

## PROJECT

Applying Chapters  
10–12

# Monte Carlo Methods

**OBJECTIVE** Use a Monte Carlo method to estimate the area of an ellipse.

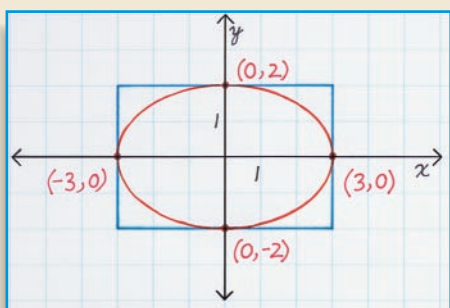
**Materials:** graph paper, calculator or computer

Some situations are too complex to be conveniently modeled by equations, graphs, or other standard mathematical tools. Even many simple-sounding problems, such as finding the average wait in line at an ATM machine, can be difficult to solve using traditional mathematics.

To solve such problems, you can often use *Monte Carlo* methods. A Monte Carlo method uses simulations based on random processes to model a real-life situation. The outcomes of these simulations are used to predict the outcomes of the real-life situation and to solve problems requiring knowledge of these outcomes. In this project you will estimate the area of an ellipse with a Monte Carlo method.

## INVESTIGATION

1. An ellipse with a horizontal major axis and center  $(0, 0)$  has an equation of the form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Write an inequality that must be satisfied if a point  $(x, y)$  is on or inside the ellipse (that is,  $(x, y)$  is part of the region bounded by the ellipse).
2. Write an equation for a particular ellipse with a horizontal major axis and center  $(0, 0)$ . Choose integer values of  $a$  and  $b$  so that the equation will be easy to work with. Graph your equation.
3. On the graph, draw a rectangle extending horizontally from  $-a$  to  $a$  and vertically from  $-b$  to  $b$  that will contain your ellipse. An example is shown below.
4. Use a random number generator on a calculator or computer to generate 20 points  $(x, y)$  where  $-a \leq x \leq a$  and  $-b \leq y \leq b$ . Check if each point satisfies your inequality from **Step 1**. Make a table to keep track of your results.



Point ( $x, y$ )	On or inside ellipse?
$(-1.2, 0)$	Yes
$(2.6, 1.9)$	No

5. Use your table to find the ratio of the number of points on or inside the ellipse to the number of points on or inside the rectangle. To estimate the area of the ellipse, multiply the area of the rectangle by this ratio.
6. Use the formula for the area of an ellipse to find the actual area. Compare this with the area you found in **Step 5**. (*Hint:* See page 611 for help with the formula for the area of an ellipse.)
7. Repeat **Steps 4 and 5** using simulations with 10, 50, and 100 randomly generated points. Make a conjecture about the relationship between the number of points generated and the accuracy of your estimate for the ellipse's area.

## PRESENT YOUR RESULTS

Write a report to present your results.

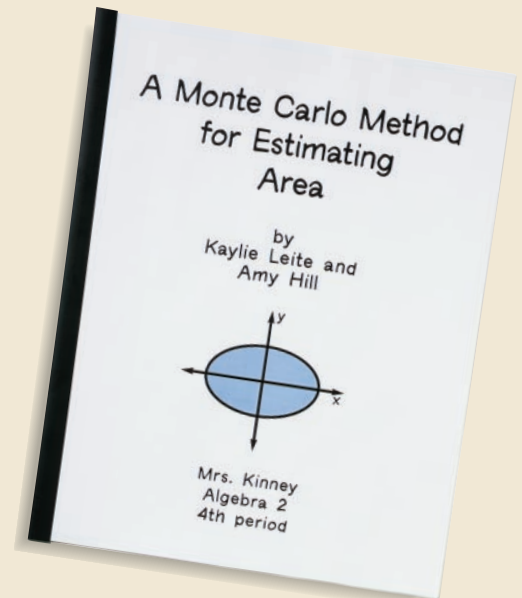
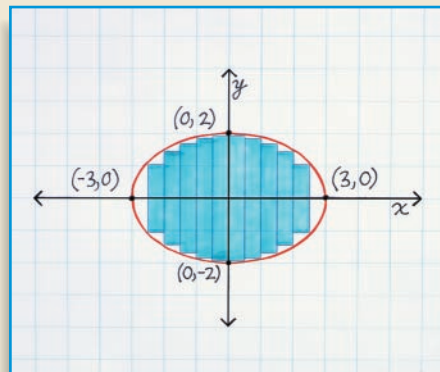
- Explain how you used a Monte Carlo method to estimate the area of an ellipse.
- Is using the Monte Carlo method the same as calculating the area of the ellipse directly? Explain any differences.
- Explain why you estimated the area of the ellipse by multiplying the area of the rectangle by the ratio of the points on or inside the ellipse to those on or inside the rectangle.
- Include your graph.
- Include your table.
- Let  $n$  be the number of points used to estimate the area of the ellipse, and let  $a_n$  be a sequence defined as follows:

$$a_n = |\text{estimated area} - \text{actual area}|$$

Use your results from the investigation to graph the sequence defined by  $a_n$  for  $n = 10, 25, 50$ , and  $100$ . Describe what your graph shows.

- Modify the Monte Carlo method so that you generate random points only in the first quadrant. Explain how you can use these points and the ellipse's symmetry to estimate the entire area of the ellipse.

Now estimate the area of your ellipse using this method:



- Copy the ellipse you drew. Sketch small rectangles to fill it, as shown.
- Find the area of each rectangle. Add these areas together to estimate the area of the ellipse.
- Compare the areas given by this method and the Monte Carlo method. How would you make this method more accurate?

## EXTENSION

Modify your Monte Carlo method to estimate the area of a region bounded (at least in part) by a different conic section. For example, you could estimate the area of a region bounded by one branch of a hyperbola and a line that intersects the branch in two points.