3.1 Solving Linear Systems by Graphing

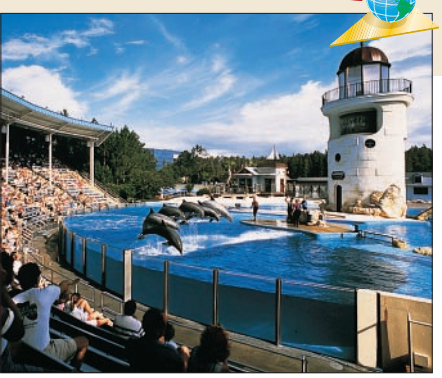
What you should learn

GOAL 1 Graph and solve systems of linear equations in two variables.

GOAL 2 Use linear systems to solve **real-life** problems, such as choosing the least expensive long-distance telephone service in **Ex. 64**.

Why you should learn it

▼ To solve **real-life** problems, such as how to stay within a budget on a vacation in Florida in **Example 4**.



GOAL 1 GRAPHING AND SOLVING A SYSTEM

A **system of two linear equations** in two variables x and y consists of two equations of the following form.

$$Ax + By = C \quad \text{Equation 1}$$
$$Dx + Ey = F \quad \text{Equation 2}$$

A **solution** of a system of linear equations in two variables is an ordered pair (x, y) that satisfies each equation.

EXAMPLE 1 Checking Solutions of a Linear System

Check whether **(a)** (2, 2) and **(b)** (0, −1) are solutions of the following system.

$$3x - 2y = 2 \quad \text{Equation 1}$$
$$x + 2y = 6 \quad \text{Equation 2}$$

SOLUTION

a. $3(2) - 2(2) = 2$ ✓ Equation 1 checks.

$2 + 2(2) = 6$ ✓ Equation 2 checks.

▶ Since (2, 2) is a solution of each equation, it is a solution of the system.

b. $3(0) - 2(-1) = 2$ ✓ Equation 1 checks.

$0 + 2(-1) = -2 \neq 6$ Equation 2 does not check.

▶ Since (0, −1) is not a solution of Equation 2, it is not a solution of the system.

EXAMPLE 2 Solving a System Graphically

Solve the system.

$$2x - 3y = 1 \quad \text{Equation 1}$$
$$x + y = 3 \quad \text{Equation 2}$$

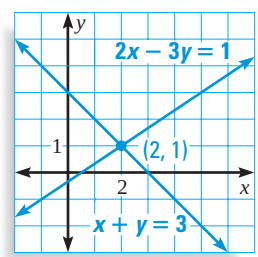
SOLUTION

Begin by graphing both equations as shown at the right. From the graph, the lines appear to intersect at (2, 1). You can check this algebraically as follows.

$2(2) - 3(1) = 1$ ✓ Equation 1 checks.

$2 + 1 = 3$ ✓ Equation 2 checks.

▶ The solution is (2, 1).



The system in Example 2 has exactly one solution. It is also possible for a system of linear equations to have infinitely many solutions or no solution.

EXAMPLE 3 *Systems with Many or No Solutions*

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

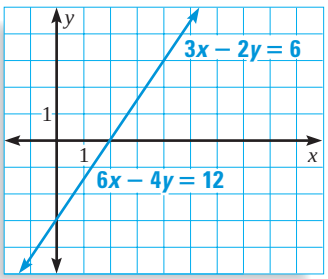
Tell how many solutions the linear system has.

a. $3x - 2y = 6$
$6x - 4y = 12$

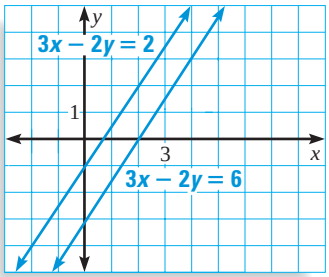
b. $3x - 2y = 6$
$3x - 2y = 2$

SOLUTION

a. The graph of the equations is the same line. So, each point on the line is a solution and the system has infinitely many solutions.



b. The graphs of the equations are two parallel lines. Because the two lines have no point of intersection, the system has no solution.

