2.2 Slope and Rate of Change

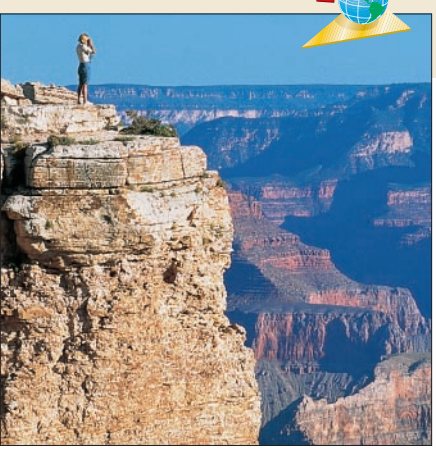
What you should learn

GOAL 1 Find slopes of lines and classify parallel and perpendicular lines.

GOAL 2 Use slope to solve **real-life** problems, such as how to safely adjust a ladder in **Example 5**.

Why you should learn it

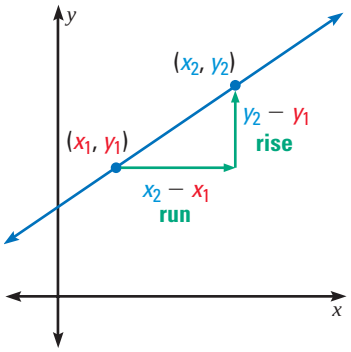
▼ To model **real-life** quantities, such as the average rate of change in the temperature of the Grand Canyon in **Ex. 52**.



GOAL 1 FINDING SLOPES OF LINES

The **slope** of a nonvertical line is the ratio of vertical change (the *rise*) to horizontal change (the *run*).

The slope of a line is represented by the letter m. Just as two points determine a line, two points are all that are needed to determine a line's slope. The slope of a line is the same regardless of which two points are used.



THE SLOPE OF A LINE

The slope of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

When calculating the slope of a line, be careful to subtract the coordinates in the correct order.

EXAMPLE 1 *Finding the Slope of a Line*

Find the slope of the line passing through $(-3, 5)$ and $(2, 1)$.

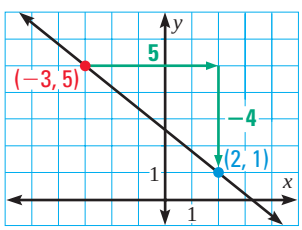
SOLUTION Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (2, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Rise: Difference of y-values ← Run: Difference of x-values

$$= \frac{1 - 5}{2 - (-3)}$$ Substitute values.

$$= \frac{-4}{2 + 3}$$ Simplify.

$$= -\frac{4}{5}$$ Simplify.



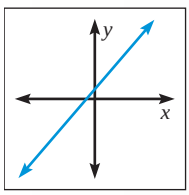
STUDENT HELP

Look Back
For help with evaluating expressions, see p. 12.

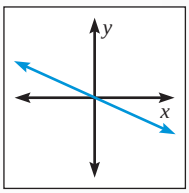
In Example 1 notice that the line *falls* from left to right and that the slope of the line is *negative*. This suggests one of the important uses of slope—to decide whether y decreases, increases, or is constant as x increases.

CONCEPT SUMMARY **CLASSIFICATION OF LINES BY SLOPE**

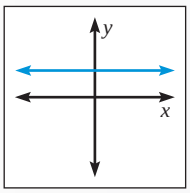
- A line with a *positive* slope *rises* from left to right. ($m > 0$)
- A line with a *negative* slope *falls* from left to right. ($m < 0$)
- A line with a slope of *zero* is *horizontal*. ($m = 0$)
- A line with an *undefined* slope is *vertical*. (m is undefined.)



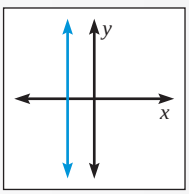
Positive slope



Negative slope



Zero slope



Undefined slope

EXAMPLE 2 *Classifying Lines Using Slope*

Without graphing tell whether the line through the given points *rises*, *falls*, *is horizontal*, or *is vertical*.

a. $(3, -4), (1, -6)$ **b.** $(2, -1), (2, 5)$

SOLUTION

a. $m = \dfrac{-6 - (-4)}{1 - 3} = \dfrac{-2}{-2} = 1$ Because $m > 0$, the line rises.

b. $m = \dfrac{5 - (-1)}{2 - 2} = \dfrac{6}{0}$ Because m is undefined, the line is vertical.

