CHAPTER 3

Chapter Summary

WHAT did you learn? / *WHY did you learn it?*

WHAT did you learn?	WHY did you learn it?
Solve systems of linear equations in two variables. • by graphing (3.1)	Plan a vacation within a budget. (p. 141)
• using algebraic methods (3.2)	Find the weights of atoms in a molecule. (p. 153)
Graph and solve systems of linear inequalities. (3.3)	Describe conditions that will satisfy nutritional requirements of wildlife. (p. 161)
Solve linear programming problems. (3.4)	Plan a meal that minimizes cost while satisfying nutritional requirements. (p. 167)
Graph linear equations in three variables. (3.5)	Find the volume of a geometric figure graphed in a three-dimensional coordinate system. (p. 174)
Model real-life problems with functions of two variables. (3.5)	Evaluate advertising costs of a commercial. (p. 175)
Solve systems of linear equations in three variables. (3.6)	Use regional data to find the number of voters for different political parties in the United States. (p. 183)
Identify the number of solutions of a linear system. (3.1, 3.2, 3.6)	See if a bus catches up to another one before arriving at a common destination. (p. 144)
Solve real-life problems. • using a system of linear equations (3.1, 3.2, 3.6)	Find the break-even point of a business. (p. 153)
• using a system of linear inequalities (3.3, 3.4)	Display possible sale prices for shoes. (p. 161)

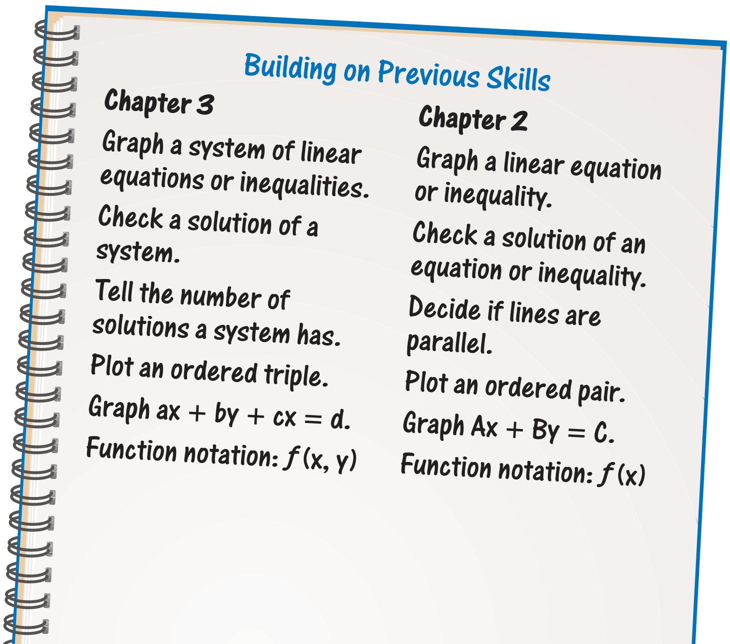
How does Chapter 3 fit into the BIGGER PICTURE of algebra?

Linear algebra is an important branch of mathematics that begins with solving linear systems. It has widespread applications to other areas of mathematics and to real-life problems, especially in business and the sciences. You will continue your study of linear algebra in the next chapter with matrices.

STUDY STRATEGY

Did you recognize when new skills related to previously learned skills?

The two-column list you made, following the **Study Strategy** on page 138, may resemble this one.



CHAPTER 3

Chapter Review

VOCABULARY

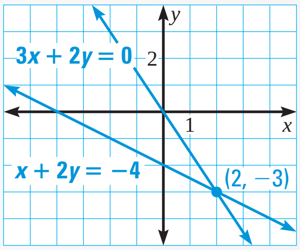
- system of two linear equations in two variables, p. 139
- solution of a system of linear equations, p. 139
- substitution method, p. 148
- linear combination method, p. 149
- System of linear inequalities in two variables, p. 156
- solution of a system of linear inequalities, p. 156
- graph of a system of linear inequalities, p. 156
- optimization, p. 163
- linear programming, p. 163
- objective function, p. 163
- constraints, p. 163
- feasible region, p. 163
- three-dimensional coordinate system, p. 170
- *z*-axis, p. 170
- ordered triple, p. 170
- octants, p. 170
- linear equation in three variables, p. 171
- function of two variables, p. 171
- system of three linear equations in three variables, p. 177
- solution of a system of three linear equations, p. 177

3.1 Solving Linear Systems by Graphing

Examples on pp. 139–141

EXAMPLE You can solve a system of two linear equations in two variables by graphing.

$x + 2y = -4$ **Equation 1**
$3x + 2y = 0$ **Equation 2**



From the graph, the lines appear to intersect at $(2, -3)$. You can check this algebraically as follows.

