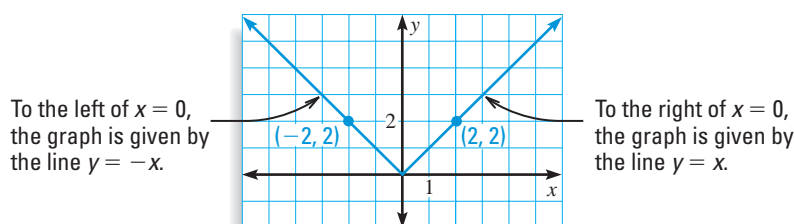# 2.8

Absolute Value Functions

GOAL ① **REPRESENTING ABSOLUTE VALUE FUNCTIONS**

In Lesson 1.7 you learned that the absolute value of x is defined by:

$$|x| = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x < 0 \end{cases}$$

The graph of this piecewise function consists of two rays, is V-shaped, and opens up. The corner point of the graph, called the **vertex**, occurs at the origin.

To the left of $x = 0$, the graph is given by the line $y = -x$.

$(-2, 2)$ $(2, 2)$

To the right of $x = 0$, the graph is given by the line $y = x$.

Notice that the graph of $y = |x|$ is symmetric in the y-axis because for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

> **► ACTIVITY**
> **Developing Concepts**
>
> ## Graphs of Absolute Value Functions
>
> **①** In the same coordinate plane, graph $y = a|x|$ for $a = -2, -\frac{1}{2}, \frac{1}{2}$, and 2. What effect does a have on the graph of $y = a|x|$? What is the vertex of the graph of $y = a|x|$?
>
> **②** In the same coordinate plane, graph $y = |x - h|$ for $h = -2, 0$, and 2. What effect does h have on the graph of $y = |x - h|$? What is the vertex of the graph of $y = |x - h|$?
>
> **③** In the same coordinate plane, graph $y = |x| + k$ for $k = -2, 0$, and 2. What effect does k have on the graph of $y = |x| + k$? What is the vertex of the graph of $y = |x| + k$?

Although in the activity you investigated the effects of a, h, and k on the graph of $y = a|x - h| + k$ separately, these effects can be combined. For example, the graph of $y = 2|x - 4| + 3$ is shown in red along with the graph of $y = |x|$ in blue. Notice that the vertex of the red graph is (4, 3) and that the red graph is narrower than the blue graph.
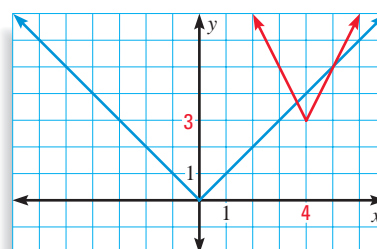
GRAPHING ABSOLUTE VALUE FUNCTIONS

The graph of $y = a|x - h| + k$ has the following characteristics.
- The graph has vertex (h, k) and is symmetric in the line $x = h$.
- The graph is V-shaped. It opens up if $a > 0$ and down if $a < 0$.
- The graph is wider than the graph of $y = |x|$ if $|a| < 1$.
 The graph is narrower than the graph of $y = |x|$ if $|a| > 1$.

To graph an absolute value function you may find it helpful to plot the vertex and one other point. Use symmetry to plot a third point and then complete the graph.

EXAMPLE 1 *Graphing an Absolute Value Function*

Graph $y = -|x + 2| + 3$.

SOLUTION

To graph $y = -|x + 2| + 3$, plot the vertex at $(-2, 3)$. Then plot another point on the graph, such as $(-3, 2)$. Use symmetry to plot a third point, $(-1, 2)$. Connect these three points with a V-shaped graph. Note that $a = -1 < 0$ and $|a| = 1$, so the graph opens down and is the same width as the graph of $y = |x|$.
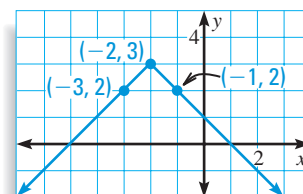
EXAMPLE 2 *Writing an Absolute Value Function*

Write an equation of the graph shown.