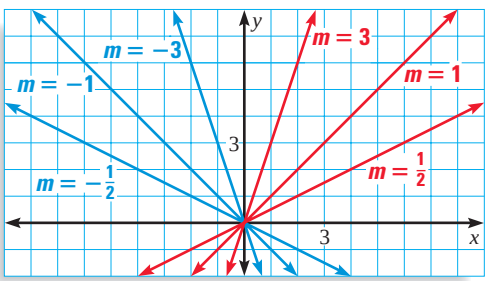
. . . . . .

STUDENT HELP

Study Tip

You can think of horizontal lines as "flat" and vertical lines as "infinitely steep."

The slope of a line tells you more than whether the line rises, falls, is horizontal, or is vertical. It also tells you the steepness of the line. For two lines with **positive slopes**, the line with the greater slope is steeper. For two lines with **negative slopes**, the line with the slope of greater absolute value is steeper.



EXAMPLE 3 *Comparing Steepness of Lines*

Tell which line is steeper.

Line 1: through $(2, 3)$ and $(4, 7)$ Line 2: through $(-1, 2)$ and $(4, 5)$

SOLUTION

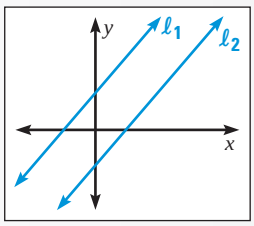
The slope of line 1 is $m_1 = \dfrac{7 - 3}{4 - 2} = 2$ and the slope of line 2 is $m_2 = \dfrac{5 - 2}{4 - (-1)} = \dfrac{3}{5}$.

▶ Because the lines have positive slopes and $m_1 > m_2$, line 1 is steeper than line 2.

Two lines in a plane are **parallel** if they do not intersect. Two lines in a plane are **perpendicular** if they intersect to form a right angle. Slope can be used to determine whether two different (nonvertical) lines are parallel or perpendicular.

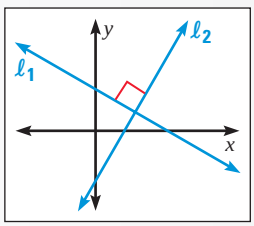
SLOPES OF PARALLEL AND PERPENDICULAR LINES

Consider two different nonvertical lines ℓ_1 and ℓ_2 with slopes m_1 and m_2.



PARALLEL LINES The lines are parallel if and only if they have the same slope.

$$m_1 = m_2$$



PERPENDICULAR LINES The lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$m_1 = -\frac{1}{m_2} \text{ or } m_1 m_2 = -1$$

EXAMPLE 4 *Classifying Parallel and Perpendicular Lines*

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Tell whether the lines are *parallel*, *perpendicular*, or *neither*.

a. Line 1: through $(-3, 3)$ and $(3, -1)$
Line 2: through $(-2, -3)$ and $(2, 3)$

b. Line 1: through $(-3, 1)$ and $(3, 4)$
Line 2: through $(-4, -3)$ and $(4, 1)$

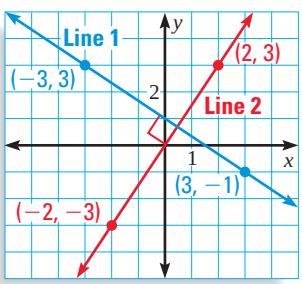
SOLUTION

a. The slopes of the two lines are:

$$m_1 = \frac{-1 - 3}{3 - (-3)} = \frac{-4}{6} = -\frac{2}{3}$$

$$m_2 = \frac{3 - (-3)}{2 - (-2)} = \frac{6}{4} = \frac{3}{2}$$

Because $m_1 m_2 = -\frac{2}{3} \cdot \frac{3}{2} = -1$, m_1 and m_2 are negative reciprocals of each other. Therefore, you can conclude that the lines are perpendicular.

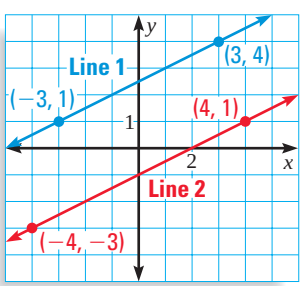


b. The slopes of the two lines are:

$$m_1 = \frac{4 - 1}{3 - (-3)} = \frac{3}{6} = \frac{1}{2}$$

$$m_2 = \frac{1 - (-3)}{4 - (-4)} = \frac{4}{8} = \frac{1}{2}$$

Because $m_1 = m_2$ (and the lines are different), you can conclude that the lines are parallel.



GOAL 2 USING SLOPE IN REAL LIFE

REAL LIFE *Ladder Safety*

EXAMPLE 5 *Geometrical Use of Slope*

In a home repair manual the following ladder safety guideline is given.

