

2.3

Quick Graphs of Linear Equations

What you should learn

GOAL 1 Use the slope-intercept form of a linear equation to graph linear equations.

GOAL 2 Use the standard form of a linear equation to graph linear equations, as applied in Example 5.

Why you should learn it

▼ To identify relationships between **real-life** variables, such as the sales of student and adult basketball tickets in Ex. 63.



GOAL 1 SLOPE-INTERCEPT FORM

In Lesson 2.1 you graphed a linear equation by creating a table of values, plotting the corresponding points, and drawing a line through the points. In this lesson you will study two quicker ways to graph a linear equation.

If the graph of an equation intersects the y -axis at the point $(0, b)$, then the number b is the **y-intercept** of the graph. To find the y -intercept of a line, let $x = 0$ in an equation for the line and solve for y .

ACTIVITY

Developing Concepts

Investigating Slope and y-intercept

Equation	Points on graph of equation	Slope	y-intercept
$y = 2x + 3$	$(0, ?), (1, ?)$?	?
$y = -x + 2$	$(0, ?), (1, ?)$?	?
$y = \frac{1}{2}x - 4$	$(0, ?), (1, ?)$?	?
$y = -2x$	$(0, ?), (1, ?)$?	?
$y = 7$	$(0, ?), (1, ?)$?	?

- 1 Copy and complete the table.
- 2 What do you notice about each equation and the slope of the line?
- 3 What do you notice about each equation and the y -intercept of the line?

The **slope-intercept form** of a linear equation is $y = mx + b$. As you saw in the activity, a line with equation $y = mx + b$ has slope m and y -intercept b .

GRAPHING EQUATIONS IN SLOPE-INTERCEPT FORM

The slope-intercept form of an equation gives you a quick way to graph the equation.

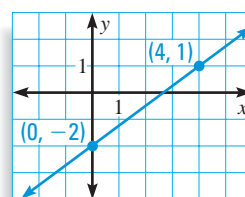
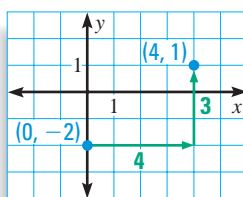
- STEP 1** Write the equation in slope-intercept form by solving for y .
- STEP 2** Find the y -intercept and use it to plot the point where the line crosses the y -axis.
- STEP 3** Find the slope and use it to plot a second point on the line.
- STEP 4** Draw a line through the two points.

EXAMPLE 1 *Graphing with the Slope-Intercept Form*

Graph $y = \frac{3}{4}x - 2$.

SOLUTION

- 1 The equation is already in slope-intercept form.
- 2 The y-intercept is -2 , so plot the point $(0, -2)$ where the line crosses the y-axis.
- 3 The slope is $\frac{3}{4}$, so plot a second point on the line by moving 4 units to the right and 3 units up. This point is $(4, 1)$.
- 4 Draw a line through the two points.



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In a real-life context the y-intercept often represents an initial amount and, as you saw in Lesson 2.2, the slope often represents a rate of change.

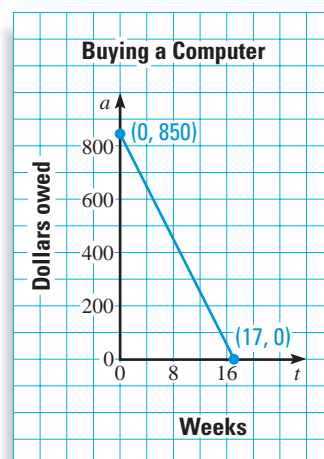
**EXAMPLE 2** *Using the Slope-Intercept Form*

You are buying an \$1100 computer on layaway. You make a \$250 deposit and then make weekly payments according to the equation $a = 850 - 50t$ where a is the amount you owe and t is the number of weeks.

- a. What is the original amount you owe on layaway?
- b. What is your weekly payment?
- c. Graph the model.

SOLUTION

- a. First rewrite the equation as $a = -50t + 850$ so that it is in slope-intercept form. Then you can see that the a -intercept is 850. So, the original amount you owe on layaway (the amount when $t = 0$) is \$850.
- b. From the slope-intercept form you can also see that the slope is $m = -50$. This means that the amount you owe is changing at a rate of $-\$50$ per week. In other words, your weekly payment is \$50.
- c. The graph of the model is shown. Notice that the line stops when it reaches the t -axis (at $t = 17$) so the computer is completely paid for at that point.



