2.4 Writing Equations of Lines

GOAL 1 WRITING LINEAR EQUATIONS

What you should learn

GOAL 1 Write linear equations.

GOAL 2 Write direct variation equations, as applied in **Example 7**.

Why you should learn it

▼ To model **real-life** quantities, such as the number of calories you burn while dancing in **Ex. 64**.

REAL LIFE

In Lesson 2.3 you learned to find the slope and y-intercept of a line whose equation is given. In this lesson you will study the reverse process. That is, you will learn to write an equation of a line using one of the following: the slope and y-intercept of the line, the slope and a point on the line, or two points on the line.

CONCEPT SUMMARY — **WRITING AN EQUATION OF A LINE**

SLOPE-INTERCEPT FORM Given the slope m and the y-intercept b, use this equation:

$$y = mx + b$$

POINT-SLOPE FORM Given the slope m and a point (x_1, y_1), use this equation:

$$y - y_1 = m(x - x_1)$$

TWO POINTS Given two points (x_1, y_1) and (x_2, y_2), use the formula

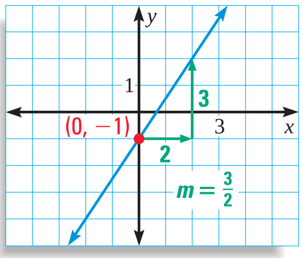
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to find the slope m. Then use the point-slope form with this slope and either of the given points to write an equation of the line.

Every nonvertical line has only one slope and one y-intercept, so the slope-intercept form is unique. The point-slope form, however, depends on the point that is used. Therefore, in this book equations of lines will be simplified to slope-intercept form so a unique solution may be given.

EXAMPLE 1 *Writing an Equation Given the Slope and the y-intercept*

Write an equation of the line shown.



SOLUTION

From the graph you can see that the slope is $m = \frac{3}{2}$. You can also see that the line intersects the y-axis at the point $(0, -1)$, so the y-intercept is $b = -1$.

Because you know the slope and the y-intercept, you should use the slope-intercept form to write an equation of the line.

$y = mx + b$ **Use slope-intercept form.**

$y = \frac{3}{2}x - 1$ **Substitute $\frac{3}{2}$ for m and -1 for b.**

▶ An equation of the line is $y = \frac{3}{2}x - 1$.

EXAMPLE 2 *Writing an Equation Given the Slope and a Point*

Write an equation of the line that passes through (2, 3) and has a slope of $-\frac{1}{2}$.

SOLUTION

Because you know the slope and a point on the line, you should use the point-slope form to write an equation of the line. Let $(x_1, y_1) = (2, 3)$ and $m = -\frac{1}{2}$.

$y - y_1 = m(x - x_1)$ **Use point-slope form.**

$y - 3 = -\frac{1}{2}(x - 2)$ **Substitute for m, x_1, and y_1.**

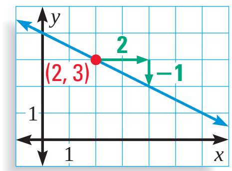
Once you have used the point-slope form to find an equation, you can simplify the result to the slope-intercept form.

$y - 3 = -\frac{1}{2}(x - 2)$ **Write point-slope form.**

$y - 3 = -\frac{1}{2}x + 1$ **Distributive property**

$y = -\frac{1}{2}x + 4$ **Write in slope-intercept form.**

✓**CHECK** You can check the result graphically. Draw the line that passes through the point (2, 3) with a slope of $-\frac{1}{2}$. Notice that the line has a y-intercept of 4, which agrees with the slope-intercept form found above.



EXAMPLE 3 *Writing Equations of Perpendicular and Parallel Lines*

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Write an equation of the line that passes through (3, 2) and is (**a**) perpendicular and (**b**) parallel to the line $y = -3x + 2$.

SOLUTION

a. The given line has a slope of $m_1 = -3$. So, a line that is perpendicular to this line must have a slope of $m_2 = -\frac{1}{m_1} = \frac{1}{3}$. Because you know the slope and a point on the line, use the point-slope form with $(x_1, y_1) = (3, 2)$ to find an equation of the line.

$y - y_1 = m_2(x - x_1)$ **Use point-slope form.**

$y - 2 = \frac{1}{3}(x - 3)$ **Substitute for m_2, x_1, and y_1.**

$y - 2 = \frac{1}{3}x - 1$ **Distributive property**

$y = \frac{1}{3}x + 1$ **Write in slope-intercept form.**

b. For a parallel line use $m_2 = m_1 = -3$ and $(x_1, y_1) = (3, 2)$.