Adjust the ladder until the distance from the base of the ladder to the wall is at least one quarter of the height where the top of the ladder hits the wall. For example, a ladder that hits the wall at a height of 12 feet should have its base at least 3 feet from the wall.

a. Find the maximum recommended slope for a ladder.

b. Find the minimum distance a ladder's base should be from a wall if you need the ladder to reach a height of 20 feet.

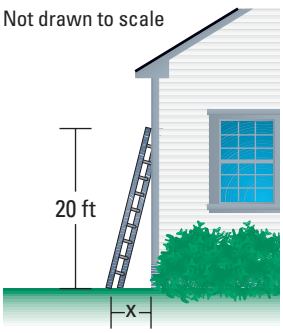
SOLUTION

a. A ladder that hits the wall at a height of 12 feet with its base about 3 feet from the wall has slope $m = \frac{\text{rise}}{\text{run}} = \frac{12}{3} = 4$. The maximum recommended slope is 4.

STUDENT HELP

Skills Review
For help with solving proportions, see p. 910.

b. Let x represent the minimum distance that the ladder's base should be from the wall for the ladder to safely reach a height of 20 feet.



$$\frac{\text{rise}}{\text{run}} = \frac{4}{1} \quad \text{Write a proportion.}$$

$$\frac{20}{x} = \frac{4}{1} \quad \text{The rise is 20 and the run is } x.$$

$$20 = 4x \quad \text{Cross multiply.}$$

$$5 = x \quad \text{Solve for } x.$$

▶ The ladder's base should be at least 5 feet from the wall.

FOCUS ON APPLICATIONS

REAL LIFE **DESERTS** Animals in the Mojave Desert must cope with extreme temperatures. Many reptiles burrow into the ground to escape high temperatures.

INTERNET APPLICATION LINK
www.mcdougallittell.com

In real-life problems slope is often used to describe an *average rate of change*. These rates involve units of measure, such as miles per hour or dollars per year.

EXAMPLE 6 *Slope as a Rate of Change*

DESERTS In the Mojave Desert in California, temperatures can drop quickly from day to night. Suppose the temperature drops from 100°F at 2 P.M. to 68°F at 5 A.M. Find the average rate of change and use it to determine the temperature at 10 P.M.

SOLUTION

$$\text{Average rate of change} = \frac{\text{Change in temperature}}{\text{Change in time}}$$

$$= \frac{68°\text{F} - 100°\text{F}}{5 \text{ A.M.} - 2 \text{ P.M.}} = \frac{-32°\text{F}}{15 \text{ hours}} \approx -2°\text{F per hour}$$

Because 10 P.M. is 8 hours after 2 P.M., the temperature changed $8(-2°\text{F}) = -16°\text{F}$. That means the temperature at 10 P.M. was about $100°\text{F} - 16°\text{F} = 84°\text{F}$.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe what is meant by the slope of a nonvertical line. Explain how your description relates to the definition of slope.

Concept Check ✓

2. What type of line has a slope of zero? What type of line has a slope that is undefined?

3. How can you decide, using slope, whether two nonvertical lines are parallel? whether two nonvertical lines are perpendicular?

Skill Check ✓

Find the slope of the line passing through the given points. Then tell whether the line *rises, falls, is horizontal,* or *is vertical*.

4. $(4, 2), (14, 3)$
5. $(8, 4), (8, 1)$
6. $(-3, 4), (3, -5)$
7. $(-2, 4), (-6, 8)$
8. $(-7, 3), (4, 3)$
9. $(6, 9), (-2, -7)$

Tell which line is steeper.

10. Line 1: through $(-5, 0)$ and $(3, 4)$
    Line 2: through $(0, 4)$ and $(1, 6)$
11. Line 1: through $(2, 4)$ and $(1, 7)$
    Line 2: through $(5, 2)$ and $(3, 12)$

Tell whether the lines are *parallel, perpendicular,* or *neither*.

12. Line 1: through $(1, 5)$ and $(-4, -2)$
    Line 2: through $(3, 0)$ and $(-2, -7)$
13. Line 1: through $(2, -2)$ and $(-2, 7)$
    Line 2: through $(4, -5)$ and $(5, 1)$
14. Line 1: through $(3, 6)$ and $(2, -1)$
    Line 2: through $(-1, 2)$ and $(6, 1)$
15. Line 1: through $(9, 0)$ and $(3, 4)$
    Line 2: through $(-5, 6)$ and $(4, 0)$

16. **AVERAGE SPEED** You are driving through Europe. At 9:00 A.M. you are 420 kilometers from Rome. At 3:00 P.M. you are 108 kilometers from Rome. Find your average speed.