SOLUTION

The vertex of the graph is $(0, -3)$, so the equation has the form:

$$y = a|x - 0| + (-3) \quad \text{or} \quad y = a|x| - 3$$

To find the value of a, substitute the coordinates of the point $(2, 1)$ into the equation and solve.

$y = a\|x\| - 3$	**Write equation.**
$1 = a\|2\| - 3$	**Substitute 1 for *y* and 2 for *x*.**
$1 = 2a - 3$	**Simplify.**
$4 = 2a$	**Add 3 to each side.**
$2 = a$	**Divide each side by 2.**

▶ An equation of the graph is $y = 2|x| - 3$.

✓**CHECK** Notice the graph opens up and is narrower than the graph of $y = |x|$, so 2 is a reasonable value for a.
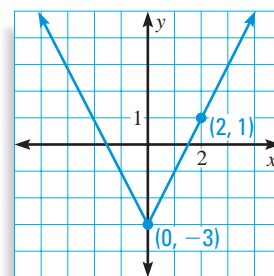
REAL LIFE

Camping

EXAMPLE 3 *Interpreting an Absolute Value Function*

The front of a camping tent can be modeled by the function

$$y = -1.4\,|x - 2.5| + 3.5$$

where x and y are measured in feet and the x-axis represents the ground.

a. Graph the function.

b. Interpret the domain and range of the function in the given context.

SOLUTION

a. The graph of the function is shown. The vertex is (2.5, 3.5) and the graph opens down. It is narrower than the graph of $y = |x|$.

b. The domain is $0 \le x \le 5$, so the tent is 5 feet wide. The range is $0 \le y \le 3.5$, so the tent is 3.5 feet tall.

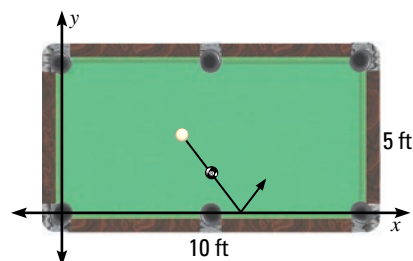**REAL LIFE**

Billiards
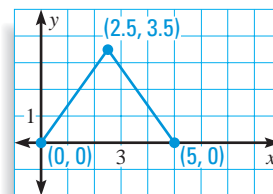
EXAMPLE 4 *Interpreting an Absolute Value Graph*

While playing pool, you try to shoot the eight ball into the corner pocket as shown. Imagine that a coordinate plane is placed over the pool table. The eight ball is at $\left(5, \dfrac{5}{4}\right)$ and the pocket you are aiming for is at (10, 5). You are going to bank the ball off the side at (6, 0).

a. Write an equation for the path of the ball.

b. Do you make your shot?

SOLUTION

a. The vertex of the path of the ball is (6, 0), so the equation has the form $y = a\,|x - 6|$. Substitute the coordinates of the point $\left(5, \dfrac{5}{4}\right)$ into the equation and solve for a.

$$\frac{5}{4} = a\,|5 - 6| \qquad \text{Substitute } \tfrac{5}{4} \text{ for } y \text{ and 5 for } x.$$

$$\frac{5}{4} = a \qquad\qquad \text{Solve for } a.$$

▶ An equation for the path of the ball is $y = \dfrac{5}{4}\,|x - 6|$.

b. You will make your shot if the point (10, 5) lies on the path of the ball.

$$5 \stackrel{?}{=} \frac{5}{4}\,|10 - 6| \qquad \text{Substitute 5 for } y \text{ and 10 for } x.$$

$$5 = 5 \checkmark \qquad\qquad \text{Simplify.}$$

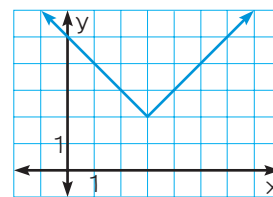▶ The point (10, 5) satisfies the equation, so you do make your shot.

GUIDED PRACTICE

Vocabulary Check ✓

1. What do the coordinates (h, k) represent on the graph of $y = a|x - h| + k$?

Concept Check ✓

2. How do you know if the graph of $y = a|x - h| + k$ opens up or down? How do you know if it is wider, narrower, or the same width as the graph of $y = |x|$?

3. **ERROR ANALYSIS** Explain why the graph shown is not the graph of $y = |x + 3| + 2$.