GOAL 2 STANDARD FORM

The **standard form** of a linear equation is $Ax + By = C$ where A and B are not both zero. A quick way to graph an equation in standard form is to plot its intercepts (when they exist). You found the y -intercept of a line in Goal 1. The **x -intercept** of a line is the x -coordinate of the point where the line intersects the x -axis.

GRAPHING EQUATIONS IN STANDARD FORM

The standard form of an equation gives you a quick way to graph the equation:

- STEP 1** Write the equation in standard form.
- STEP 2** Find the x -intercept by letting $y = 0$ and solving for x . Use the x -intercept to plot the point where the line crosses the x -axis.
- STEP 3** Find the y -intercept by letting $x = 0$ and solving for y . Use the y -intercept to plot the point where the line crosses the y -axis.
- STEP 4** Draw a line through the two points.

EXAMPLE 3 Drawing Quick Graphs

Graph $2x + 3y = 12$.

SOLUTION

Method 1 USE STANDARD FORM

- 1 The equation is already written in standard form.

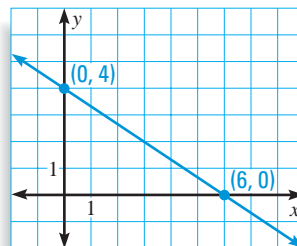
2 $2x + 3(0) = 12$ **Let $y = 0$.**
 $x = 6$ **Solve for x .**

The x -intercept is 6, so plot the point $(6, 0)$.

3 $2(0) + 3y = 12$ **Let $x = 0$.**
 $y = 4$ **Solve for y .**

The y -intercept is 4, so plot the point $(0, 4)$.

- 4 Draw a line through the two points.



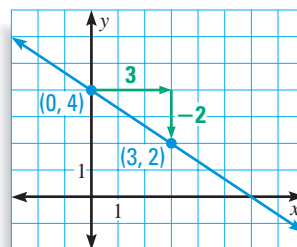
Method 2 USE SLOPE-INTERCEPT FORM

1 $2x + 3y = 12$
 $3y = -2x + 12$
 $y = -\frac{2}{3}x + 4$ **Slope-intercept form**

- 2 The y -intercept is 4, so plot the point $(0, 4)$.

- 3 The slope is $-\frac{2}{3}$, so plot a second point by moving 3 units to the right and 2 units down. This point is $(3, 2)$.

- 4 Draw a line through the two points.



STUDENT HELP

Look Back

For help with solving an equation for y , see p. 26.

The equation of a vertical line cannot be written in slope-intercept form because the slope of a vertical line is not defined. Every linear equation, however, can be written in standard form—even the equation of a vertical line.

HORIZONTAL AND VERTICAL LINES

HORIZONTAL LINES The graph of $y = c$ is a horizontal line through $(0, c)$.

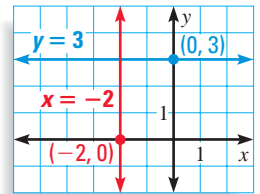
VERTICAL LINES The graph of $x = c$ is a vertical line through $(c, 0)$.

EXAMPLE 4 Graphing Horizontal and Vertical Lines

Graph (a) $y = 3$ and (b) $x = -2$.

SOLUTION

- The graph of $y = 3$ is a horizontal line that passes through the point $(0, 3)$. Notice that every point on the line has a y -coordinate of 3.
- The graph of $x = -2$ is a vertical line that passes through the point $(-2, 0)$. Notice that every point on the line has an x -coordinate of -2 .



EXAMPLE 5 Using the Standard Form

The school band is selling sweatshirts and T-shirts to raise money. The goal is to raise \$1200. Sweatshirts sell for a profit of \$2.50 each and T-shirts for \$1.50 each. Describe numbers of sweatshirts and T-shirts the band can sell to reach the goal.