PRACTICE AND APPLICATIONS

STUDENT HELP

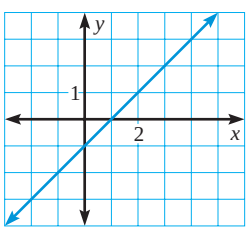
Extra Practice to help you master skills is on p. 941.

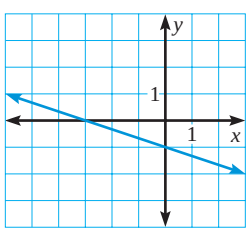
STUDENT HELP

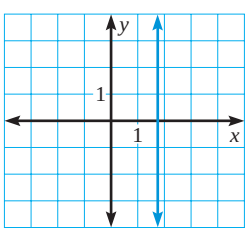
HOMEWORK HELP

Example 1: Exs. 17–31
Example 2: Exs. 20–31
Example 3: Exs. 32–35, 37–40
Example 4: Exs. 41–44
Example 5: Exs. 48–50
Example 6: Exs. 45–47, 51, 52

ESTIMATING SLOPE **Estimate the slope of the line.**

17. 

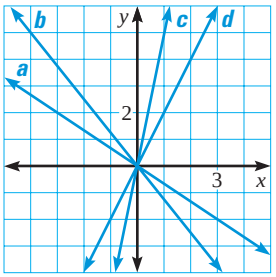
18. 

19. 

FINDING SLOPE **Find the slope of the line passing through the given points. Then tell whether the line *rises, falls, is horizontal,* or *is vertical*.**

20. $(3, 2), (-4, 3)$
21. $(1, -4), (2, 6)$
22. $(14, -3), (4, 11)$
23. $(-10, -12), (2, -6)$
24. $(-7, 3), (-2, 3)$
25. $(6, -6), (-6, 6)$
26. $(4, 2), (-18, 1)$
27. $(-9, 8), (-9, 2)$
28. $(3, 4), \left(2, -\frac{5}{4}\right)$
29. $\left(0, \frac{7}{2}\right), \left(2, \frac{5}{2}\right)$
30. $\left(\frac{1}{5}, -1\right), \left(\frac{3}{5}, -2\right)$
31. $\left(\frac{4}{3}, -\frac{9}{5}\right), \left(\frac{4}{3}, -\frac{8}{5}\right)$

MATCHING SLOPES AND LINES **Match the given slopes with the given lines.**



32. $-\frac{5}{4}$ **33.** 5

34. 2 **35.** $-\frac{2}{3}$

36. LOGICAL REASONING Use the formula for slope to verify that a horizontal line has a slope of zero and that a vertical line has an undefined slope.

DETERMINING STEEPNESS **Tell which line is steeper.**

37. Line 1: through $(-2, 6)$ and $(2, 8)$
Line 2: through $(0, -4)$ and $(5, -3)$

38. Line 1: through $(4, 1)$ and $(-8, 6)$
Line 2: through $(-2, 4)$ and $(-1, -8)$

39. Line 1: through $(3, -10)$ and $(2, -10)$
Line 2: through $(-6, 8)$ and $(2, 12)$

40. Line 1: through $(-5, 6)$ and $(-2, -9)$
Line 2: through $\left(1, \frac{1}{2}\right)$ and $\left(\frac{5}{4}, 1\right)$

TYPES OF LINES **Tell whether the lines are *parallel*, *perpendicular*, or *neither*.**

41. Line 1: through $(-1, 9)$ and $(-6, -6)$
Line 2: through $(-7, -23)$ and $(0, -2)$

42. Line 1: through $(4, -3)$ and $(-8, 1)$
Line 2: through $(5, 11)$ and $(8, 20)$

43. Line 1: through $(0, 3)$ and $(0, -7)$
Line 2: through $(-6, -4)$ and $(12, -4)$

44. Line 1: through $(1, 10)$ and $(5, 15)$
Line 2: through $\left(\frac{3}{2}, \frac{3}{2}\right)$ and $(4, 2)$

AVERAGE RATE OF CHANGE **Find the average rate of change in *y* for the given *xy*-pairs. State the unit of measure for the average rate of change.**

45. $(4, 3)$ and $(8, 27)$ x is measured in hours and y is measured in dollars

46. $(0, 5)$ and $(3, 17)$ x is measured in seconds and y is measured in meters

47. $(2, 10)$ and $(4, 16)$ x is measured in years and y is measured in inches

48. HISTORY CONNECTION Aqueducts were once used to carry water from rivers using gravity. Water flowing too quickly might damage an aqueduct, but water flowing too slowly might not keep the aqueduct clear. One of the best and most common designs for an aqueduct was to raise it 3 meters for every kilometer in length. What is the slope of an aqueduct built with this design?