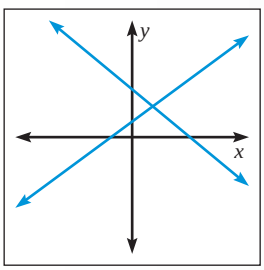


CONCEPT SUMMARY

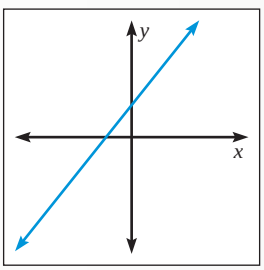
NUMBER OF SOLUTIONS OF A LINEAR SYSTEM

The relationship between the graph of a linear system and the system's number of solutions is described below.

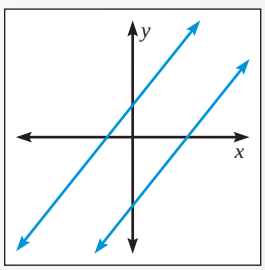
GRAPHICAL INTERPRETATION	ALGEBRAIC INTERPRETATION
The graph of the system is a pair of lines that intersect in one point.	The system has exactly one solution.
The graph of the system is a single line.	The system has infinitely many solutions.
The graph of the system is a pair of parallel lines so that there is no point of intersection.	The system has no solution.



Exactly one solution



Infinitely many solutions



No solution

GOAL 2 USING LINEAR SYSTEMS IN REAL LIFE

EXAMPLE 4 *Writing and Using a Linear System*

VACATION COSTS Your family is planning a 7 day trip to Florida. You estimate that it will cost \$275 per day in Tampa and \$400 per day in Orlando. Your total budget for the 7 days is \$2300. How many days should you spend in each location?

SOLUTION

You can use a verbal model to write a system of two linear equations in two variables.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Time spent in Tampa + Time spent in Orlando = Total vacation time

Daily rate in Tampa · Time spent in Tampa + Daily rate in Orlando · Time spent in Orlando = Total 7 day budget

LABELS

Equation 1

Time spent in Tampa = x (days)

Time spent in Orlando = y (days)

Total vacation time = 7 (days)

Equation 2

Daily rate in Tampa = 275 (dollars per day)

Time spent in Tampa = x (days)

Daily rate in Orlando = 400 (dollars per day)

Time spent in Orlando = y (days)

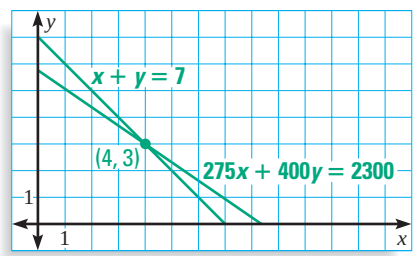
Total 7 day budget = 2300 (dollars)

ALGEBRAIC MODEL

Equation 1: $x + y = 7$ Total vacation time

Equation 2: $275x + 400y = 2300$ Total 7 day budget

To solve the system, graph each equation as shown at the right.



Notice that you need to graph the equations only in the first quadrant because only positive values of x and y make sense in this situation.

The lines appear to intersect at the point (4, 3). You can check this algebraically as follows.

$4 + 3 = 7$ ✓ Equation 1 checks.

$275(4) + 400(3) = 2300$ ✓ Equation 2 checks.

▶ The solution is (4, 3), which means that you should plan to spend 4 days in Tampa and 3 days in Orlando.

FOCUS ON APPLICATIONS

VACATION COSTS The daily costs given in Example 4 take into account money spent at tourist attractions. *Amusement Business* estimates that a family of four would spend about \$188.50 at a theme park in Florida.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: A(n) ? of a system of linear equations in two variables is an ordered pair (x, y) that satisfies each equation.

Concept Check ✓

2. How can you use the graph of a linear system to decide how many solutions the system has?

3. Explain why a linear system in two variables cannot have exactly two solutions.

Skill Check ✓

Check whether the ordered pair (5, 6) is a solution of the system.

4. $-2x + 4y = -14$
   $3x + y = 21$

5. $7x - 2y = 23$
   $-x + 3y = 13$

6. $x + y = 11$
   $-x - y = -11$

Graph the linear system. How many solutions does it have?