$2 + 2(-3) = -4$ ✓ **Equation 1 checks.**

$3(2) + 2(-3) = 0$ ✓ **Equation 2 checks.**

Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

1. $x + y = 2$
$-3x + 4y = 36$

2. $x - 5y = 10$
$-2x + 10y = -20$

3. $2x - y = 5$
$2x + 3y = 9$

4. $y = \frac{1}{3}x$
$y = \frac{1}{3}x - 2$

3.2 Solving Linear Systems Algebraically

Examples on pp. 148–151

EXAMPLE 1 You can use the substitution method to solve a system algebraically.

1 Solve the first equation for x.

2 Substitute the value of x into the second equation and solve for y.

$x - 4y = -25$ → $x = 4y - 25$ → $2(4y - 25) + 12y = 10$
$2x + 12y = 10$ $\qquad y = 3$

When you substitute $y = 3$ into one of the original equations, you get $x = -13$.

EXAMPLE 2 You can also use the linear combination method to solve a system of equations algebraically.

1 Multiply the first equation by 3 and add to the second equation. Solve for x.

$$x - 4y = -25 \quad\rightarrow\quad 3x - 12y = -75$$
$$2x + 12y = 10 \quad\rightarrow\quad 2x + 12y = 10$$
$$5x = -65$$
$$x = -13$$

2 Substitute $x = -13$ into the original first equation and solve for y.

$$-13 - 4y = -25$$
$$-4y = -12$$
$$y = 3$$

Solve the system using any algebraic method.

5. $9x - 5y = -30$
$x + 2y = 12$

6. $x + 3y = -2$
$x + y = 2$

7. $2x + 3y = -7$
$-4x - 5y = 13$

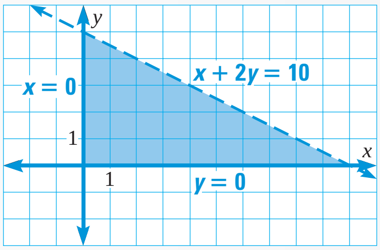
8. $3x + 3y = 0$
$-2x + 6y = -24$

3.3 Graphing and Solving Systems of Linear Inequalities

Examples on pp. 156–158

EXAMPLE You can use a graph to show all the solutions of a system of linear inequalities.

$$x \geq 0$$
$$y \geq 0$$
$$x + 2y < 10$$



Graph each inequality. The graph of the system is the region common to *all* of the shaded half-planes and includes any solid boundary line.

Graph the system of linear inequalities.

9. $y < -3x + 3$
$y > x - 1$

10. $x \geq 0$
$y \geq 0$
$-x + 2y < 8$

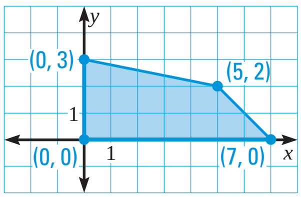
11. $x \geq -2$
$x \leq 5$
$y \geq -1$
$y \leq 3$

12. $x + y \leq 8$
$2x - y > 0$
$y \leq 4$

3.4 Linear Programming

Examples on pp. 163–165

EXAMPLE You can find the minimum and maximum values of the objective function $C = 6x + 5y$ subject to the constraints graphed below. They must occur at vertices of the feasible region.



At (0, 0): $C = 6(0) + 5(0) = 0$ ← **Minimum**

At (0, 3): $C = 6(0) + 5(3) = 15$

At (5, 2): $C = 6(5) + 5(2) = 40$

At (7, 0): $C = 6(7) + 5(0) = 42$ ← **Maximum**

3.4 continued

Find the minimum and maximum values of the objective function $C = 5x + 2y$ subject to the given constraints.

13. $x \geq 0$
$y \geq 0$
$x + y \leq 10$

14. $x \geq 0$
$y \geq 0$
$4x + 5y \leq 20$

15. $x \geq 1; x \leq 4$
$y \geq 0; y \leq 9$

16. $y \leq 6; x + y \leq 10$
$x \geq 0; x - y \leq 0$

3.5 GRAPHING LINEAR EQUATIONS IN THREE VARIABLES

Examples on pp. 170–172

EXAMPLE You can sketch the graph of an equation in three variables in a three-dimensional coordinate system.

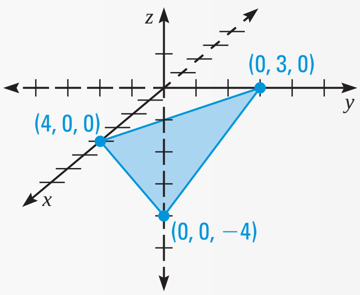
To graph $3x + 4y - 3z = 12$, find x-, y-, and z-intercepts.

If $y = 0$ and $z = 0$, then $x = 4$. Plot $(4, 0, 0)$.

