CHAPTER 12

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Count the number of ways an event can happen.	
• using the fundamental counting principle **(12.1)**	Find the number of possible license plates. **(p. 702)**
• using permutations **(12.1)**	Find the number of ways skiers can finish in an Olympic event. **(p. 703)**
• using combinations **(12.2)**	Find the number of combinations of plays you can attend. **(p. 709)**
Expand a binomial that is raised to a power. **(12.2)**	Apply Pascal's triangle to algebra. **(p. 710)**
Find theoretical, experimental, and geometric probabilities. **(12.3)**	Find the probability that an archer hits the center of a target. **(p. 721)**
Find probabilities of unions and intersections of two events. **(12.4)**	Find the probability that it will rain on both Saturday and Sunday. **(p. 728)**
Use complements to find probabilities. **(12.4)**	Find the probability that friends will be in the same college dormitory. **(p. 729)**
Find probabilities of independent and dependent events. **(12.5)**	Find the probability that a baseball team wins three games in a row. **(p. 730)**
Find binomial probabilities and analyze binomial distributions. **(12.6)**	Find the most likely number of people who will give type O− blood. **(p. 743)**
Test a hypothesis. **(12.6)**	Test the claim that only 5% of computers will fail in a month. **(p. 743)**
Use normal distributions to calculate probabilities and to approximate binomial distributions. **(12.7)**	Find the probability that certain numbers of patients are nearsighted. **(p. 751)**
Use probability and statistics to solve real-life problems. **(12.1–12.7)**	Find the probability of winning a lottery. **(p. 720)**

How does Chapter 12 fit into the BIGGER PICTURE of algebra?

In this chapter you saw how algebra is used in probability and statistics. In fact, every branch of mathematics uses algebra. You can use what you have learned in this and other chapters to make everyday decisions.

STUDY STRATEGY

How did you connect to your life?

Here is an example of a connection, following the **Study Strategy** on page 700.

Connect to Your Life

I just got a bank card and chose my 4-digit personal identification number (PIN).

There are $10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$ different PINs possible. (fundamental counting principle)

The probability that someone who finds my card will guess my PIN on the first try is $\frac{1}{10{,}000}$. (theoretical probability)

CHAPTER 12

Chapter Review

VOCABULARY

- permutation, p. 703
- combination, p. 708
- Pascal's triangle, p. 710
- binomial theorem, p. 710
- probability, p. 716
- theoretical probability, p. 716
- experimental probability, p. 717
- geometric probability, p. 718
- compound event, p. 724
- mutually exclusive events, p. 724
- complement, p. 726
- independent events, p. 730
- dependent events, p. 732
- conditional probability, p. 732
- binomial experiment, p. 739
- binomial distribution, p. 739
- symmetric distribution, p. 740
- skewed distribution, p. 740
- hypothesis testing, p. 741
- normal curve, p. 746
- normal distribution, p. 746
- expected value, p. 753
- fair game, p. 753

12.1 THE FUNDAMENTAL COUNTING PRINCIPLE AND PERMUTATIONS

Examples on pp. 701–704

EXAMPLES You can use the fundamental counting principle and permutations to count the number of ways an event can happen.

The number of possible outfits you can make with 2 pairs of jeans and 5 shirts is:

$$2 \cdot 5 = 10 \text{ outfits}$$

The number of ways 4 members from a family of 5 can line up for a photo is:

$$_5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = \frac{120}{1} = 120$$

1. How many different 5-digit zip codes are there if any of the digits 0–9 can be used?
2. How many different ways can 4 friends stand in a cafeteria line?

Find the number of permutations.

3. $_6P_6$
4. $_8P_4$
5. $_5P_1$
6. $_9P_3$
7. $_{10}P_6$
8. $_4P_4$

12.2 COMBINATIONS AND THE BINOMIAL THEOREM

Examples on pp. 708–711

EXAMPLES You can use combinations to find the number of ways an event can happen when order is not important.

You must write reports on 3 of the 12 most recent Presidents of the United States for history class. The number of possible combinations of reports is:

$$_{12}C_3 = \frac{12!}{9! \cdot 3!} = \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!} \cdot 3!} = \frac{1320}{6} = 220$$

You can use the binomial theorem to expand a binomial raised to a power.

$$\begin{aligned}(x+6)^4 &= {}_4C_0x^46^0 + {}_4C_1x^36^1 + {}_4C_2x^26^2 + {}_4C_3x^16^3 + {}_4C_4x^06^4 \\ &= (1)(x^4)(1) + (4)(x^3)(6) + (6)(x^2)(36) + (4)(x)(216) + (1)(1)(1296) \\ &= x^4 + 24x^3 + 216x^2 + 864x + 1296\end{aligned}$$

Find the number of combinations.