7. $2x - y = 4$
   $-6x + 3y = -18$

8. $14x + 3y = 16$
   $7x - 5y = 34$

9. $21x - 7y = 7$
   $-3x + y = -1$

10. **SCHOOL OUTING** Your school is planning a 5 hour outing at the community park. The park rents bicycles for \$8 per hour and inline skates for \$6 per hour. The total budget per person is \$34. How many hours should students spend doing each activity?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 943.

CHECKING A SOLUTION **Check whether the ordered pair is a solution of the system.**

11. $(6, -1)$
    $4x - y = 25$
    $-3x - 2y = -16$

12. $(3, 0)$
    $-x + 2y = 3$
    $10x + y = 30$

13. $(-2, -8)$
    $2x - y = 52$
    $9x - y = -10$

14. $(-3, -5)$
    $-x - y = 8$
    $2x + 5y = -31$

15. $(-4, 1)$
    $-4x + 3y = 19$
    $5x - 7y = -27$

16. $(10, 8)$
    $-3x - y = -38$
    $-8x + 8y = -16$

17. $(1, -1)$
    $-3x + y = -4$
    $7x + 2y = -5$

18. $(-2, -7)$
    $5x - y = -3$
    $x + 3y = -23$

19. $(0, 2)$
    $17x + 8y = 16$
    $-x - 4y = 8$

GRAPH AND CHECK **Graph the linear system and estimate the solution. Then check the solution algebraically.**

20. $2x + y = 13$
    $5x - 2y = 1$

21. $x + 2y = 9$
    $-x + 6y = -1$

22. $-2x + y = 5$
    $x + y = 2$

23. $3x + 4y = -10$
    $-7x - y = -10$

24. $2x + y = -11$
    $-6x - 3y = 33$

25. $y = 5x$
    $y = x + 4$

26. $-x + 3y = 3$
    $2x - 6y = -6$

27. $2x + y = -2$
    $x - 2y = 19$

28. $3x - y = 12$
    $-x + 8y = -4$

29. $3x - y = 8$
    $\frac{1}{3}x - \frac{1}{6}y = 1$

30. $y = \frac{1}{6}x - 2$
    $y = -\frac{1}{6}x + 2$

31. $-x + 4y = 10$
    $4x - y = -10$

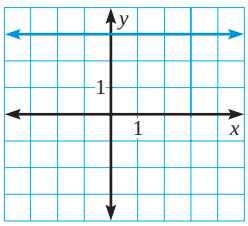
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 11–19
Example 2: Exs. 20–31
Example 3: Exs. 32–52
Example 4: Exs. 54–59

INTERPRETING A GRAPH **The graph of a system of two linear equations is shown. Tell whether the linear system has *infinitely many solutions*, *one solution*, or *no solution*. Explain your reasoning.**

32.



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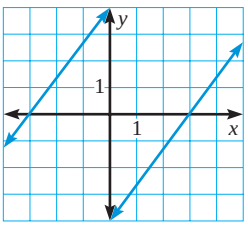

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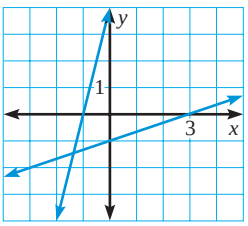


MATCHING GRAPHS **Match the linear system with its graph. Tell how many solutions the system has.**

35. $2x - y = -5$
$x + 2y = 0$

36. $-2x + 3y = 12$
$2x - 3y = 6$

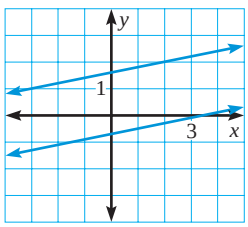
37. $2x - y = 5$
$-4x + 2y = -10$

38. $x + 5y = -12$
$x - 5y = 8$

39. $-x + 5y = 8$
$2x - 10y = 7$

40. $4x - 7y = 27$
$-6x - 9y = -21$

A.