If $x = 0$ and $z = 0$, then $y = 3$. Plot $(0, 3, 0)$.

If $x = 0$ and $y = 0$, then $z = -4$. Plot $(0, 0, -4)$.

Draw the plane that contains $(4, 0, 0)$, $(0, 3, 0)$, and $(0, 0, -4)$.



Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

17. $x + y + z = 5$

18. $5x + 3y + 6z = 30$

19. $3x + 6y - 4z = -12$

3.6 SOLVING SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

Examples on pp. 177–180

EXAMPLE You can use algebraic methods to solve a system of linear equations in three variables. First rewrite it as a system in two variables.

1 Add the first and second equations.

$$\begin{aligned} x - 3y + z &= 22 \\ 2x - 2y - z &= -9 \\ x + y + 3z &= 24 \end{aligned} \qquad \begin{aligned} x - 3y + z &= 22 \\ 2x - 2y - z &= -9 \\ \hline 3x - 5y &= 13 \end{aligned}$$

2 Multiply the second equation by 3 and add to the third equation.

$$\begin{aligned} 6x - 6y - 3z &= -27 \\ x + y + 3z &= 24 \\ \hline 7x - 5y &= -3 \end{aligned}$$

3 Solve the new system.

$$\begin{aligned} 3x - 5y &= 13 \\ -7x + 5y &= 3 \\ \hline -4x &= 16 \end{aligned}$$

$x = -4$ and $y = -5$

When you substitute $x = -4$ and $y = -5$ into one of the original equations, you get the value of the last variable: $z = 11$.

Solve the system using any algebraic method.

20. $x + 2y - z = 3$
$-x + y + 3z = -5$
$3x + y + 2z = 4$

21. $2x - 4y + 3z = 1$
$6x + 2y + 10z = 19$
$-2x + 5y - 2z = 2$

22. $x + y + z = 3$
$x + y - z = 3$
$2x + 2y + z = 6$

CHAPTER 3

Chapter Test

Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.

1. $x + y = 1$
$2x - 3y = 12$

2. $y = -\frac{1}{3}x + 4$
$y = 6$

3. $y = 2x + 2$
$y = 2x - 3$

4. $\frac{1}{2}x + 5y = 2$
$-x - 10y = -4$

Solve the system using any algebraic method.

5. $3x + 6y = -9$
$x + 2y = -3$

6. $x - y = -5$
$x + y = 11$

7. $7x + y = -17$
$3x - 10y = 24$

8. $8x + 3y = -2$
$-5x + y = -3$

Graph the system of linear inequalities.

9. $2x + y \geq 1$
$x \leq 3$

10. $x \geq 0$
$y < x$
$y > -x$

11. $x + 2y \geq -6$
$x + 2y \leq 2$
$y \geq -1$

12. $x + y < 7$
$2x - y \geq 5$
$x \geq -2$

Find the minimum and maximum values of the objective function subject to the given constraints.

13. **Objective function:** $C = 7x + 4y$
Constraints: $x \geq 0$
$y \geq 0$
$4x + 3y \leq 24$

14. **Objective function:** $C = 3x + 4y$
Constraints: $x + y \leq 10$
$-x + y \leq 5$
$2x + 4y \leq 32$

Plot the ordered triple in a three-dimensional coordinate system.

15. $(-1, 3, 2)$
16. $(0, 4, -2)$
17. $(-5, -1, 2)$
18. $(6, -2, 1)$

Sketch the graph of the equation. Label the points where the graph crosses the *x*-, *y*-, and *z*-axes.

19. $2x + 3y + 5z = 30$
20. $4x + y + 2z = 8$
21. $3x + 12y - 6z = 24$

22. Write the linear equation $2x - 5y + z = 9$ as a function of x and y. Then evaluate the function when $x = 10$ and $y = 3$.

Solve the system using any algebraic method.

23. $x + 2y - 6z = 23$
$x + 3y + z = 4$
$2x + 5y - 4z = 24$

24. $x + y + 2z = 1$
$x - y + z = 0$
$3x + 3y + 6z = 4$

25. $x + 3y - z = 1$
$-4x - 2y + 5z = 16$
$7x + 10y + 6z = -15$

26. **CRAFT SUPPLIES** You are buying beads and string to make a necklace. The string costs \$1.50, a package of 10 decorative beads costs \$.50, and a package of 25 plain beads costs \$.75. You can spend only \$7.00 and you need 150 beads. How many packages of each type of bead should you buy?

27. **BUSINESS** An appliance store manager is ordering chest and upright freezers. One chest freezer costs \$250 and delivers a \$40 profit. One upright freezer costs \$400 and delivers a \$60 profit. Based on previous sales, the manager expects to sell at least 100 freezers. Total profit must be at least \$4800. Find the least number of each type of freezer the manager should order to minimize costs.