CHAPTER 12

Extension

Expected Value

What you should learn

GOAL Find expected values of collections of outcomes.

Why you should learn it

▼ To solve **real-life** problems, such as finding the expected value of insurance coverage in **Example 2**.

REAL LIFE

GOAL FINDING AN EXPECTED VALUE

Suppose you and a friend are playing a game. You flip a coin. If the coin lands heads up, then your friend scores 1 point and you lose 1 point. If the coin lands tails up, then you score 1 point and your friend loses 1 point. After playing the game many times, would you expect to have more, fewer, or the same number of points as when you started? The answer is that you should expect to end up with about the same number of points. You can expect to lose a point about half the time and win a point about half the time. Therefore, the *expected value* for this game is 0.

EXPECTED VALUE

A collection of outcomes is partitioned into n events, no two of which have any outcomes in common. The probabilities of the n events occurring are $p_1, p_2, p_3, \ldots, p_n$ where $p_1 + p_2 + p_3 + \cdots + p_n = 1$. The values of the n events are $x_1, x_2, x_3, \ldots, x_n$. The **expected value** V of the collection of outcomes is the sum of the products of the events' probabilities and their values.

$$V = p_1x_1 + p_2x_2 + p_3x_3 + \cdots + p_nx_n$$

EXAMPLE 1 *Finding the Expected Value of a Game*

You and a friend each flip a coin. If both coins land heads up, then your friend scores 3 points and you lose 3 points. If one or both of the coins land tails up, then you score 1 point and your friend loses 1 point. What is the expected value of the game from your point of view?

SOLUTION

When two coins are tossed, four outcomes are possible: HH, HT, TH, and TT. Let event A be HH and event B be HT, TH, and TT. Note that all possible outcomes are listed, but no outcome is listed twice. The probabilities of the events are:

$$P(A) = \frac{1}{4} \qquad\qquad P(B) = \frac{3}{4}$$

From your point of view the values of the events are:

value of event $A = -3$ value of event $B = 1$

Therefore, the expected value of the game is:

$$V = \frac{1}{4}(-3) + \frac{3}{4}(1) = 0$$

. . . .

If the expected value of the game is 0, as in Example 1, then the game is called a **fair game**.

REAL LIFE
Insurance

EXAMPLE 2 *Finding an Expected Value in Real Life*

In 1996 there were 124,600,000 cars in use in the United States. That year there were 13,300,000 automobile accidents. The average premium paid in 1996 for automobile collision insurance was $685 per car, and the average automobile collision claim paid by insurance companies was $2100 per car. What was the expected value of insurance coverage for a car with collision insurance in 1996?

SOLUTION

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Let event A be having an automobile accident and event B be not having an automobile accident. Note that events A and B are mutually exclusive and that $B = A'$, so all outcomes are accounted for but no outcome is counted twice. The probabilities of the events are:

$$P(A) = \frac{13{,}300{,}000}{124{,}600{,}000} \approx 0.107 \qquad P(B) = 1 - P(A) \approx 1 - 0.107 = 0.893$$

You can calculate the values of the events as follows. If an insured car had an accident, the owner paid an average of $685 and received an average of $2100. If a car did not have an accident, the owner only paid an average of $685. So, the values of the events for an insured person are:

$$\text{value of event } A = -685 + 2100 = 1415 \qquad \text{value of event } B = -685$$

Therefore, the expected value of insurance coverage was:

$$V \approx 0.107(1415) + 0.893(-685) \approx -\$460$$

EXERCISES

EXPECTED VALUE In Exercises 1 and 2, consider a game in which two people each choose an integer from 1 to 3. Find the expected value of the game for each player. Is the game fair? (*Hint:* There may be more than two events to consider.)

1. If the two numbers are equal, then no points are received. If the numbers differ by one, then the player with the higher number wins 1 point and the other player loses 1 point. If the numbers differ by two, then the player with the lower number wins 1 point and the other player loses 1 point.

2. If the sum of the two numbers is odd, then player A loses that sum of points and player B wins that sum. If the sum of the two numbers is even, then player B loses 4 points and player A wins 4 points.

3. **LOTTERY** To win a certain state's weekly lottery, you must match 5 different numbers chosen from the integers 1 to 49 plus an additional number chosen from the integers 1 to 42. You purchase a ticket for $1. If the jackpot for that week is $45,000,000, what is the expected value of your ticket?

4. **CONTESTS** A fast-food restaurant chain is having a contest with five prizes. No purchase is necessary to enter. What is the expected value of a contest ticket?

Prize	Value	Probability of winning
Gift certificate	$5	0.0002
Home theater system	$3,000	0.0000004
Hawaiian vacation	$7,000	0.00000008
Car	$50,000	0.000000003
Cash	$1,000,000	0.000000002