$y - y_1 = m_2(x - x_1)$ **Use point-slope form.**

$y - 2 = -3(x - 3)$ **Substitute for m_2, x_1, and y_1.**

$y - 2 = -3x + 9$ **Distributive property**

$y = -3x + 11$ **Write in slope-intercept form.**

FOCUS ON PEOPLE

REAL LIFE **BARBARA JORDAN** was the first African-American woman elected to Congress from a southern state. She was a member of the House of Representatives from 1973 to 1979.

EXAMPLE 4 *Writing an Equation Given Two Points*

Write an equation of the line that passes through $(-2, -1)$ and $(3, 4)$.

SOLUTION

The line passes through $(x_1, y_1) = (-2, -1)$ and $(x_2, y_2) = (3, 4)$, so its slope is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{3 - (-2)} = \frac{5}{5} = 1$$

Because you know the slope and a point on the line, use the point-slope form to find an equation of the line.

$y - y_1 = m(x - x_1)$ **Use point-slope form.**

$y - (-1) = 1[x - (-2)]$ **Substitute for m, x_1, and y_1.**

$y + 1 = x + 2$ **Simplify.**

$y = x + 1$ **Write in slope-intercept form.**

EXAMPLE 5 *Writing and Using a Linear Model*

POLITICS In 1970 there were 160 African-American women in elected public office in the United States. By 1993 the number had increased to 2332. Write a linear model for the number of African-American women who held elected public office at any given time between 1970 and 1993. Then use the model to predict the number of African-American women who will hold elected public office in 2010.

INTERNET **DATA UPDATE** of Joint Center for Political and Economic Studies data at www.mcdougallittell.com

SOLUTION

The average rate of change in officeholders is $m = \frac{2332 - 160}{1993 - 1970} \approx 94.4$.

You can use the average rate of change as the slope in your linear model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Number of officeholders $=$ Number in 1970 $+$ Average rate of change $\cdot$ Years since 1970

LABELS

Number of officeholders $= y$ (people)

Number in 1970 $= \mathbf{160}$ (people)

Average rate of change $= \mathbf{94.4}$ (people per year)

Years since 1970 $= t$ (years)

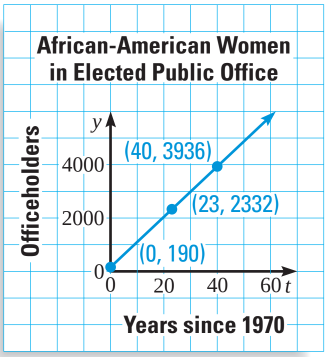
ALGEBRAIC MODEL

$y = \mathbf{160} + \mathbf{94.4}\, t$

In 2010, which is 40 years since 1970, you can predict that there will be

$$y = 160 + 94.4(40) \approx 3936$$

African-American women in elected public office. You can graph the model to check your prediction visually.



GOAL 2 WRITING DIRECT VARIATION EQUATIONS

Two variables x and y show **direct variation** provided $y = kx$ and $k \neq 0$. The nonzero constant k is called the **constant of variation**, and y is said to *vary directly* with x. The graph of $y = kx$ is a line through the origin.

EXAMPLE 6 *Writing and Using a Direct Variation Equation*

The variables x and y vary directly, and $y = 12$ when $x = 4$.

a. Write and graph an equation relating x and y. **b.** Find y when $x = 5$.

SOLUTION

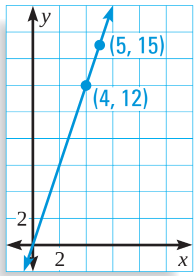
a. Use the given values of x and y to find the constant of variation.

$y = kx$ — **Write direct variation equation.**

$12 = k(4)$ — **Substitute 12 for *y* and 4 for *x*.**

$3 = k$ — **Solve for *k*.**

The direct variation equation is $y = 3x$. The graph of $y = 3x$ is shown.



b. When $x = 5$, the value of y is $y = 3(5) = 15$.

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The equation for direct variation can be rewritten as $\frac{y}{x} = k$. This tells you that a set of data pairs (x, y) shows direct variation if the quotient of y and x is constant.

REAL LIFE Jewelry

EXAMPLE 7 *Identifying Direct Variation*

Tell whether the data show direct variation. If so, write an equation relating x and y.

a.

14-karat Gold Chains (1 gram per inch)					
Length, *x* (inches)	16	18	20	24	30
Price, *y* (dollars)	288	324	360	432	540

b.