SOLUTION

First write a model for the problem.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Profit per sweatshirt}} \cdot \boxed{\text{Number of sweatshirts}} + \boxed{\text{Profit per T-shirt}} \cdot \boxed{\text{Number of T-shirts}} = \boxed{\text{Total Profit}}$$

LABELS

Profit per sweatshirt = \$2.50

Number of sweatshirts = s

Profit per T-shirt = \$1.50

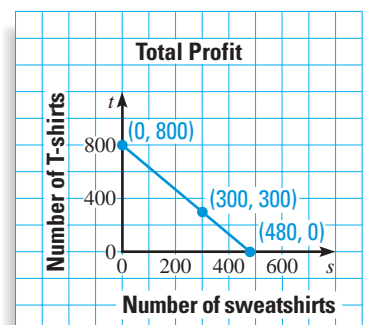
Number of T-shirts = t

Total profit = \$1200

ALGEBRAIC MODEL

$$2.5s + 1.5t = 1200$$

The graph of $2.5s + 1.5t = 1200$ is a line that intersects the s -axis at $(480, 0)$ and intersects the t -axis at $(0, 800)$. Points with integer coordinates on the line segment joining $(480, 0)$ and $(0, 800)$ represent ways to reach the goal. For instance, the band can sell 300 sweatshirts and 300 T-shirts.



STUDENT HELP

Study Tip

Finding the intercepts of a line before you draw the line can help you determine reasonable scales for the x -axis and the y -axis.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

- What are the slope-intercept and standard forms of a linear equation?
- Which of the two quick-graph techniques discussed in the lesson would you use to graph $y = -2x + 4$? Explain.
- Which of the two quick-graph techniques discussed in the lesson would you use to graph $3x + 4y = 24$? Explain.

Find the slope and y -intercept of the line.

- $y = x + 10$
- $y = -2x - 7$
- $2x - 3y = 18$

Find the intercepts of the line.

- $x - y = 11$
- $5x - 2y = 20$
- $y = 5x - 15$

Graph the equation.

- $y = 2x + 1$
- $y = \frac{1}{3}x - 4$
- $y = 7$
- $x = -5$
- $2x - 6y = 6$
- $5x + 3y = -15$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 941.

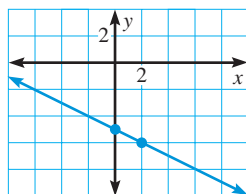
MATCHING GRAPHS Match the equation with its graph.

16. $y = -5x + 10$

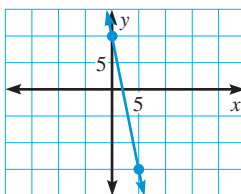
17. $y = -\frac{1}{2}x - 5$

18. $y = 4x - 12$

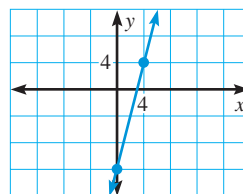
A.



B.



C.



USING SLOPE AND y -INTERCEPT Draw the line with the given slope and y -intercept.

19. $m = 3, b = -2$

20. $m = -2, b = 0$

21. $m = 1, b = 1$

22. $m = \frac{1}{2}, b = 5$

23. $m = 0, b = -7$

24. $m = -\frac{3}{7}, b = 14$

SLOPE-INTERCEPT FORM Graph the equation.

25. $y = -x + 5$

26. $y = 4x + 1$

27. $y = \frac{4}{5}x - 1$

28. $y = 2x - 3$

29. $y = -\frac{5}{2}x - 3$

30. $y = 5x - \frac{5}{2}$

FINDING SLOPE AND y -INTERCEPT Find the slope and y -intercept of the line.

31. $y = 6x + 10$

32. $y = -9x$

33. $y = 100$

34. $2x + y = 14$

35. $8x - 2y = 14$

36. $x + 10y = 7$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–36,
52–57

Example 2: Exs. 58–60

Example 3: Exs. 37–57

Example 4: Exs. 49–57

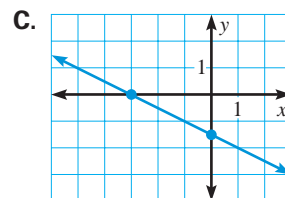
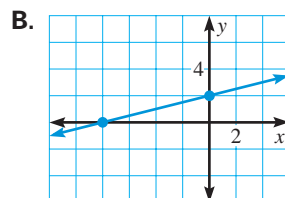
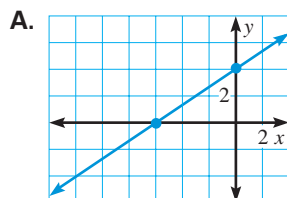
Example 5: Exs. 61–63

MATCHING GRAPHS Match the equation with its graph.