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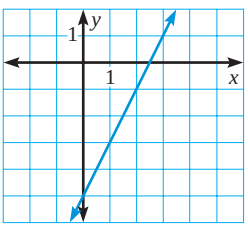

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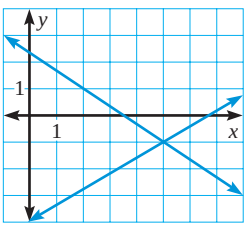

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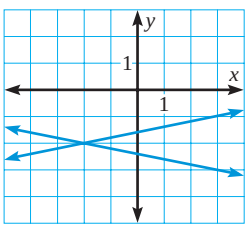

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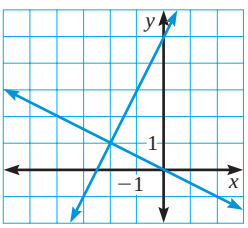

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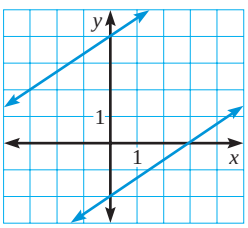


STUDENT HELP

Skills Review
For help with graphing, see p. 933.

NUMBER OF SOLUTIONS **Graph the linear system and tell how many solutions it has. If there is exactly one solution, estimate the solution and check it algebraically.**

41. $x = 5$
$x + y = 1$

42. $7x + y = 10$
$3x - 2y = -3$

43. $y = \frac{1}{2}x - 5$
$y = \frac{1}{2}x + 3$

44. $y = -5 - x$
$x + 3y = -15$

45. $\frac{1}{3}x + 7y = 2$
$\frac{2}{3}x + 14y = 4$

46. $-4y = 24x + 4$
$y = -6x - 1$

47. $2x - y = 7$
$y = 2x + 8$

48. $y = \frac{3}{4}x + 3$
$y = 3x - 6$

49. $6x - 2y = -2$
$-3x - 7y = 17$

50. $\frac{1}{2}x + 3y = 6$
$\frac{1}{3}x - 5y = -3$

51. $-6x + 2y = 8$
$y = 3x + 4$

52. $\frac{3}{4}x + y = 5$
$3x + 4y = 2$

FOCUS ON APPLICATIONS

PEDOMETER Worn at the hip, a pedometer counts the steps taken and multiplies by stride length to calculate the distance traveled. In Ex. 56 you would need to change the stride length setting when switching from walking to jogging.

53. CRITICAL THINKING Write a system of two linear equations that has the given number of solutions.

a. one solution **b.** no solution **c.** infinitely many solutions

54. BOOK CLUB You enroll in a book club in which you can earn bonus points to use toward the purchase of books. Each paperback book you order costs $6.95 and earns you 2 bonus points. Each hardcover book costs $19.95 and earns you 4 bonus points. The first order you place comes to a total of $60.75 and earns you 14 bonus points. How many of each type of book did you order? Use the verbal model to write and solve a system of linear equations.

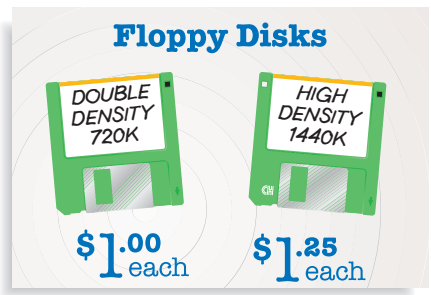
Price of paperback book	·	Number of paperback books	+	Price of hardcover book	·	Number of hardcover books	=	Total cost of order

Bonus points for paperback book	·	Number of paperback books	+	Bonus points for hardcover book	·	Number of hardcover books	=	Total number of bonus points

55. DECORATION COSTS You are on the prom decorating committee and are in charge of buying balloons. You want to use both latex and mylar balloons. The latex balloons cost $.10 each and the mylar balloons cost $.50 each. You need 125 balloons and you have $32.50 to spend. How many of each can you buy? Use a verbal model to write and solve a system of linear equations.

56. FITNESS For 30 minutes you do a combination of walking and jogging. At the end of your workout your pedometer displays a total of 2.5 miles. You know that you walk 0.05 mile per minute and jog 0.1 mile per minute. For how much time were you walking? For how much time were you jogging? Use a verbal model to write and solve a system of linear equations.