Ex. 3

Skill Check ✓

Graph the function. Then identify the vertex, tell whether the graph opens up or down, and tell whether the graph is wider, narrower, or the same width as the graph of $y = |x|$.

4. $y = \frac{1}{2}|x|$

5. $y = |x + 5|$

6. $y = |x| - 10$

7. $y = |x| + 5$

8. $y = 2|x + 6| - 10$

9. $y = -\left|x - \frac{1}{2}\right| - 14$

10. Write an equation for the function whose graph is shown.

11. 🌎 **CAMPING** Suppose that the tent in Example 3 is 7 feet wide and 5 feet tall. Write a function that models the front of the tent. Let the x-axis represent the ground. Then graph the function and identify the domain and range of the function.
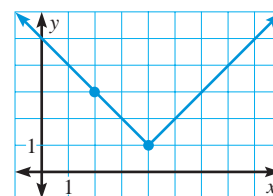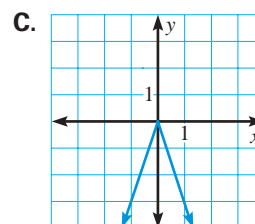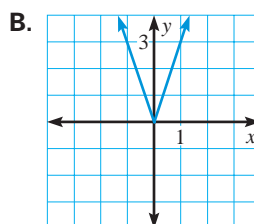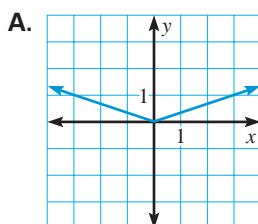
Ex. 10

PRACTICE AND APPLICATIONS

EXAMINING THE EFFECT OF a Match the function with its graph.

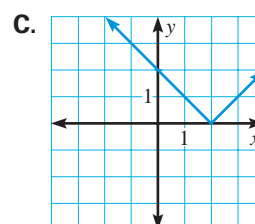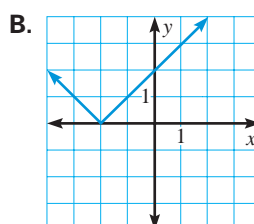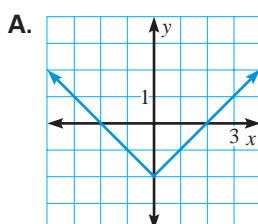12. $f(x) = 3|x|$

13. $f(x) = -3|x|$

14. $f(x) = \frac{1}{3}|x|$

A.

B.

C.

EXAMINING THE EFFECTS OF h AND k Match the function with its graph.

15. $y = |x - 2|$

16. $y = |x| - 2$

17. $y = |x + 2|$

A.

B.

C.

GRAPHING ABSOLUTE VALUE FUNCTIONS Graph the function. Then identify the vertex, tell whether the graph opens up or down, and tell whether the graph is wider, narrower, or the same width as the graph of $y = |x|$.

18. $y = 6|x - 7|$

19. $y = |x| + 9$

20. $y = -|x - 8| + 1$

21. $y = -|x + 2| + 11$

22. $y = \frac{1}{3}|x - 3| + 4$

23. $y = -2|x + 9| + 3$

24. $y = |x| - \frac{5}{2}$

25. $y = -\frac{1}{2}|x + 6|$

ABSOLUTE VALUE On many graphing calculators $|x|$ is denoted by ABS(x). Use a graphing calculator to graph the absolute value function. Then use the *Trace* feature to find the corresponding x-value(s) for the given y-value.