Loose Diamonds (round, colorless, very small flaws)					
Weight, *x* (carats)	0.5	0.7	1.0	1.5	2.0
Price, *y* (dollars)	2250	3430	6400	11,000	20,400

SOLUTION For each data set, check whether the quotient of y and x is constant.

a. For the 14-karat gold chains, $\frac{288}{16} = \frac{324}{18} = \frac{360}{20} = \frac{432}{24} = \frac{540}{30} = 18$. The data do show direct variation, and the direct variation equation is $y = 18x$.

b. For the loose diamonds, $\frac{2250}{0.5} = 4500$, but $\frac{3430}{0.7} = 4900$. The data do not show direct variation.

Guided Practice

Vocabulary Check ✓

1. Define the constant of variation for two variables x and y that vary directly.

Concept Check ✓

2. How can you find an equation of a line given the slope and the y-intercept of the line? given the slope and a point on the line? given two points on the line?

3. Give a real-life example of two quantities that vary directly.

Skill Check ✓

Write an equation of the line that has the given properties.

4. slope: $\frac{2}{5}$, y-intercept: 2
5. slope: 2, passes through $(0, -4)$
6. slope: -3, passes through $(5, 2)$
7. slope: $-\frac{3}{4}$, passes through $(-7, 0)$
8. passes through $(4, 8)$ and $(1, 2)$
9. passes through $(0, 2)$ and $(-5, 0)$
10. Write an equation of the line that passes through $(1, -6)$ and is perpendicular to the line $y = 3x + 7$.
11. Write an equation of the line that passes through $(3, 9)$ and is parallel to the line $y = 5x - 15$.
12. **Law of Supply** The *law of supply* states that the quantity supplied of an item varies directly with the price of that item. Suppose that for \$4 per tape 5 million cassette tapes will be supplied. Write an equation that relates the number c (in millions) of cassette tapes supplied to the price p (in dollars) of the tapes. Then determine how many cassette tapes will be supplied for \$5 per tape.

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 942.

Slope-Intercept Form **Write an equation of the line that has the given slope and y-intercept.**

13. $m = 5, b = -3$
14. $m = -3, b = -4$
15. $m = -4, b = 0$
16. $m = 0, b = 4$
17. $m = \frac{3}{5}, b = 6$
18. $m = -\frac{3}{4}, b = \frac{7}{3}$

Point-Slope Form **Write an equation of the line that passes through the given point and has the given slope.**

19. $(0, 4), m = 2$
20. $(1, 0), m = 3$
21. $(-6, 5), m = 0$
22. $(9, 3), m = -\frac{2}{3}$
23. $(3, -2), m = -\frac{4}{3}$
24. $(7, -4), m = \frac{2}{5}$
25. Write an equation of the line that passes through $(1, -1)$ and is perpendicular to the line $y = -\frac{1}{2}x + 6$.
26. Write an equation of the line that passes through $(6, -10)$ and is perpendicular to the line that passes through $(4, -6)$ and $(3, -4)$.
27. Write an equation of the line that passes through $(2, -7)$ and is parallel to the line $x = 5$.
28. Write an equation of the line that passes through $(4, 6)$ and is parallel to the line that passes through $(6, -6)$ and $(10, -4)$.

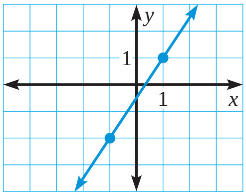
Student Help

Homework Help

Example 1: Exs. 13–18
Example 2: Exs. 19–24
Example 3: Exs. 25–28
Example 4: Exs. 29–40
Example 5: Exs. 59–62
Example 6: Exs. 43–54
Example 7: Exs. 55–58, 63–68

VISUAL THINKING **Write an equation of the line.**

29.



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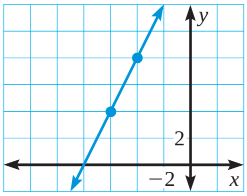

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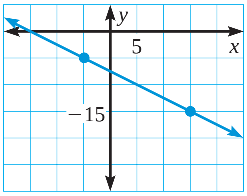

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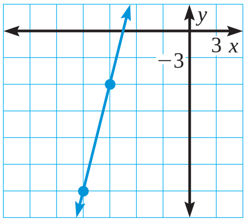

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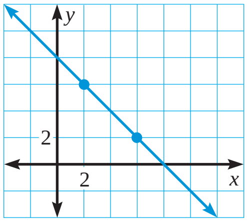