37. $x - 4y = -8$

38. $3x + 6y = -9$

39. $2x - 3y = -12$



USING INTERCEPTS Draw the line with the given intercepts.

40. x -intercept: 3
 y -intercept: 5

41. x -intercept: 2
 y -intercept: -6

42. x -intercept: -4
 y -intercept: $-\frac{1}{2}$

STANDARD FORM Graph the equation. Label any intercepts.

43. $2x + y = 8$

44. $x + 2y = 8$

45. $3x + 4y = -10$

46. $3x - y = 3$

47. $5x - 6y = -2$

48. $3x + 0.2y = 2$

49. $y = 6$

50. $x = -5$

51. $y = -\frac{1}{2}$

CHOOSE A METHOD Graph the equation using any method.

52. $y = 3x + 7$

53. $x = -10$

54. $2x - 7y = 14$

55. $y = \frac{3}{4}$

56. $5x + 10y = 30$

57. $y = \frac{5}{2}x - 2$

58. **IRS** The amount a (in billions of dollars) of annual taxes collected by the Internal Revenue Service can be modeled by $a = 57.1t + 488$ where t represents the number of years since 1980. Graph the equation.

► Source: *Statistical Abstract of the United States*

59. **PLACING AN AD** The cost C (in dollars) of placing a color advertisement in a newspaper can be modeled by $C = 7n + 20$ where n is the number of lines in the ad. Graph the equation. What do the slope and C -intercept represent?

60. **RAINFORESTS** The area A (in millions of hectares) of land covered by rainforests can be modeled by $A = 718.3 - 4.6t$ where t represents the number of years since 1990. Graph the equation. What are three predicted future areas of land covered by rainforests? ► Source: Food and Agriculture Organization

61. **CAR WASH** A car wash charges \$8 per wash and \$12 per wash-and-wax. After a busy day sales totaled \$3464. Use the verbal model to write an equation that shows the different numbers of washes and wash-and-waxes that could have been done. Then graph the equation.

Price per wash	•	Number of washes	+	Price per wash-and-wax	•	Number of wash-and-waxes	=	Total sales
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62. **SAILING** The owner of a sailboat takes passengers to an island 5 miles away to go snorkeling. A sailboat averages about 9 miles per hour when using its sails and about 14 miles per hour when using its motor. Write an equation that shows the numbers of *minutes* the sailboat can use its sails and its motor to get to the island. Then graph the equation.

FOCUS ON APPLICATIONS



RAINFORESTS

In Brazil the rate of rainforest destruction is 2.2 million hectares per year. Brazil recently passed a law giving its government the authority to protect forests.

STUDENT HELP**HOMEWORK HELP**

Visit our Web site
www.mcdougallittell.com
 for help with problem
 solving in Ex. 63.

Test Preparation



63. **TICKET PRICES** Student tickets at a high school basketball game cost \$2.50 each. Adult tickets cost \$6.00 each. The ticket sales at the first game of the season totaled \$7000. Write a model that shows the numbers of student and adult tickets that could have been sold. Then graph the model and determine three combinations of student and adult tickets that satisfy the model.
64. **Writing** Explain how to find the intercepts of a line if they exist. What kind of line has no x -intercept? What kind of line has no y -intercept?
65. **MULTIPLE CHOICE** You have an individual retirement account (IRA). The amount a you have deposited into your account after t years can be modeled by $a = 4500 + 2000t$. How much money do you put into your IRA every year?
- (A) \$1000 (B) \$2000 (C) \$2500 (D) \$4500 (E) \$6500
66. **MULTIPLE CHOICE** What is the slope-intercept form of $4x - 6y = 18$?
- (A) $x = \frac{3}{2}y + \frac{9}{2}$ (B) $y = \frac{2}{3}x - 3$ (C) $-y = \frac{4}{6}x + 3$
- (D) $6y = -4x + 18$ (E) $4x = 6y + 18$
67. **CALCULATING SLOPE** For the line $y = 7x + 6$, show that the slope is 7 regardless of the points (x_1, y_1) and (x_2, y_2) you use to calculate the slope. (Hint: Substitute x_1 and x_2 into the equation to obtain expressions for y_1 and y_2 .)