26. $y = |x| + 4; y = 10$

27. $y = |x + 14|; y = 9$

28. $y = 15|x|; y = \frac{3}{2}$
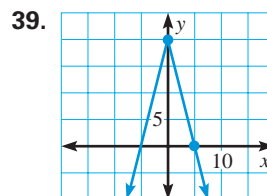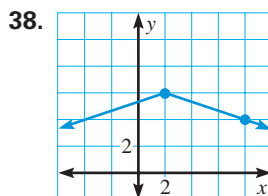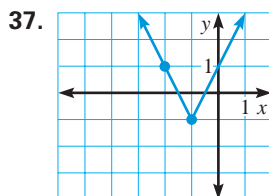
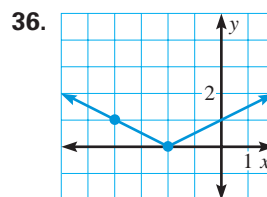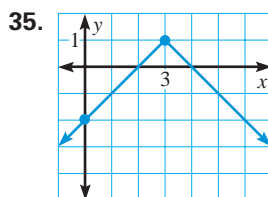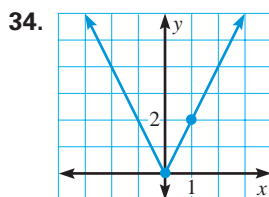**29.** $y = |x + \frac{4}{7}| - 5; y = 0$

30. $y = -|x - 2| + 5; y = 0.5$

31. $y = -3.2|x| + 7; y = -2$

32. $y = -3.75|x + 1.5| - 5; y = -5$

33. $y = 1.5|x - 3| + 6; y = 8.25$

WRITING EQUATIONS Write an equation of the graph shown.

34.

35.

36.

37.

38.

39.

MUSIC SINGLES In Exercises 40 and 41, use the following information.
A musical group's new single is released. Weekly sales s (in thousands) increase steadily for a while and then decrease as given by the function $s = -2|t - 20| + 40$ where t is the time (in weeks).

40. Graph the function.

41. What was the maximum number of singles sold in one week?

RAINSTORMS In Exercises 42 and 43, use the following information.
A rainstorm begins as a drizzle, builds up to a heavy rain, and then drops back to a drizzle. The rate r (in inches per hour) at which it rains is given by the function $r = -0.5|t - 1| + 0.5$ where t is the time (in hours).

42. Graph the function.

**43.** For how long does it rain and when does it rain the hardest?

🌐 **SOUND LEVELS** In Exercises 44 and 45, use the following information.
Suppose a musical piece calls for an orchestra to start at *fortissimo* (about 90 decibels), decrease in loudness to *pianissimo* (about 50 decibels) in four measures, and then increase back to *fortissimo* in another four measures. The sound level *s* (in decibels) of the musical piece can be modeled by the function $s = 10|m - 4| + 50$ where *m* is the number of measures.

44. Graph the function for $0 \le m \le 8$.

45. After how many measures should the orchestra be at the loudness of *mezzo forte* (about 70 decibels)?

46. 🌐 **MINIATURE GOLF** You are trying to make a hole-in-one on the miniature golf green shown. Imagine that a coordinate plane is placed over the golf green. The golf ball is at (2.5, 2) and the hole is at (9.5, 2). You are going to bank the ball off the side wall of the green at (6, 8). Write an equation for the path of the ball and determine if you make your shot.
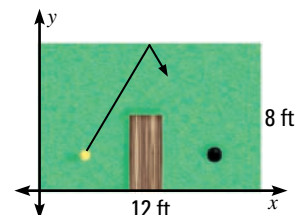
47. 🌐 **REFLECTING SUNLIGHT** You are sitting in a boat on a lake. You can get a sunburn from sunlight that hits you directly and from sunlight that reflects off the water. Sunlight reflects off the water at the point (2, 0) and hits you at the point (3.5, 3). Write and graph the function that shows the path of the sunlight.

**48.** 🌐 **TRANSAMERICA PYRAMID** The Transamerica Pyramid, shown at the right, is an office building in San Francisco. It stands 853 feet tall and is 145 feet wide at its base. Imagine that a coordinate plane is placed over a side of the building. In the coordinate plane, each unit represents one foot, and the origin is at the center of the building's base. Write an absolute value function whose graph is the V-shaped outline of the sides of the building, ignoring the "shoulders" of the building.
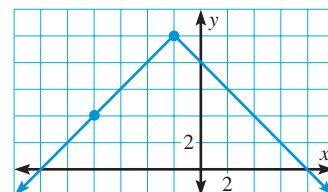
Test Preparation
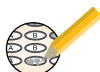
49. MULTIPLE CHOICE Which statement is true about the graph of the function $y = -|x + 2| + 3$?

Ⓐ Its vertex is at (2, 3).   Ⓑ Its vertex is at (−2, −3).