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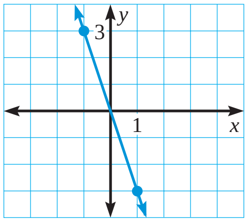


WRITING EQUATIONS **Write an equation of the line that passes through the given points.**

35. $(8, 5), (11, 14)$ **36.** $(-5, 9), (-4, 7)$ **37.** $(-8, 8), (0, 1)$

38. $(2, 0), (4, -6)$ **39.** $(-20, -10), (5, 15)$ **40.** $(-2, 0), (0, 6)$

41. LOGICAL REASONING Redo Example 2 by substituting the given point and slope into $y = mx + b$. Then solve for b to write an equation of the line. Explain why using this method does not change the equation of the line.

42. LOGICAL REASONING Redo Example 4 by substituting (3, 4) for (x_1, y_1) into $y - y_1 = m(x - x_1)$. Then rewrite the equation in slope-intercept form. Explain why using the point (3, 4) does not change the equation of the line.

RELATING VARIABLES **The variables *x* and *y* vary directly. Write an equation that relates the variables. Then find *y* when *x* = 8.**

43. $x = 2, y = 7$ **44.** $x = -6, y = 15$ **45.** $x = -3, y = 9$

46. $x = 24, y = 4$ **47.** $x = 1, y = \frac{1}{2}$ **48.** $x = 0.8, y = 1.6$

RELATING VARIABLES **The variables *x* and *y* vary directly. Write an equation that relates the variables. Then find *x* when *y* = −5.**

49. $x = 6, y = 3$ **50.** $x = 9, y = 15$ **51.** $x = -5, y = -1$

52. $x = 100, y = 2$ **53.** $x = \frac{5}{2}, y = \frac{5}{4}$ **54.** $x = -0.3, y = 2.2$

IDENTIFYING DIRECT VARIATION **Tell whether the data show direct variation. If so, write an equation relating *x* and *y*.**

55.

x	2	4	6	8	10
y	1	2	3	4	5

56.

x	1	2	3	4	5
y	5	4	3	2	1

57.

x	3	6	9	12	15
y	−3	−6	−9	−12	−15

58.

x	−5	−4	−3	−2	−1
y	10	8	6	4	2

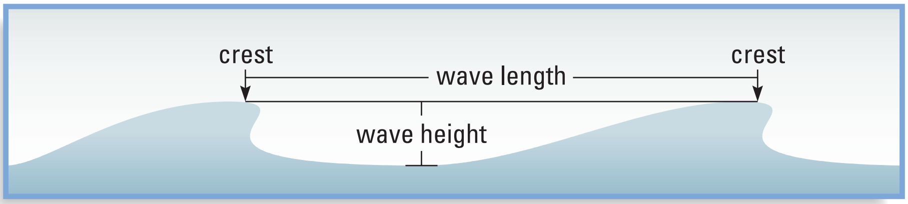
59. **POPULATION OF OREGON** From 1990 to 1996 the population of Oregon increased by about 60,300 people per year. In 1996 the population was about 3,204,000. Write a linear model for the population P of Oregon from 1990 to 1996. Let t represent the number of years since 1990. Then estimate the population of Oregon in 2014. ▶ Source: *Statistical Abstract of the United States*

60. **AIRFARE** In 1998 an airline offered a special airfare of $201 to fly from Cincinnati to Washington, D.C., a distance of 386 miles. Special airfares offered for longer flights increased by about $.138 per mile. Write a linear model for the special airfares a based on the total number of miles t of the flight. Estimate the airfare offered for a flight from Boston to Sacramento, a distance of 2629 miles.

61. **BOOKSTORE SALES** In 1990 retail sales at bookstores were about $7.4 billion. In 1997 retail sales at bookstores were about $11.8 billion. Write a linear model for retail sales s (in billions of dollars) at bookstores from 1990 through 1997. Let t represent the number of years since 1990. Then estimate the retail sales at bookstores in 2012. ▶ Source: American Booksellers Association

62. **SCIENCE CONNECTION** The velocity of sound in dry air increases as the temperature increases. At 40°C sound travels at a rate of about 355 meters per second. At 49°C it travels at a rate of about 360 meters per second. Write a linear model for the velocity v (in meters per second) of sound based on the temperature T (in degrees Celsius). Then estimate the velocity of sound at 60°C.
▶ Source: *CRC Handbook of Chemistry and Physics*

63. **BREAKING WAVES** The height h (in feet) at which a wave breaks varies directly with the wave length l (in feet), which is the distance from the crest of one wave to the crest of the next. A wave that breaks at a height of 4 feet has a wave length of 28 feet. Write a linear model that gives h as a function of l. Then estimate the wave length of a wave that breaks at a height of 5.5 feet.
▶ Source: Rhode Island Sea Grant



FOCUS ON APPLICATIONS

REAL LIFE **HAILSTONES** The largest hailstone ever recorded fell at Coffeyville, Kansas. It weighed 1.67 pounds and had a radius of about 2.75 inches.