57. FLOPPY DISK STORAGE You want to copy some documents on your friend's computer. The documents use 6480 kilobytes(K) of disk space. You go to a store and see a sign advertising double-density disks and high-density disks. If you have $6 to spend, how many of each type of disk can you buy to get the disk space you need? Use a verbal model to write and solve a system of linear equations.



58. BATTERY POWER Your portable stereo requires 10 size D batteries. You have $25 to spend on 5 packages of 2 batteries each and would like to maximize your battery power. Each regular package of batteries costs $4.25 and each alkaline package of batteries costs $5.50 (because alkaline batteries last longer). How many packages of each type of battery should you buy? Use a verbal model to write and solve a system of linear equations.

59. AIRPORT SHUTTLE A bus station 15 miles from the airport runs a shuttle service to and from the airport. The 9:00 A.M. bus leaves for the airport traveling 30 miles per hour. The 9:05 A.M. bus leaves for the airport traveling 40 miles per hour. Write a system of linear equations to represent distance as a function of time for each bus. Graph and solve the system. How far from the airport will the 9:05 A.M. bus catch up to the 9:00 A.M. bus?

TYPES OF SYSTEMS **In Exercises 60–62, use the following definitions to tell whether the system is *consistent and independent, consistent and dependent,* or *inconsistent*.**

A system that has at least one solution is *consistent*. A consistent system that has exactly one solution is *independent*, and a consistent system that has infinitely many solutions is *dependent*. If a system has no solution, the system is *inconsistent*.

60. $-5x + 2y = 12$
$10x - 4y = -24$

61. $-3x - 3y = -6$
$7x + 4y = 20$

62. $2x - y = -12$
$-6x + 3y = 8$

63. **GEOMETRY CONNECTION** Graph the equations $x + y = 2$, $-5x + y = 20$, and $-\frac{5}{7}x + y = -\frac{10}{7}$. What geometric figure do the graphs of the equations form? What are the coordinates of the vertices of the figure? Explain the steps you used to find the coordinates.

Test Preparation

64. **MULTI-STEP PROBLEM** You are choosing between two long-distance telephone companies. Company A charges \$.09 per minute plus a \$4 monthly fee. Company B charges \$.11 per minute with no monthly fee.

a. Let x be the number of minutes you call long distance in one month, and let y be the total cost of long-distance phone service. Write and graph two equations representing the cost of each company's service for one month.

b. Estimate the coordinates of the point where the two graphs intersect. Check your estimate algebraically.

c. *Writing* What does the point of intersection you found in part (b) represent? How can it help you decide which long-distance company to use?

Challenge

65. **BUYING A DIGITAL CAMERA** The school yearbook staff is purchasing a digital camera. Recently the staff received two ads in the mail. The ad for Store 1 states that all digital cameras are 15% off. The ad for Store 2 gives a \$300 coupon to use when purchasing any digital camera. Assume that the lowest priced digital camera is \$700. Write and graph two equations that describe the prices at both stores. When does Store 1 have a better deal than Store 2?

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3 for 3.2)

66. $4x + 11 = 39$

67. $\frac{1}{2}x - 10 = 8$

68. $6x - 8 = 3x + 16$

69. $-9x - 2 = x + 1$

70. $2(3x - 5) = 7(x + 2)$

71. $10(x + 1) = \frac{1}{2}(x - 18)$

CHECKING SOLUTIONS **Check whether the ordered pairs are solutions of the inequality.** (Review 2.6)

72. $12x + 4y \geq 3$; $(1, -3)$, $(0, 2)$

73. $-x - y \leq -10$; $(-3, -7)$, $(5, 4)$

74. $15 > 2x - 2y$; $(10, 3)$, $(-5, 7)$

75. $6x + \frac{1}{2}y \leq -5$; $(2, -6)$, $(-1, 7)$

GRAPHING ABSOLUTE VALUE FUNCTIONS **Graph the function.** (Review 2.8)

76. $y = |x| - 5$

77. $y = |x - 9|$

78. $y = -|x - 8| + 3$

79. $y = |7 - x| + 4$