Ⓒ It opens down.   Ⓓ It is wider than the graph of $y = |x|$.

50. MULTIPLE CHOICE Which function is represented by the graph shown?

Ⓐ $y = -|x - 10| + 2$

Ⓑ $y = -|x + 10| - 2$

Ⓒ $y = -|x - 2| - 10$

Ⓓ $y = -|x + 2| + 10$

★ **Challenge**

GRAPHING Graph the functions.

51. $y = |2x|$ and $y = 2|x|$

52. $y = |-5x|$ and $y = 5|x|$

53. $y = |x + 6|$ and $y = |x| + 6$

54. $y = |x + (-3)|$ and $y = |x| + 3$

55. Based on your answers to Exercises 51–54, do you think $|ab| = |a| \cdot |b|$ and $|a + b| = |a| + |b|$ are true statements? Explain.

MIXED REVIEW

REWRITING EQUATIONS Solve the equation for *y*. **(Review 1.4)**

56. $3x - 5y = 8$ **57.** $6x + 2y = -9$ **58.** $-\frac{1}{5}x - \frac{3}{2}y = 1$

GRAPHING EQUATIONS Graph the equation. **(Review 2.3 for 3.1)**

59. $y = x - 5$ **60.** $y = 6x + 7$ **61.** $y = -\frac{1}{2}x + 10$

62. $x + y = 8$ **63.** $4x + y = 2$ **64.** $3x - y = -1$

FITTING A LINE TO DATA Draw a scatter plot of the data. Then approximate the best-fitting line for the data. **(Review 2.5)**

65.

x	−2	−1.5	−1	−0.5	0.5	1	1	1.5	2	2
y	−5	−3	−1	−2	1	−1	2	4	3	3

66.

x	−2	−1	0	0.5	1	2	2.5	3.5	4	4.5
y	5	3	3.5	1.5	2	0	−2	−3.5	−2	−3.5

QUIZ 3

Self-Test for Lessons 2.6–2.8

Graph the inequality in a coordinate plane. (Lesson 2.6)

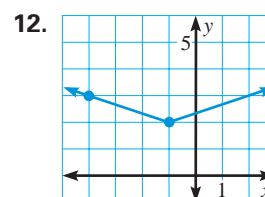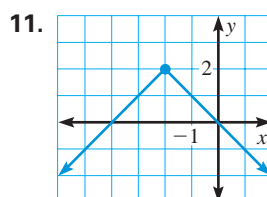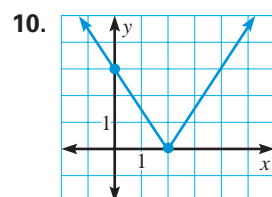**1.** $y \le -8$ **2.** $2x \ge -5$ **3.** $y > 3x - 4$ **4.** $2x + 5y < 15$

Evaluate the function for the given value of x. (Lesson 2.7)

5. $f(5)$ where $f(x) = \begin{cases} 3x + 9, & \text{if } x \le 3 \\ 2x - 3, & \text{if } x > 3 \end{cases}$ **6.** $f(0)$ where $f(x) = \begin{cases} 10, & \text{if } -1 \le x < 0 \\ 5, & \text{if } 0 \le x < 1 \\ 0, & \text{if } 1 \le x < 2 \end{cases}$

Graph the function. (Lesson 2.8)

7. $y = 3\lvert x - 2 \rvert$ **8.** $y = -\lvert x \rvert + 6$ **9.** $y = -5\lvert x + 3 \rvert - 8$

Write an equation of the graph shown. (Lesson 2.8)

10.

11.

12.

13. 🌐 **BEACH SNACKS** You and four friends have $15 to spend on snacks at the beach. A medium box of popcorn costs $2.50 and a medium soft drink costs $1.25. Write and graph an inequality that represents the numbers of medium boxes of popcorn and medium soft drinks you can buy. **(Lesson 2.6)**

14. 🌐 **RENTING A CAR** A local car rental company charges a weekly rate of $200 with 1000 free miles. Each additional mile is $.20. Write and graph a piecewise function that shows the car rental charge. If you drive 1200 miles in one week, how much will the rental car cost? **(Lesson 2.7)**