64. **DANCING** The number C of calories a person burns performing an activity varies directly with the time t (in minutes) the person spends performing the activity. A 160 pound person can burn 73 Calories by dancing for 20 minutes. Write a linear model that gives C as a function of t. Then estimate how long a 160 pound person should dance to burn 438 Calories. ▶ Source: *Health Journal*

65. **HAILSTONES** Hailstones are formed when frozen raindrops are caught in updrafts and carried into high clouds containing water droplets. As a rule of thumb, the radius r (in inches) of a hailstone varies directly with the time t (in seconds) that the hailstone is in a high cloud. After a hailstone has been in a high cloud for 60 seconds, its radius is 0.25 inch. Write a linear model that gives r as a function of t. Then estimate how long a hailstone was in a high cloud if its radius measures 2.75 inches. ▶ Source: National Oceanic and Atmospheric Administration

66. **GEOMETRY CONNECTION** When the length of a rectangle is fixed, the area A (in square inches) of the rectangle varies directly with its width w (in inches). When the width of a particular rectangle is 12 inches, its area is 36 square inches. Write an equation that gives A as a function of w. Then find A when w is 7.5 inches.

STATISTICS CONNECTION **Tell whether the data show direct variation. If so, write an equation relating *x* and *y*.**

67.

Applesauce					
Ounces, *x*	8	16	24	36	48
Price, *y*	\$.89	\$1.25	\$1.39	\$2.09	\$2.49

68.

Fresh Apples					
Pounds, *x*	1	1.5	2	2.5	3
Price, *y*	\$.89	\$1.34	\$1.78	\$2.23	\$2.49

Test Preparation

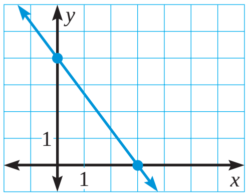
69. MULTI-STEP PROBLEM Besides slope-intercept and point-slope forms, another form that can be used to write equations of lines is *intercept form*: $\frac{x}{a} + \frac{y}{b} = 1$

a. Graph $\frac{x}{5} + \frac{y}{3} = 1.$

b. Graph $\frac{x}{-2} + \frac{y}{9} = 1.$

c. *Writing* Geometrically, what do a and b represent in the intercept form of a linear equation?

d. Write an equation of the line shown using intercept form.



e. Write an equation of the line with x-intercept -5 and y-intercept -8 using intercept form.

f. Write an equation of the line that passes through $(0, -3)$ and $(2, 0)$ using intercept form.

★ Challenge

70. SLOPE-INTERCEPT FORM Derive the slope-intercept form of a linear equation from the slope formula using $(0, b)$ as the coordinates of the point where the line crosses the y-axis and an arbitrary point (x, y).

MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** **(Review 1.7)**

71. $|x - 10| = 17$ **72.** $|7 - 2x| = 5$ **73.** $|-x - 9| = 1$

74. $|4x + 1| = 0.5$ **75.** $|22x + 6| = 9.2$ **76.** $|5.2x + 7| = 3.8$

FINDING SLOPE **Find the slope of the line passing through the given points.** **(Review 2.2 for 2.5)**

77. $(1, -7), (2, 7)$ **78.** $(-1, -1), (-5, -4)$ **79.** $(2, 4), (5, 10)$

80. $(5, -2), (-3, -1)$ **81.** $(-2, 4), (2, 4)$ **82.** $(-4, -1), (5, -4)$

83. $(0, -8), (-9, 10)$ **84.** $(6, 11), (6, -5)$ **85.** $(-11, 4), (-4, 11)$

GRAPHING EQUATIONS **Graph the equation.** **(Review 2.3 for 2.5)**

86. $y = \frac{3}{4}x - 5$ **87.** $y = -\frac{1}{5}x + 2$ **88.** $y = -\frac{3}{7}x + 2$

89. $3x + 7y = 42$ **90.** $2x - 8y = -15$ **91.** $-5x + 3y = 10$

92. $x = 0$ **93.** $y = -3$ **94.** $y = x$