★ Challenge

MIXED REVIEW

SOLVING INEQUALITIES Solve the inequality. Then graph your solution.
 (Review 1.6)

68. $9 + x \leq 21$ 69. $-\frac{2}{3}x + 3 < 11$
70. $2x - 11 > 34 - x$ 71. $64 - 3x \geq 19 - 2x$
72. $-5 < 2x - 0.5 \leq 23$ 73. $x + 12 \leq 5$ or $3x - 21 \geq 0$

EVALUATING FUNCTIONS Evaluate the function for the given value of x .
 (Review 2.1)

74. $f(x) = \frac{1}{2}x - 13$; $f(8)$ 75. $f(x) = x^2 - 3x + 2$; $f(5)$
76. $f(x) = -x^3 + 8x^2 + 3$; $f(-7)$ 77. $f(x) = 10 - 2x$; $f(1)$
78. $f(x) = |x + 17|$; $f(-5)$ 79. $f(x) = 12x^2 - 19$; $f\left(\frac{1}{2}\right)$

FINDING SLOPE Find the slope of the line passing through the given points.
 (Review 2.2 for 2.4)

80. $(3, 2), (7, 2)$ 81. $(16, -3), (2, 9)$
82. $(-12, -9), (1, -8)$ 83. $(-1, -1), (-1, -5)$
84. $(5, -2), (-3, 2)$ 85. $(-4, 7), (2, -5)$

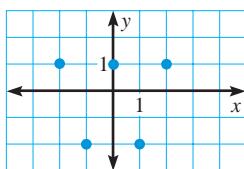
86. **READING SPEED** You can read a novel at a rate of 2 pages per minute. Write a model that shows the number of pages you can read in h hours. Then find how long it will take you to read a 1048 page novel. (Review 1.5 for 2.4)

QUIZ 1

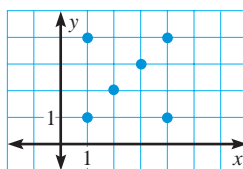
Self-Test for Lessons 2.1–2.3

Identify the domain and range. Then tell whether the relation is a function.
(Lesson 2.1)

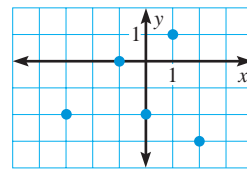
1.



2.



3.



Evaluate the function for the given value of x . (Lesson 2.1)

4. $f(x) = -2x - 13$; $f(4)$

5. $f(x) = 5x^2 - x + 9$; $f(-5)$

Tell whether the lines are *parallel*, *perpendicular*, or *neither*. (Lesson 2.2)

6. Line 1: through $(2, 10)$ and $(1, 5)$

7. Line 1: through $(4, 5)$ and $(9, -2)$

Line 2: through $(3, -7)$ and $(8, -8)$

Line 2: through $(6, -6)$ and $(-2, -1)$

Graph the equation. (Lesson 2.3)

8. $y = 3x + 5$

9. $2x - 3y = 10$

10. $y = -11$

11. **BICYCLING** There is an annual seven day bicycle ride across Iowa that covers about 468 miles. If a participant rides each day from 8:00 A.M. to 5:00 P.M., stopping only 1 hour for lunch, what is the rider's average speed in miles per hour? (Lesson 2.2)

MATH & History

Transatlantic Voyages



APPLICATION LINK

www.mcdougallittell.com

THEN

AT 2:00 P.M. ON APRIL 11, 1912, the *Titanic* left Cobh, Ireland, on her maiden voyage to New York City. At 11:40 P.M. on April 14, the *Titanic* struck an iceberg and sank, having covered only about 2100 miles of the approximately 3400 mile trip.



- What was the total length of the *Titanic*'s maiden voyage in hours?
- What was the *Titanic*'s average speed in miles per hour?
- Write an equation relating the *Titanic*'s distance from New York City and the number of hours traveled. Identify the domain and range.
- Graph the equation from Exercise 3.

NOW

TODAY, ocean liners still cross the Atlantic Ocean. The *Queen Elizabeth 2*, or *QE2*, is one of the fastest with a top speed of 32.5 knots (about 37 miles per hour).

Titanic's maiden voyage



1912



1927

Charles Lindbergh makes the first solo transatlantic flight.



1969

QE2's maiden voyage

Benoit Lecomte swims across the Atlantic.



1998