9. ${}_9C_2$ **10.** ${}_7C_1$ **11.** ${}_5C_3$ **12.** ${}_8C_7$ **13.** ${}_{10}C_{10}$ **14.** ${}_{13}C_5$

Use the binomial theorem to write the binomial expansion.

15. $(x + 4)^3$ **16.** $(x - 10)^5$ **17.** $(x - 3y)^7$ **18.** $(2x + y^2)^4$

12.3 AN INTRODUCTION TO PROBABILITY

Examples on pp. 716–718

EXAMPLES You can find the probability that an event will occur.

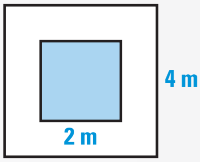
You toss two six-sided dice. The *theoretical* probability that the sum of the dice is 4 is

$$\frac{\text{number of ways sum can be 4}}{\text{number of possible outcomes}} = \frac{3}{36} = \frac{1}{12}.$$

You toss two 6-sided dice 100 times and record 8 times that the sum is 4. The *experimental* probability that the sum of the dice is 4 is

$$\frac{\text{number of times sum is 4}}{\text{number of times dice are tossed}} = \frac{8}{100} = \frac{2}{25}.$$

A dart thrown at the square target shown is equally likely to hit any point inside the target. The *geometric* probability that the dart hits the shaded square is $\frac{\text{area of shaded square}}{\text{area of entire target}} = \frac{4}{16} = \frac{1}{4}.$



You toss a coin 3 times. Find the probability of the given event.

19. You toss exactly 1 tail.

20. You toss at least 1 tail.

21. You toss a coin 200 times and get heads 90 times. Find the experimental probability of getting heads. Compare this with the theoretical probability.

22. What is the probability that a dart hits the unshaded region of the target above?

12.4 PROBABILITY OF COMPOUND EVENTS

Examples on pp. 724–726

EXAMPLES You can find the probability that compound events will occur and the probability that the complement of an event will occur.

If A and B are two events and $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$, and $P(A \text{ and } B) = \frac{1}{4}$,

then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{3}{4} + \frac{2}{5} - \frac{1}{4} = \frac{18}{20} = \frac{9}{10}.$

The probability of the complement of A is $P(A') = 1 - P(A) = 1 - \frac{3}{4} = \frac{1}{4}.$

Find the indicated probability.

23. $P(A) = 0.25$, $P(B) = 0.2$, $P(A \text{ and } B) = 0.15$, $P(A \text{ or } B) = \underline{\ ?\ }$

24. $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{10}$, $P(A \text{ and } B) = \underline{\ ?\ }$, $P(A \text{ or } B) = \frac{1}{2}$

25. $P(A) = 99\%$, $P(A') = \underline{\ ?\ }$

12.5 PROBABILITY OF INDEPENDENT AND DEPENDENT EVENTS

Examples on pp. 730–733

EXAMPLES You can find the probability that independent events will occur and the probability that dependent events will occur.

Nine slips of paper numbered 1–9 are placed in a hat. You randomly draw two slips. What is the probability that the first number is odd (A) and the second is even (B)?

If you replace the first slip of paper before selecting the second, A and B are *independent* events, and $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{5}{9} \cdot \frac{4}{9} = \frac{20}{81} \approx 0.247$.

If you do not replace the first slip of paper before selecting the second, A and B are *dependent* events, and $P(A \text{ and } B) = P(A) \cdot P(B \mid A) = \frac{5}{9} \cdot \frac{4}{8} = \frac{20}{72} = \frac{5}{18} \approx 0.278$.

Find the probability of randomly drawing the given marbles from a bag of 4 red, 6 green, and 2 blue marbles (a) with replacement and (b) without replacement.

26. a red, then a green **27.** a blue, then a red **28.** a red, then a red

12.6 BINOMIAL DISTRIBUTIONS

Examples on pp. 739–741

EXAMPLE You can find the probability of getting exactly k successes for a binomial experiment.

The probability of tossing a coin 10 times and getting exactly 7 heads is:

$$P(k = 7) = {}_{10}C_7(0.5)^7(1 - 0.5)^3 = \frac{10!}{3! \cdot 7!}(0.5)^7(0.5)^3 \approx 0.117$$

Calculate the probability of tossing a coin 10 times and getting the given number of tails.

