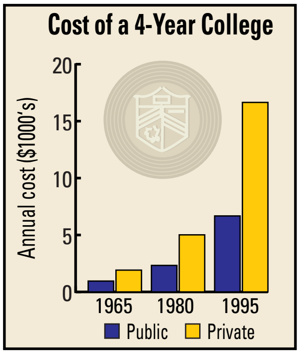
1997 *Number of units—CDs: 26,277; cassettes: 17,799; and music videos: 70.*
Dollar value—CDs: 344,697; cassettes: 144,645; and music videos: 1,260.

39. Use matrices to organize the information.

40. Write a matrix that gives the total numbers of units shipped and total values for both years.

41. Write a matrix that gives the change in units shipped and dollar value from 1996 to 1997.

FOCUS ON APPLICATIONS



REAL LIFE **COLLEGE COSTS** Since 1980, college costs have risen substantially. After adjusting for inflation, costs rose by 48% at public 4-year colleges and 71% at private 4-year colleges between 1980 and 1995.

42. **COLLEGE COSTS** The matrices below show the average yearly cost (in dollars) of tuition and room and board at colleges in the United States from 1995 through 1997. Use matrix addition to write a matrix showing the totals of these costs. ▶ Source: U.S. Department of Education

TUITION

	1995	1996	1997
Public 2-year college	1,192	1,239	1,283
Public 4-year college	2,681	2,848	2,986
Private 2-year college	6,914	7,094	7,190
Private 4-year college	11,481	12,243	12,920

ROOM AND BOARD

	1995	1996	1997
Public 2-year college	2,944	2,978	3,128
Public 4-year college	3,990	4,166	4,345
Private 2-year college	4,256	4,469	4,699
Private 4-year college	5,121	5,368	5,555

PSAT SCORES In Exercises 43 and 44, use the following information. Eligibility for a National Merit Scholarship is based on a student's PSAT score. Through 1996, this total score was found by doubling a student's verbal score and adding this value to the student's mathematics score. Let V represent the average verbal scores and let M represent the average mathematics scores earned by sophomores and juniors at Central High for tests taken in 1993 through 1996.

VERBAL SCORES (V)

	Sophomores	Juniors
1993	48.9	49.0
1994	48.9	48.9
1995	48.7	48.8
1996	48.2	48.6

MATHEMATICS SCORES (M)

	Sophomores	Juniors
1993	49.0	50.4
1994	48.3	50.0
1995	49.4	50.8
1996	49.8	50.9

43. Write an expression in terms of V and M that you could use to determine the average total PSAT scores for sophomores and juniors at Central High from 1993 through 1996. Then evaluate the expression.

44. Use the matrix from Exercise 43 to determine the average total PSAT score for juniors at Central High in 1996.

U.S. POPULATION In Exercises 45–47, use the following information. The matrices show the number of people (in thousands) who lived in each region of the United States in 1991 and the number of people (in thousands) projected to live in each region in 2010. The regional populations are separated into three age categories.

INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

1991

	0–17	18–65	Over 65
Northeast	12,142	31,791	7,043
Midwest	15,814	36,554	7,857
South	22,504	53,471	10,942
Mountain	3,993	8,461	1,580
Pacific	10,693	25,001	4,331

2010

	0–17	18–65	Over 65
Northeast	12,493	33,822	7,377
Midwest	15,840	41,095	8,980
South	25,428	67,337	14,832
Mountain	5,094	12,420	2,707
Pacific	13,655	31,125	5,511

45. The total population in 1991 was 252,177,000 and the projected total population in 2010 is 297,716,000. Rewrite the matrices to give the information as percents of the total population. (*Hint:* Multiply each matrix by the reciprocal of the total population (in thousands), and then multiply by 100.)

46. Write a matrix that gives the projected change in the percent of the population in each region and age group from 1991 to 2010.

47. Based on the result of Exercise 46, which region(s) and age group(s) are projected to show relative growth from 1991 to 2010?

Test Preparation

48. MULTI-STEP PROBLEM The matrices show the number of hardcover volumes sold and the average price per volume (in dollars) for different subject areas.

▶ Source: *The Bowker Annual*

$$\begin{array}{r} \textbf{1995 }(A) \\ \end{array}\qquad\qquad \begin{array}{r} \textbf{1996 }(B) \end{array}$$

$$\begin{array}{rc} & \begin{array}{cc} \text{Volumes sold} & \text{Average price per volume} \end{array} \\ \begin{array}{r} \text{Art} \\ \text{Law} \\ \text{Music} \\ \text{Travel} \end{array} & \begin{bmatrix} 1{,}116{,}000 & 41.23 \\ 716{,}000 & 73.09 \\ 251{,}000 & 43.27 \\ 199{,}000 & 38.30 \end{bmatrix} \end{array} \qquad \begin{array}{c} \begin{array}{cc} \text{Volumes sold} & \text{Average price per volume} \end{array} \\ \begin{bmatrix} 1{,}070{,}000 & 53.40 \\ 827{,}000 & 88.51 \\ 253{,}000 & 39.21 \\ 179{,}000 & 33.92 \end{bmatrix} \end{array}$$

a. Calculate $B - A$. How many more (or fewer) law volumes were sold in 1996 than in 1995? How much more (or less) did the average music book cost in 1996 than in 1995?

b. Calculate $B + A$. Does the "volumes sold" column in $B + A$ give you meaningful information? Does the "average price per volume" column in $B + A$ give you meaningful information? Explain.

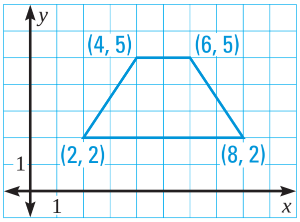
c. *Writing* What conclusions can you make about the number of volumes sold and the average price per volume of these books from 1995 to 1996?

★ **Challenge**

49. GEOMETRY CONNECTION A triangle has vertices (2, 2), (8, 2), and (5, 6). Assign a letter to each vertex and organize the triangle's vertices in a matrix. When you multiply the matrix by 4, what does the "new" triangle look like? How are the two triangles related? Use a graph to help you.

MIXED REVIEW

TRANSFORMING FIGURES **Draw the figure produced by each transformation of the figure shown.** (Skills Review, p. 921)



50. a 270° counterclockwise rotation about the origin

51. a translation by 2 units right and 4 units down

52. a reflection over the y-axis

MULTIPLYING REAL NUMBERS **Find the product.** (Skills Review, p. 905)

53. $-4(-5)$ **54.** $8(-2)$ **55.** $-7(-1)$

56. $\frac{1}{2}(-7)$ **57.** $\frac{5}{6} \cdot \frac{3}{7}$ **58.** $3.2(2.4 + 8.1)$

CHECKING SOLUTIONS **Check whether the ordered pairs are solutions of the inequality.** (Review 2.6)

59. $x + 2y \le -3$; $(0, 3)$, $(-5, 1)$ **60.** $5x - y > 2$; $(-5, 0)$, $(5, 23)$

61. $-8x - 3y < 5$; $(-1, 1)$, $(3, -9)$ **62.** $21x - 10y > 4$; $(2, 3)$, $(-1, 0)$

FINDING A SOLUTION **Give an ordered pair that is a solution of the system.** (Lesson 3.3)

63. $x + y < 10$
$y > 1$

64. $x - y \ge 3$
$y < 12$

65. $3x > y$
$x \le 15$