▶ Source: *Roman Aqueducts and Water Supply*

49. LEANING TOWER OF PISA The top of the Leaning Tower of Pisa is about 55.9 meters above the ground. As of 1997 its top was leaning about 5.2 meters off-center. Approximate the slope of the tower.

▶ Source: Endex Engineering

50. PITCH OF A ROOF Building codes require the minimum slope, or pitch, of a roof with asphalt shingles to be such that it rises at least 4 feet for every 12 feet of horizontal distance. A 72 foot wide apartment building has a 12 foot high roof. Does it meet the building code? Explain.

51. OCEANOGRAPHY Loihi is the name of an underwater volcano that has formed twenty miles off the coast of Hawaii. The peak of the volcano is currently 3100 feet below sea level. Oceanographers estimate that it will take about 50,000 years before the peak breaks the water. If this holds true, what will be the rate of change in the volcano's height? Explain.

▶ Source: United States Geological Survey

FOCUS ON PEOPLE

REAL LIFE **JACQUES COUSTEAU** was famous for his work in oceanography, which is discussed in Ex. 51. Cousteau invented the aqua-lung, the one-man submarine, and the first underwater diving station.

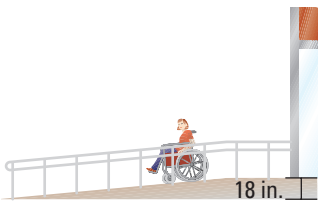
52. GRAND CANYON You are camping at the Grand Canyon. When you pitch your tent at 1:00 P.M. the temperature is 81°F. When you wake up at 6:00 A.M. the temperature is 47°F. What is the average rate of change in the temperature? Estimate the temperature when you went to sleep at 9:00 P.M.

53. CRITICAL THINKING Does it make a difference what two points on a line you choose when finding slope? Does it make a difference which point is (x_1, y_1) and which point is (x_2, y_2) in the formula for slope? Draw a line and calculate its slope using several pairs of points to support your answer.

Test Preparation

54. MULTI-STEP PROBLEM You are in charge of building a wheelchair ramp for a doctor's office. Federal regulations require that the ramp must extend 12 inches for every 1 inch of rise. The ramp needs to rise to a height of 18 inches.

▶ Source: *Uniform Federal Accessibility Standards*



STUDENT HELP

Skills Review
For help with the Pythagorean theorem, see p. 917.

a. How far should the end of the ramp be from the base of the building?

b. Use the Pythagorean theorem to determine the length of the ramp.

c. Some northern states require that outdoor ramps extend 20 inches for every 1 inch of rise because of the added problems of winter weather. Under this regulation, what should be the length of the ramp?

d. *Writing* How does changing the slope of the ramp affect the required length of the ramp?

Challenge

MISSING COORDINATES **Find the value of *k* so that the line through the given points has the given slope. Check your solution.**

55. $(5, k)$ and $(k, 7)$, $m = 1$

56. $(-3, 2k)$ and $(k, 6)$, $m = 4$

57. $(-2, k)$ and $(k, 4)$, $m = 3$

58. $(9, -k)$ and $(3k, -1)$, $m = -\frac{1}{3}$

MIXED REVIEW

IDENTIFYING PROPERTIES **Identify the property shown.** (Review 1.1)

59. $12 + (-12) = 0$

60. $(16 + 5) + 10 = 16 + (5 + 10)$

61. $8(2 + 13) = 8 \cdot 2 + 8 \cdot 13$

62. $22 \cdot \frac{1}{22} = 1$

REWRITING EQUATIONS **Solve the equation for *y*.** (Review 1.4 for 2.3)

63. $8x + y = 15$

64. $-2x - y = 11$

65. $\frac{8}{3}x + 2y = 16$

66. $-6y + \frac{4}{5}x = 10$

SOLVING EQUATIONS **Solve the equation.** (Review 1.7)

67. $|9 + 2x| = 7$

68. $|4 - 6x| = 2$

69. $|-3x + 1| = 4$

70. $|0.25x - 9| = 6$

71. MIXED NUTS A 16 ounce can of mixed nuts costs $5.82, but peanuts cost only $.25 per ounce. The can contains 7 ounces of peanuts and 9 ounces of other nuts. What is the cost per ounce of the other nuts? (Review 1.5 for 2.3)