29. 3 **30.** 5 **31.** 9 **32.** 6 **33.** 1 **34.** 10

12.7 NORMAL DISTRIBUTIONS

Examples on pp. 746–748

EXAMPLE You can use normal distributions to approximate binomial distributions.

In 1990 about 1 in 43 births resulted in twins. If a town had 2157 births that year, what is the probability that between 29 and 50 of them were twins?

$$\bar{x} = np = 2157\left(\frac{1}{43}\right) \approx 50 \text{ and } \sigma = \sqrt{np(1-p)} = \sqrt{(2157)\left(\frac{1}{43}\right)\left(\frac{42}{43}\right)} \approx 7$$

So, $P(29 \le x \le 50) = P(\bar{x} - 3\sigma \le x \le \bar{x}) = 0.0235 + 0.135 + 0.34 = 0.4985$, referring to the diagram on page 746.

A binomial distribution consists of 100 trials with probability 0.9 of success. Approximate the probability of getting the given numbers of successes.

35. between 87 and 93 **36.** greater than 90 **37.** less than 84 **38.** between 81 and 84

CHAPTER 12

Chapter Test

Find the number of permutations or combinations.

1. ${}_{4}P_{3}$ **2.** ${}_{11}P_{5}$ **3.** ${}_{14}P_{2}$ **4.** ${}_{9}C_{6}$ **5.** ${}_{17}C_{3}$ **6.** ${}_{5}C_{4}$

7. Find the number of distinguishable permutations of the letters in MONTANA.

Expand the power of the binomial.

8. $(x + 4)^6$ **9.** $(2x - 2)^5$ **10.** $(x + 8)^3$ **11.** $(x^2 + 1)^4$ **12.** $(x + y^2)^5$ **13.** $(3x - y)^3$

A card is drawn randomly from a standard 52-card deck. Find the probability of drawing the given card. (For a listing of the deck, see page. 708.)

14. a black card **15.** an ace **16.** a black ace **17.** a king **18.** a heart **19.** the king of hearts

Find the indicated probability.

20. $P(A) = 80\%$
$P(B) = 20\%$
$P(A \text{ or } B) = 100\%$
$P(A \text{ and } B) = \underline{\ ?\ }$

21. $P(A) = \underline{\ ?\ }$
$P(B) = 0.7$
$P(A \text{ or } B) = 0.82$
$P(A \text{ and } B) = 0.05$

22. $P(A) = \frac{1}{4}$
$P(A') = \underline{\ ?\ }$

23. A and B are independent events.
$P(A) = 0.25$
$P(B) = 0.75$
$P(A \text{ and } B) = \underline{\ ?\ }$

24. A and B are dependent events.
$P(A) = 30\%$
$P(B \mid A) = 40\%$
$P(A \text{ and } B) = \underline{\ ?\ }$

25. A and B are dependent events.
$P(A) = \underline{\ ?\ }$
$P(B \mid A) = 0.8$
$P(A \text{ and } B) = 0.32$

26. Calculate the probability of randomly guessing at least 7 correct answers on a 10-question true-or-false quiz to get a passing grade.

27. What percent of the area under a normal curve lies within 1 standard deviation of the mean? What percent lies within 2 standard deviations of the mean?

28. **SCHOOL SHIRTS** A school shirt is available either long-sleeved or short-sleeved, in sizes small, medium, large, or extra large, and in one of two colors. How many different choices for a school shirt are there?

29. **SUPREME COURT** The Supreme Court of the United States has 9 justices. On a certain case the justices voted 5 to 4 in favor of the defendant. In how many ways could this have happened?

30. **ASTRONOMY** The surface area of Earth is about 197 million square miles. The land area is about 57 million square miles and the rest is water. What is the probability that a meteorite falling to Earth will hit land? What is the probability that it will hit water?

31. **EMPLOYMENT AGENCY** A temporary employment agency claims that it has a "no-show" rate of 1 out of 1000 workers. If fewer employees show up for a job than are requested, the difference is the number of "no-shows." A company hires the employment agency to supply 200 workers and only 198 show up. Would you reject the agency's claim about its "no-show" rate? Explain.

32. **HEALTH** Health officials who have studied a particular virus say that 50% of all Americans have had the virus. If a random sample of 144 people is taken, what is the probability that fewer than 60 have had the virus?